Appendix A

Electromagnetic wave equations

Using Euler formula [1], we can express as electric flux density D and magnetic flux density B in root mean squared electromagnetic wave form.

$$D = A(\mathrm{D})\frac{Exp\left(2\pi j\theta\right)}{\sqrt{2}}\left[\frac{As}{m^2}\right]\left(A-1a\right)$$

$$B = A(\mathrm{B})\frac{Exp\left(2\pi j\theta\right)}{\sqrt{2}}\left[\frac{Vs}{m^2}\right]\left(A-1b\right)$$

where,

A(D) and A(B) are amplitude in them, respectively

j is imaginary unit

π is the famous number pi per unit of radian

$\sqrt{2}$ means constant coefficient for root mean square

θ is phase angle [rad] below

$$\theta = ft - kz\left[rad\right]\left(A-2\right)$$

Where,

f is frequency [1/s]

k is wave number [1/m]

z: space or space interval on a z-axis for light to travel [m] in the rectangular Cartesian coordinate system

t: time or time interval on a time-axis [s], orthogonal to the Cartesian coordinate system.

Applying the phase angle to the exact differential equation with respect to variable space z and time t in the equation (A-2), we can get the derivative of the phase angle,

$$d\theta = 0\left(A-3a\right)$$

$$d\theta = \left(\frac{\partial\theta}{\partial t}\right)dt + \left(\frac{\partial\theta}{\partial z}\right)\partial z = 0\left(A-3b\right)$$

$$\frac{\partial\theta}{\partial t} = \left(-\frac{\partial z}{\partial t}\right)\frac{\partial\theta}{\partial z}\left(A-3c\right)$$

where,

$$\frac{\partial\theta}{\partial t} = f\left[\frac{1}{s}\right](A-3d)$$

$$\frac{\partial\theta}{\partial z} = -k\left[\frac{1}{m}\right](A-3e)$$

So that, we can get an equation

$$fdt = kdz[rad](A-3f)$$

$$\frac{dz}{dt} = \frac{f}{k}\left[\frac{m}{s}\right](A-3g)$$

Applying the speed of beam in equation (A-3g) to the exact differential equation under conditions that frequency and wavenumber constant, respectively, we can obtain constancy of speed c, frequency f, and wave number k in one spectrum of the beam with fully spectrum,

$$df = 0(A-3h)$$

$$dk = 0(A-3i)$$

So, under conditions that discrete frequency and wavenumber is constant in equations (A-3h) and (A-3i), according to applying the speed of beam to exact differential equation, we get constant of the speed of beam.

$$d\left(\frac{dz}{dt}\right) = \left(\frac{d\left(\frac{dz}{dt}\right)}{dt}\right)df + \left(\frac{d\left(\frac{dz}{dt}\right)}{dz}\right)dk = 0$$

using each equation (A3g) and (A3h),

$$d\left(\frac{dz}{dt}\right) = 0(A-4j)$$

Appendix B

Electric flux density D and magnetic flux density B Besides, applying exact differential equation [2] to one-dimensional Maxwell’s equation [3],

$$\frac{\partial D}{\partial z} = -\varepsilon \frac{\partial B}{\partial z} \left[\frac{As}{m^3} \right] (B-1a)$$

$$\frac{\partial B}{\partial z} = -\mu \frac{\partial D}{\partial z} \left[\frac{As}{m^3} \right] (B-1b)$$

Changing left hand and right hand in the above equations, respectively,

$$\frac{\partial B}{\partial t} = \frac{-1}{\varepsilon} \frac{\partial D}{\partial z} \left[\frac{V}{m^2} \right] (B-2a)$$

$$\frac{\partial D}{\partial t} = \frac{-1}{\mu} \frac{\partial B}{\partial z} \left[\frac{VA}{m^2} \right] (B-2a)$$

Spatial current density J is,

$$J = \frac{\partial D}{\partial t} = \frac{-1}{\mu} \frac{\partial B}{\partial z} \left[\frac{A}{m^2} \right] (B-3)$$

where,

J is defined as scalar spatial current density [A/Sq(m)] hereafter, replacing the so-called displacement current defined by Maxwell in reference to (P-2), a term of the displacement current in this paper does not use for vacuum space has no media to displace.

