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Symmetry-Based Dynamics-Independent Restrictions on Possible Measurements

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Abstract

The restrictions on possible measurements termed "superselection" are reexamined. My analysis is based on aspects of these restrictions that have not been taken into account in the literature, with essentially different results: the *scope* of superselection is shown to be narrower and the *nature* of the symmetry-based restrictions to be broader than generally claimed. The fundamental restriction is *unconditional*: *incompatibility* of a symmetry operation and measurability of the subset of Hermitian operators connecting states distinguished by essentially different values of the phase $e^{i\alpha}$ of a unimodular multiple of the identity operator generated by this symmetry; it is a purely theoretical restriction. Consequent are two mutually-exclusive *conditional* restrictions: (1) *exclusion* of Hermitian operators connecting states with essentially different values of $e^{i\alpha}$ from the subset of observables consistent with the symmetry operation, and (2) dynamics-independent *symmetry breaking* upon measurement of such operators; each has both theoretical and empirical contexts. The theoretical contexts of both conditional restrictions and the empirical context of exclusion apply without exception. The empirical context of dynamics-independent symmetry breaking has been realized selectively: observed in the case of Galilean invariance but, to date, not for rotational invariance. These two symmetries collectively exemplify all aspects of my analysis.

Keywords:** **Superselection; Measurement-generated symmetry breaking; Symmetry-generated restrictions

Abbreviations:** **UR: Uncertainty Relations; SR: Symmetry-Generated Restrictions

1. Introduction

Dynamics-independent restrictions on possible measurements have been an inherent element of quantum mechanics since its inception: the uncertainty relations do not depend on the dynamics of the measurement of complementary observables. Even earlier, the exclusion principle placed a dynamics-independent restriction on states that can be physically realized. This principle was eventually shown, within the framework of local quantum field theory, to be a consequence of special relativity and causality. About 25 years after the discovery of quantum mechanics, Wick, Wightman and Wigner (WWW) considered another dynamics-independent restriction on possible measurements – one based on symmetry considerations – that they termed "superselection" [1]. The current view of these restrictions is analyzed in Sect. 2.

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My view [2] of the physical content of these symmetry-based dynamics-independent restrictions on possible measurements differs significantly from those found in the literature, as detailed in Sect. 3. The origin of the differences traces back to the conclusion that Hermitian operators connecting specific subsets of states in the Hilbert space H of state vectors are excluded from the set of all observables in H. The rationale against the absolute nature of this conclusion – exclusion from all observables – is given in Sect. 3. The differences between my view and the prevailing one are due to aspects of these restrictions that underlie my analysis but have not been taken into account in the literature on superselection. These aspects are specified in Sect. 3.

The results are summarized in Sect. 4.

2. Superselection – A critical overview

WWW analyzed the restrictions on the nature and scope of possible measurements, consequent to general symmetry considerations, that are independent of the dynamical structure of the system. They considered the effect of a unimodular multiple of the identity operator I, $U = e^{i\alpha}I$, generated by a symmetry and acting on the states in a Hilbert space H of state vectors.

They showed that a specific symmetry can split H into distinct sets of states distinguished by essentially different values of the phase $e^{i\alpha}$ of the symmetry operator U. They termed such sets sectors; the subset of states within a given sector, a coherent subspace. I will hereinafter refer to sets characterized by essentially different values of $e^{i\alpha}$ as distinct sectors. WWW considered the case of spin states and showed that integral and half-odd-integral spin states belong to distinct sectors. Their original analysis [1] made use of invariance under double time reversal; their result was subsequently derived using invariance under rotation by 2π [3].

They observed that application of U to a pure state should reproduce it up to a phase, but that a coherent superposition of a spin-½ and a spin-0 state does not transform under U into a multiple of the original state. They concluded that such a coherent superposition cannot be physically realized, and that Hermitian operators connecting two such states must therefore be excluded from the set of all observables in H. They termed such exclusion from the set of observables of the subset of Hermitian operators connecting states in distinct sectors a superselection rule. Their analysis dealt with a system with a finite number of degrees of freedom.

Since then, analysis of superselection has been extended to quantum field theory. The term "superselection" has come to be applied in various – ostensibly equivalent – guises, including as a limitation on the applicability of the superposition principle, on the measurability of Hermitian operators connecting distinct sectors, and on the physical realization of Hilbert space vectors with components in distinct sectors. All variants share the absolutism inherent to exclusion from all observables in H. One also finds claims that breaking of the sector-generating symmetry invalidates the proof of the attendant superselection.