Both electric flux density D and magnetic flux density B, using equations (B-1a) and (B-1b), we can get wave forms below.

$$\frac{\partial \left(\frac{\partial D}{\partial z} \right)}{\partial z} = \varepsilon\mu \frac{\partial \left(\frac{\partial D}{\partial t} \right)}{\partial t} (B-4a)$$

$$\frac{\partial \left(\frac{\partial B}{\partial z} \right)}{\partial z} = \varepsilon\mu \frac{\partial \left(\frac{\partial B}{\partial t} \right)}{\partial t} (B-4b)$$

Besides, applying electric flux density D and magnetic flux density B to the exact differential equation,

$$dD = 0 \left(B-5a\right)$$

$$\frac{\partial D}{\partial t} = \left(-\frac{dz}{dt}\right)\left(\frac{\partial D}{\partial z}\right)\left(B-5b\right)$$

$$\frac{\partial\left(\frac{\partial D}{\partial t}\right)}{\partial z} = \frac{\partial\left(\frac{\partial D}{\partial z}\right)}{\partial t}\left(B-5c\right)$$

In addition, connecting left hand in equation (B-1a) and right hand in equation (B-5b),

so that,

$$\frac{\partial D}{\partial t} = \left(\frac{dz}{dt}\right)\left(\varepsilon\frac{\partial B}{\partial t}\right)\left(B-5d\right)$$

Under condition of constancy of $\frac{dz}{dt}$ and ε,

we can obtain an equation,

$$D = \left(\frac{dz}{dt}\right)\left(\varepsilon B\right)\left[\frac{As}{m^2}\right]\left(B-5e\right)$$

Moreover, the time derivative of $\frac{\partial D}{\partial t}$ in equation (B-5b), and using equation (B5c), we can get the derivative equation as wave function

$$\frac{\partial\left(\frac{\partial D}{\partial t}\right)}{\partial t} = \left(\frac{dz}{dt}\right)^2\frac{\partial\left(\frac{\partial D}{\partial z}\right)}{\partial z}\left(B-6\right)$$

On the other hand, applying the same process as the electric flux density D to the magnetic flux density B,

$$dB = 0 \left(B - 7a\right)$$

$$\frac{\partial B}{\partial t} = -\left(\frac{dz}{dt}\right)\left(\frac{dB}{dz}\right)\left(B - 7b\right)$$

$$\frac{\partial\left(\frac{\partial B}{\partial t}\right)}{\partial z} = \frac{\partial\left(\frac{\partial B}{\partial z}\right)}{\partial t}\left(B - 7c\right)$$

$$\frac{\partial B}{\partial t} = \left(\frac{dz}{dt}\right)\left(\mu\frac{dD}{dt}\right)\left(B - 7d\right)$$

$$B = \left(\frac{dz}{dt}\right)\left(\mu D\right)\left(B - 7e\right)$$

$$\frac{\partial\left(\frac{\partial B}{\partial t}\right)}{\partial t} = \left(\frac{dz}{dt}\right)^2 \frac{\partial\left(\frac{\partial B}{\partial z}\right)}{\partial z}\left(B - 8\right)$$

Next, each speed factor in both equations (B-6d) and (B-7d), is equal to the speed of the factor in equation (B-4a) and (B-4b) each other, we can obtain an equation.

$$\left(\frac{dz}{dt}\right)^2 = \frac{1}{\varepsilon\mu}\left(B - 9a\right)$$

The speed of beam is defined as below.

$$c = \frac{dz}{dt}\left[\frac{m}{s}\right]\left(B - 9b\right)$$

So that,

$$c^2 = \frac{1}{\varepsilon\mu}\left[\left(\frac{m}{s}\right)^2\right]\left(B - 9c\right)$$

Substituting those waveforms into wave equation (A-1a) and (A-1b), respectively, we can get the speed of beam.

$$f^2 = \left(\frac{dz}{dt}\right)^2 k^2 \left(B - 9d\right)$$

Under conditions that both f and k are positive, respectively,

$$f = ck\left[\frac{1}{s}\right]\left(B - 9e\right)$$

Furthermore, using the speed factor in equation (B-5e) and in equation (B-7e),

$$\frac{dz}{dt} = \frac{D}{\varepsilon B} = \frac{B}{\mu D}\left[\frac{m}{s}\right]\left(B - 9f\right)$$

Using equation (B8c), $Sq\left(dz / dt\right) = 1/\varepsilon\mu$, we can get a relationship between frequency, wave number, the speed of the beam, permittivity and permeability,

$$\left(\frac{f}{k}\right)^2 = \left(\frac{dz}{dt}\right)^2 = \frac{1}{\varepsilon\mu}\left[\left(\frac{m}{s}\right)^2\right]\left(B - 11\right)$$

Appendix C

Relationships between electromagnetic mass, momentum and energy We know that one-dimensional Poynting vector S is defined as a vector product of electric flux density D [As/m^2] and magnetic flux density B [Vs/m^2] So that,

$$S = Sk = D \times B = DBk\left[Ns / m^3\right] \ \left(C - 1\right)$$

Unit vector k is only on z-axis. Scalar expression of the Poynting vector is equal to one-dimensional scalar momentum density defined.

$$\rho\left(M\right) = DB = S\left[\frac{Ns}{m^3}\right]\left(C - 2\right)$$

Besides, given ρ(M) subject to the exact differential equation and one-dimensional Maxwell's equations,

we can get those equations:

From one-dimensional Maxwell's equations,

$$\frac{\partial D}{\partial z}=\left(-\varepsilon\right)\frac{\partial B}{\partial t}\left(C-3a\right)$$

$$\frac{\partial B}{\partial z}=\left(-\mu\right)\frac{\partial D}{\partial t}\left(C-3b\right)$$

Multiplying equation (C-3a) by B and (C-3b) by D, respectively,

$$B\frac{\partial D}{\partial z}=-0.5\frac{\partial \varepsilon B^2}{\partial t}\left(C-3c\right)$$

$$D\frac{\partial B}{\partial z}=-0.5\frac{\partial \mu D^2}{\partial t}\left(C-3d\right)$$

Adding the above-equations, respectively,

$$\frac{\partial S}{\partial z}=\frac{\partial DB}{\partial z}=D\frac{\partial B}{\partial z}+\frac{\partial D}{\partial z}B=\left(\frac{-1}{2}\right)\frac{\left(\partial\left(\varepsilon B\right)^2+\left(\mu D\right)^2\right)}{\partial t}\left(C-3e\right)$$

On the other hand, applying one-dimensional scalar volumetric momentum density to one-dimensional exact differential equation, using equation (A-4j),

$$d\left(dz/dt\right)=dc=0$$

So that,

$$\frac{\partial S}{\partial t}=\left(\frac{-dz}{dt}\right)\frac{\partial S}{\partial z}=\frac{1}{2}\frac{\partial\left(\left(\frac{-dz}{dt}\right)\varepsilon B^2+\mu D^2\right)}{\partial t}\left(C-4a\right)$$

For the right hand in the above-equation, making shift to the left hand,

$$\frac{\partial\left(S-0.5\left(\frac{dz}{dt}\right)\left(\varepsilon B^2+\mu D^2\right)\right)}{dt}=0\left(C-4b\right)$$

So that,

$$S=\left(\frac{1}{2}\right)\left(\frac{dz}{dt}\right)\left(\varepsilon B^{2}+\mu D^{2}\right)\left[\frac{Ns}{m^{3}}\right](C-5a)$$