It is important to bear in mind that at issue is the scope and nature of symmetry-based dynamics-independent restrictions on possible measurements consequent to the division of H into distinct sectors, not the lexicology of "superselection". It is my contention that, in the context of quantum field theory, the generally held notion of the exclusion of a subset of Hermitian operators from all observables is flawed on four counts: logical, empirical, epistemological and conceptual. I also claim that the proof of superselection, consistently defined, is not voided by breaking of the sector-generating symmetry.

The *logical flaw* is the false conclusion that Hermitian operators connecting distinct sectors are excluded from the set of all observables in H. Logically, such operators are not excluded from the subset of observables connecting distinct sectors, only from the subset whose measurement is consistent with the operation of the sector-generating symmetry, i.e. whose only nonvanishing matrix elements lie within coherent subspaces.

The *empirical flaw* derives from symmetry operation being a contingent empirical fact; not only are Hermitian operators with finite matrix elements between distinct sectors logically measurable, they are known in at least one case to have been measured. Specifically, what is generally referred to as Bargmann's superselection rule on mass – the limitation imposed by Galilean invariance on the measurability of processes connecting states with different mass [4] – has contradicted the absolutist claim from the outset of Bargmann's proof of the theoretical limitation on the measurability of these operators, as Hermitian operators connecting states of different mass had already been measured in processes such as

$$\pi + N \rightarrow 2\pi + N' \qquad (1)$$

significantly before WWW opened the door to superselection. This process manifestly contradicts the contention that the restriction on allowed measurements generated by Galilean invariance excludes the Hermitian operators connecting states of different mass from the set of all observables. The absolutist view disregards reality in this case, as it leads to the oxymoron that Hermitian operators connecting states that have been labeled as being separated by a superselection rule, and are therefore excluded from the set of all observables in H, have in fact been measured.

The *epistemological flaw* is the notion that proof of the exclusion ceases to operate upon the observation of processes connecting states in distinct sectors. Thus, the apparent violation of T invariance has instigated re-derivation of the originally found restriction, but using invariance under rotation by 2π [3] rather than invariance under T^2, as originally invoked [1]. As detailed in Sect.3, the consistently defined exclusion, as well as its physical content, are operative irrespective of the possible violation of the sector-generating symmetry by the measurement of operators connecting states in distinct sectors.

Most consequential is the *conceptual flaw*. My view is that not only is the scope of the exclusion encompassed by the generally accepted notion of superselection too broad in excluding Hermitian operators connecting distinct sectors from the set of all observables, but that the nature of the restrictions consequent to the division of H into distinct sectors is undesirably too narrow on physical grounds. The full nature of these restrictions is shown in Sect. 3 to be significantly more comprehensive and richer in physical content than presently considered. Limiting them to their exclusion aspect is shown to leave out an entire class of dynamics-independent restrictions on possible measurements: the symmetry-breaking aspect of the measurement of Hermitian operators connecting distinct sectors.

3. Distinct aspects of restrictions on possible measurements

My analysis of the physical content of the symmetry-based dynamics-independent restrictions on possible measurements first considered by WWW is based on the confluence and physical contents of four distinct, albeit interrelated, concepts: (1) the *unconditional* consequence of the existence of sectors characterized by *relative* values of $e^{i\alpha}$ that cannot be removed by a permissible phase change; (2) the distinction between *unconditional* and *conditional* restrictions on possible measurements; (3) the distinction between the *theoretical* and *empirical* contexts of the two mutually-exclusive conditional restrictions; and (4) the distinction between *potential* and *actual* observables. To my knowledge, none of these has been folded into the mainstream analyses of superselection.

3.1 Sectors: Central to the symmetry-based dynamics-independent restrictions on possible measurements is the notion of operationally defined sectors: sets of state vectors in Hilbert space H characterized operationally by essentially different values of $e^{i\alpha}$. The concepts of sectors and superselection underlie the algebraic formulation of quantum field theory [5]; a consistent definition of superselection may be relevant to that role.

Distinct sectors generated by a given symmetry divide the set A of Hermitian operators A_i in H into two mutually-exclusive subsets: (i) the subset $A_1 = \{A_{1i}\}$ comprising all Hermitian operators whose only nonvanishing matrix elements connect states *within* coherent subspaces; and (ii) the complementary subset $A_2 = A - A_1$, comprising those Hermitian operators whose only nonvanishing matrix elements connect states in *distinct* sectors.

3.2 Unconditional and conditional restrictions on possible measurements: The dynamics-independent restrictions follow from consideration of the effect of the operation U, which should yield a physically equivalent state if the symmetry that generates U is operative. The elements of A_1 satisfy $UA_{1i}U^{-1} = A_{1i}$; those of A_2 satisfy $UA_{2i}U^{-1} \neq A_{2i}$, i.e. their measurement is inconsistent with the concurrent operation of the sector-generating symmetry.

Distinct sectors thus generate a dynamics-independent *unconditional* restriction on possible measurements: elements of A_2 cannot be realized concurrently with the operation of the sector-generating symmetry S. In my view, this *unconditional incompatibility* of the concurrent operation of S and measurability of the subset of Hermitian operators A_2 connecting distinct sectors constitutes the fundamental sector-dictated restriction, as it follows from general principles of quantum mechanics and symmetry considerations *alone*, without additional constraints.

Measurements that realize elements of the subset A_1 will be termed to be *compatible* with the operation of the sector-generating symmetry. As the operation of a symmetry is a contingent empirical fact, elements of the subset A_2 are *potential observables*. Their measurement converts potential observables into *actual observables*. As measurement of elements of A_2 is incompatible with the operation of the sector-generating symmetry, symmetry violation by their measurement will be referred to as *measurement-generated symmetry breaking*.

The unconditional restriction thus engenders two mutually-exclusive *conditional* restrictions that characterize two physically distinguishable types of physical phenomena. The condition that confines the set of observables to the subset A_1(Condition 1) results in the exclusion of the potential observables A_2 from the subset of observables compatible with the sector-generating symmetry S (superselection). The condition that confines the set of observables to the subset of potential observables A_2 (Condition 2) results in measurement-generated symmetry breaking of S upon measurement of processes connecting distinct sectors.

The dynamics-independent restriction on possible measurements attendant to measurement of elements of A_2 has not been considered in the literature analysis of the consequences of distinct sectors. Broadly, literature discussion of the scope and nature of the sector-dictated restrictions on possible measurements has been confined to the conditional restriction that entails the exclusion of elements of A_2 from the subset of observables compatible with the operation of the sector-generating symmetry, albeit without identifying the conditional nature of the exclusion and its applicability asserted to extend to *all* observables. This disregards the contingent nature of symmetry operation, the attendant potential for measuring operators connecting distinct sectors, and its resulting physical consequence: measurement-generated symmetry breaking.

The distinction between unconditional and conditional dynamics-independent restrictions on possible measurements is already inherent to the uncertainty relations, though I have not found this identified explicitly in the literature. The mutually exclusive aspect of the conditional restrictions on possible measurements is analogous to that in the uncertainty relations, but with a fundamental difference: the restrictions are quantitative for the uncertainty relations, qualitative for the symmetry generated ones. The analogy between the unconditional restrictions on possible measurements in the two types of restrictions is summarized in **TABLE 1**; that between the conditional restrictions, in **TABLE 2**.

TABLE 1**. Dynamics-independent unconditional restrictions on possible measurements.**

Fundamental restriction	**Physical foundation**
UR $\Delta x\ \Delta p \geq ½$; the two noncommuting observables cannot be measured concurrently with unlimited accuracy; in principle, either one can be measured with unlimited accuracy	General principles of quantum theory
SR the subset A_2 of operators connecting states in distinct sectors cannot be measured concurrently with the operation of the sector-generating symmetry S; in principle, either S is good or elements of A_2 are measurable	General principles of quantum theory imposed on symmetry-generated distinct sectors

UR: Uncertainty Relations; SR: Symmetry-Generated Restrictions

3.3 Theoretical and empirical contexts of conditional restrictions: The dynamics-independent *incompatibility* of the operation of the sector-generating symmetry and the measurability of Hermitian operators connecting distinct sectors is a logical consequence of general principles of quantum mechanics imposed on these sectors. This unconditional restriction on possible measurements is thus a purely *theoretical* limitation.

The two *conditional* restrictions consequent to Conditions 1 and 2 have both *theoretical* and *empirical* contexts. It is essential to distinguish between the two. Apart from a terse comment by Wightman [6] in the context of superselection, I am unaware of any reference to this duality. This context duality of the conditional restrictions is also inherent to the uncertainty relations.

The mutually-exclusive aspect of the two conditional restrictions obtains for both their empirical and theoretical contexts. The former are operationally distinguishable; the latter, logically distinct. Empirical realization of the symmetry breaking consequent to the measurement of Hermitian operators connecting distinct sectors converts potential into actual observables.

The *theoretical* context of both conditional restrictions applies without exception.