Using equation (B-10), we can get a simple equation

$$S=\left(\frac{dz}{dt}\right)\varepsilon B^{2}=\left(\frac{dz}{dt}\right)\mu D^{2}\left[\frac{Ns}{m^{3}}\right](C-5b)$$

Next, in reference to equation (A-1a), (A-1b), (C-5b), using equation (A-2), we can get a waveform.

$$S=\rho(M)=0.5A(M)Exp(4\pi j\theta)\left[\frac{kg}{m^{3}}\right](C-5c)$$

In addition, in reference to the same equations, electromagnetic mass density is defined.

$$\rho(m)=0.5\left(\varepsilon B^{2}+\mu D^{2}\right)\left[\frac{kg}{m^{3}}\right](C-6a)$$

Using equation (B-10),

$$\rho(m)=\varepsilon B^{2}=\mu D^{2}\left[\frac{kg}{m^{3}}\right](C-6b)$$

$$\rho(m)=0.5A(m)Exp(4\pi j\theta)\left[\frac{kg}{m^{3}}\right](C-6c)$$

Besides, in reference to the same equations, electromagnetic energy density is defined.

$$\rho(E)=0.5\left(\frac{B^{2}}{\mu}+\frac{D^{2}}{\varepsilon}\right)\left[\frac{J}{m^{3}}\right](C-7a)$$

Using equation (B-10),

$$\rho(E)=\frac{B^{2}}{\mu}=\frac{D^{2}}{\varepsilon}\left[\frac{J}{m^{3}}\right](C-7b)$$

$$\rho(E) = 0.5A(E)\,Exp(4\pi j\theta)\left[\frac{J}{m^3}\right](C-7c)$$

Next, we can get electromagnetic equation relationships, mass, momentum and energy.

$$\rho(M) = \left(\frac{dz}{dt}\right)\rho(m)(C-8a)$$

$$\rho(M) = \left(\frac{dz}{dt}\right)\varepsilon\mu\rho(E)\left[\frac{Ns}{m^2}\right](C-8b)$$

$$\rho(m) = \varepsilon\mu\rho(E)\left[\frac{kg}{m^3}\right](C-9)$$

Using equation $\frac{dz}{dt} = c$ and $\varepsilon\mu = \frac{1}{c^2}$, we can get equations below.

$$\rho(E) = c^2\rho(m)\left[\frac{J}{m^3}\right](C-10)$$

$$\rho(M) = c\rho(m)\left[\frac{Ns}{m^3}\right](C-11)$$

Appendix D

A lifting force per unit of length and an intrinsic repulsive force in beam structure

(D.1) Lifting force

According to the statements in the introduction, the beam has a momentum, so that when two beams collide with each other, the momentum increases two times in case of perfect elasticity. In other words, when given two beams bisecting a beam radiated from the same source, we can assume a thought experiment of perfect elasticity collision due to two beams: a beam has the lateral side element of area per unit of length, and the other beam normal to the area collides with the lateral beam. Concretely, two beams radiated from the same source: A beam directly radiated from a source, and the other beam reflected on a surface. The lifting force density ρ (Flift) is generated due to the time derivative of momentum when the direct beam collides with a lateral side of the other beam.

$$\rho(Flift) = 2\frac{\partial\rho(M)}{\partial t}\left[\frac{N}{m^3}\right]\ (D-1a)$$

Using equation (C-8a) and constancy of the speed of beam, we can get an equation.

$$\rho\left(Flift\right)=2\frac{dz}{dt}\frac{\partial\rho\left(m\right)}{\partial t}\left[\frac{N}{m^3}\right]\left(D-1b\right)$$