TABLE 2**. Mutually exclusive conditional restriction.**

Restriction	**Condition and conditional restriction**
Uncertainty relations (UR)	If $\Delta x = a$, then $\Delta p \geq 1/(2a)$; if $\Delta p = b$, then $\Delta x \geq 1/(2b)$; no restriction on magnitude of either a or b
Symmetry-generated restrictions (SR)	If the subset of observables is restricted to those compatible with the sector-generating symmetry, then Hermitian operators connecting distinct sectors are excluded from that subset. If the subset of observables is restricted to those that connect distinct sectors, then measurement of these observables must break the sector-generating symmetry.
Nature of complementary concepts	
UR *Quantitative*: accuracy of measurement of either of the complementary variables can be increased *continuously*, subject to a compensating decrease in accuracy with which the complementary variable can be measured concurrently.	SR *Qualitative*: either the symmetry is operative, which dictates exclusion (superselection), or a measurement connects distinct sectors, which dictates symmetry breaking.

The *empirical* context of the exclusion is realized without exception, as the condition that engenders it is the sector-generating symmetry. The empirical context of measurement-generated symmetry breaking differs essentially from that of exclusion, in that it operates selectively. The empirical contexts of symmetry-generated conditional restrictions also differ fundamentally on this count from that of the uncertainty relations, for which the empirical context is realizable without exception for both conditional restrictions.

The unconditional restriction is a purely theoretical result, independent of the realization of the empirical contexts of the two conditional restrictions, which are independent of each other, as they involve mutually-exclusive conditions and experimental phenomena. Thus, the physical content of the exclusion of the subset of observables connecting distinct sectors from the subset of observables compatible with the sector-generating symmetry remains operative even if a measurement breaks this symmetry.

The case of Galilean invariance exemplifies all these features of the restrictions.

Galilean invariance generates distinct sectors characterized by essentially different values of mass [4]. The set of Hermitian operators divides into two mutually-exclusive subsets: those whose only nonvanishing matrix elements connect states with the same mass, and those whose only nonvanishing matrix elements connect states with different mass. The fundamental dynamics-independent restriction is the *unconditional* incompatibility of the operation of Galilean invariance and the measurability of processes connecting states of different mass.

The *conditional* mutually-exclusive, operationally-distinguishable restrictions are (a) exclusion of processes connecting states of different mass from the subset of observables that can be realized by measurements compatible with Galilean invariance (Bargmann superselection), and (b) measurement-generated breaking of Galilean invariance in processes connecting states with different mass.

Empirical realization of the exclusion is ensured by the subset of observables that generates distinct sectors characterized by different mass.

Measurement-generated breaking of a sector-generating symmetry has been realized in the case of Galilean invariance by processes connecting states of different mass, e.g. in a process such as (1). This process converts a potential observable into an actual observable by realizing the empirical context of this conditional restriction. Such symmetry breaking has not been realized to date in the case of rotational invariance, e.g. in a potential process such as $\pi^+ \rightarrow 2e^+ + e^-$.

The fact that processes connecting states of different mass have been measured (condition) and that their measurement breaks Galilean invariance (conditional restriction) does not invalidate either the proof of the incompatibility of the operation of this symmetry and measurability of Hermitian operators connecting states of different mass (unconditional theoretical restriction) or the fact that the subset of processes connecting states with different mass is excluded from the subset of processes compatible with Galilean invariance (conditional restriction complementary to symmetry breaking). The processes (1) and $\pi + N \rightarrow \pi + N$ are elements of subsets that exemplify the mutually-exclusive aspects of symmetry-breaking and exclusion: breaking of Galilean invariance and excluding processes connecting states with different mass, respectively.

These features of the theoretical and empirical contexts of the conditional restrictions of rotational and Galilean invariance, and their present status, are summarized in **TABLE 3.**

TABLE 3**. Theoretical and empirical contexts of conditional restrictions.**

Restriction or symmetry	**Theoretical**	**Empirical**
Uncertainty relations	hold without exception	hold without exception
Symmetry-based restrictions	hold without exception	hold selectively
Galilean invariance	holds	measurement-generated breaking of Galilean invariance has been observed
Invariance under rotations	holds	measurement-generated breaking of rotational invariance not observed to date

4. Results and Discussion

The dynamics-independent restrictions on possible measurements first considered by Wick, Wightman and Wigner (WWW) for a system with a finite number of degrees of freedom [1] have been reexamined. The results differ fundamentally from those in the literature on superselection in quantum field theory on two counts. The *scope* of superselection is narrower than generally claimed, in that the subset of Hermitian operators connecting distinct sectors is excluded only from the subset of observables compatible with the sector-generating symmetry, not from all observables; and the *nature* of the symmetry-based restrictions is broader, in that there is another dynamics-independent restriction on possible measurements, complementary to the exclusion: dynamics-independent symmetry breaking. The differences between my view and the prevailing one are due to aspects of these restrictions that underlie my analysis but have not been taken into account in the literature.