(2) An intrinsic repulsive force

The beam has an intrinsic repulsive force confined in the self-medium, so the repulsive force is equivalent to a driving force for the beam to go forward in a vacuum space. Under a thought experiment of replacing the eddy current worked in a metal like an induction motor system with the spatial current in the self-medium, the intrinsic force is a force product of the spatial current density vector J and magnetic flux density vector B in the self-medium. Therefore, we can get an intrinsic repulsive force ρ(Frep) defined and described as

$$\rho\left(Frep\right)k=J\times B=JBk\left[\frac{N}{m^3}\right]\left(D-2a\right)$$

$$\rho\left(Frep\right)=JB\left[\frac{N}{m^3}\right]\left(D-2b\right)$$

where,

Spatial current density: $J=\frac{\partial D}{\partial t}$

k is unit vector on the z-axis.

So that, scalar electromagnetic force density ρ(Frep) is.

$$\rho\left(Frep\right)=\left(\frac{\partial D}{\partial t}\right)B\left[\frac{N}{m^3}\right]\left(3-a\right)$$

Using equations (B-3), (B-8c), and (C-6b), we can get an equation of the force volumetric density and flux density,

$$\rho\left(Frep\right)=-\left(\frac{\partial D}{\partial t}\right)B=0.5\left(\frac{dz}{dt}\right)\frac{\partial\rho\left(m\right)}{\partial t}\left[\frac{N}{m^3}\right]\left(D-3b\right)$$

Appendix E

Catenary equation with constant negative

Given the beam with a lateral side area A per unit of length, and the beam with volume V product of the area A and an elemental length equal to the speed of beam per unit of second which each beam with discrete frequency makes sure to assemble at a node over full spectrum, the relationship is,

$$V = AC\left[\frac{m^3}{s}\right](E-1)$$

Next, applying electromagnetic mass density ρ(m) to exact differential equation,
so that,

$$d\rho(m) = 0(E-2a)$$

$$d\rho(m) = \left(\frac{\partial\rho(m)}{\partial t}\right)dt + \left(\frac{\partial\rho(m)}{\partial z}\right)dz(E-2b)$$

$$\frac{\left(\frac{\partial\rho(m)}{\partial z}\right)}{\left(\frac{\partial\rho(m)}{\partial t}\right)} = \frac{-dz}{dt}(E-2c)$$

Furthermore, using equations (D-1b), (D-3b), and (E-2c), we can get a ratio β of the lifting force density ρ(Flift) divided by the repulsive force density ρ(Frep) is described.
Unit of the lifting force density ρ(Flift) is Newton per unit of area, and unit of the repulsive force is Newton per unit of volume per unit of second, so that unit of ratio β is per unit of length, though we can seem the ratio β as the unit of the reciprocal of the speed of beam squared, so the ratio β is.

$$\beta = \left(\frac{A}{V}\right)\left(\frac{\rho(Flift)}{\rho(Frep)}\right) = \frac{-4}{c^2}(E-3a)$$

where,
A [Sq(m)]: An area product of a lateral side cross section and 1 m for per unit of length in the lateral side, and denotes the cross-section collision between the beam reflected from the Mercury and the beam directly radiated from the Sun.

V[Cub(m)/s]: a volume product of the lateral side cross-section of the beam and c length per unit of the speed of beam for each electromagnetic wave has a node point at the c length, the unit is per unit of Cub(m) per unit of second.

(E-A) Ratio β is 4 divided by the speed of beam squared in equation (E-3a), so constant for the speed of beam is constant.
(E-B) Ratio β is the same value over fully-spectrum so that each mass density ρ(m) has a discrete frequency. The conventional catenary theory has positive constant for gravitational force [4], in reverse, using catenary theory with constant negative β, so a versine y in the catenary is described.

$$y = \left(\frac{1}{\beta}\right) coch\left(\beta X\right)\left(E-4\right)$$

where cosh is the hyperbolic cosine function.

The hyperbolic cosine function under condition of approximation equation changes a quadratic equation with respect to x variable, so the equation (E-4) is.

$$y = 0.5\beta x^2 \left(E-5\right)$$

Where, y is a versine in the quadratic equation.

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