The operation of selected symmetries generates distinct sectors, sets of states distinguished by essentially different values of the phase of operators that are phase multiples of the identity. States within a coherent sector have the same value of this phase.

Distinct sectors give rise to dynamics-independent restrictions on possible measurements.

In my view, the fundamental such restriction is the *unconditional* one: the incompatibility of the operation of the sector-generating symmetry and the measurability of operators connecting states in distinct sectors. This is a purely theoretical restriction that follows from general principles of quantum mechanics imposed on symmetry operation. The prevailing view does not consider the unconditional restriction.

The unconditional restriction engenders two mutually-exclusive *conditional* restrictions: one resulting in an exclusion aspect, and one resulting in a symmetry-breaking aspect. The prevailing view considers only the conditional restriction that results in exclusion, though without identifying its conditional nature.

Two elements enter into the analysis that leads to the exclusion: the unconditional restriction and the condition to confine observables to the subset comprising those compatible with the sector-generating symmetry, i.e. those within coherent sectors. The resulting restriction excludes the subset of Hermitian operators connecting states in distinct sectors from the designated subset of observables. The prevailing view excludes this subset of operators from *all* observables, which contradicts fact.

The complementary condition confines observables to the subset connecting distinct sectors. The resulting restriction engenders dynamics-independent breaking of the sector-generating symmetry as a necessary consequence of the measurement of operators connecting such sectors. The prevailing view does not consider this measurement-generated symmetry breaking.

In general, the conditional restrictions have both a theoretical and an empirical context. The theoretical context of both conditional restrictions holds without exception, as does the empirical context of the exclusion. The prevailing view does not distinguish between the theoretical and empirical contexts of the conditional restriction it considers.

To date, the empirical context of dynamics-independent symmetry-breaking measurements has been realized selectively; it is a contingent empirical fact for some symmetries. Specifically, dynamics-independent symmetry breaking has been observed in the case of Galilean invariance, but not in the case of rotational invariance. The Hermitian operators that connect integral and half-odd-integral spin states remain potential observables. The prevailing view does not consider the notion of potential observables.

The prevailing view regards the apparent breaking of T invariance to invalidate the original proof of univalent superselection. Both the purely theoretical nature of the unconditional restriction and the mutually-exclusive nature of the superselection and symmetry-breaking aspects of the conditional restrictions are at odds with this conclusion. The proof, based on this symmetry, of the exclusion of Hermitian operators connecting distinct sectors from the subset of observables compatible with the sector-generating symmetry is independent of its possible breaking.

5. Summary and Conclusion

Collectively, Galilean and rotational symmetries exemplify all salient aspects of my analysis: the distinctions between unconditional and conditional symmetry-based dynamics-independent restrictions, between the two conditional restrictions, between the theoretical and empirical contexts of the conditional restrictions, and between potential and actual observables; the mutually-exclusive nature of the two conditional restrictions; the symmetry-breaking aspect of the restriction complementary to the exclusion; the exceptionless nature of the unconditional restriction, of the theoretical contexts of the conditional restrictions, and of the empirical context of the exclusion; and the selective nature of the empirical context of the symmetry-breaking restriction: realized for Galilean invariance, but not realized to date for rotational invariance.

It is my contention that the concepts discussed in this paper bear upon the interpretation of a theory that accounts for a presently unaccounted-for empirical aspect of particle physics: the equality of the domains of validity of associated production and parity conservation observed in strong hadron processes. The theory that accounts for this equality will be developed in a future paper.

REFERENCES

1. Wick G, Wightman AS, and Wigner E. The intrinsic parity of elementary particles. Phys. Rev. 1952; 88: 101-105.
2. Spitzer R. Some comments on superselection rules. Nucl Phys. 1967; 87:823-833.
3. Hegerfeldt GC, Kraus K , Wigner EP. Proof of the Fermion Superselection rule without the Assumption of Time-Reversal Invariance. J Math Phys. 1968;9:2029-2031.
4. Bargmann V. On unitary ray representations of continuous groups. Ann of Math. 1954;59:1-46.
5. Haag R, Kastler D. An algebraic approach to quantum field theory. J Math Phys. 1964;5: 848-861.
6. Wightman A. Some comments on the quantum theory of measurement. Probabilistic Methods in Mathematical Physics. 1991; 411-438. Eds. Guerra F., Loffredo M.I., Marchioro C., World Scientific.