

Space-Time curvature values using the Schwarzschild Model.

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Abstract

In our last report published in this Journal and entitled "Calculation of the space-time curvature in the vicinity of Schwarzschild black holes. Application to Sagittarius A*" we had developed a method for calculating space-time curvatures based on the Schwarzschild model. and we had studied the results of curvatures at distances between 1 and 7 Schwarzschild radii. Now we extend our study calculating curvature values in a wider range of distances, between 1 and 1400 Schwarzschild radii, and we establish an equation by regression methods to calculate curvatures, at distances, from the center of black hole, between 1 and 1400 Schwarzschild radii. Our results allow us to calculate absolute values of curvature where the Schwarzschild model is valid.

Keywords: Black holes; Curvature of space-time; Cosmology; Generalized relativity

Introduction

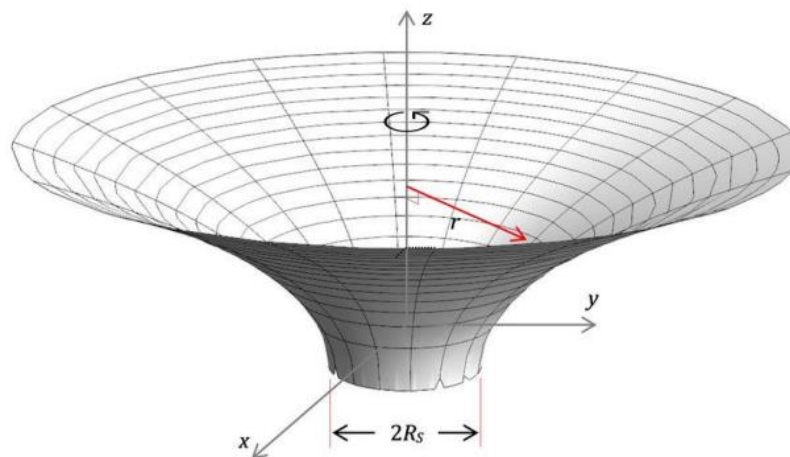


FIG 1. Space-time in the Schwarzschild model.

Schwarzschild solves Einstein's generalized relativity equations for an assumption of a gravitational mass and an empty space surrounding it [1]. The result of the determination of the space-time equations is a 2D surface, which is known as the Flamm paraboloid, and which is represented in **FIG. 1**. Since it is a 2D surface, it can be applied to the case of calculating curvatures, differential geometry techniques of surfaces and so we have done [2]. The Gaussian curvature and the mean curvature have been

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calculated at 20 points between 1 and 1400 Schwarzschild radii and an adjustment equation has been obtained using an Excel program by regression methods throughout this wide range of distances. The degree of quality of the fit obtained by calculating the parameter R^2 is very high, 0.9999. For this reason, it is to be expected that this equation allows the calculation of Gaussian curvature values, in this wide range of distances, with high accuracy without the need to carry out the laborious calculations that would otherwise have to be done. These results are detailed below in **TABLE 1**.

Results of Curvature Values

According to the exposed we have calculated curvature values at 20 points located at a distance between 1 and 1400 Schwarzschild radii (R_s) [3].

TABLE 1. Curvature values according to the Schwarzschild model.

Distance to the center of black hole	Value of Gauss Curvature k	Value of Mean Curvature H
1 R_s	$-0.5000 \times R_s^{-2}$	$-0.2500 \times R_s^{-1}$
1,2 R_s	$-0.2873 \times R_s^{-2}$	$-0.1888 \times R_s^{-1}$
1,4 R_s	$-0.1821 \times R_s^{-2}$	$-0.1509 \times R_s^{-1}$
1,6 R_s	$-0.1220 \times R_s^{-2}$	$-0.1236 \times R_s^{-1}$
1,8 R_s	$-0.0790 \times R_s^{-2}$	$-0.1119 \times R_s^{-1}$
2 R_s	$-0.0625 \times R_s^{-2}$	$-0.0543 \times R_s^{-1}$
3 R_s	$-0.0186 \times R_s^{-2}$	$-0.0485 \times R_s^{-1}$
4 R_s	$-0.0078 \times R_s^{-2}$	$-0.0312 \times R_s^{-1}$
5 R_s	$-0.0030 \times R_s^{-2}$	$-0.0292 \times R_s^{-1}$
6 R_s	$-0.0023 \times R_s^{-2}$	$-0.0168 \times R_s^{-1}$
60 R_s	$2,325.10^{-6} \times R_s^{-2}$	$-5,342.10^{-4} \times R_s^{-1}$
80 R_s	$-9,596.10^{-7} \times R_s^{-2}$	$-3,488.10^{-4} \times R_s^{-1}$
100 R_s	$-4,925.10^{-7} \times R_s^{-2}$	$-2,500.10^{-4} \times R_s^{-1}$
200 R_s	$-5,963.10^{-8} \times R_s^{-2}$	$-9,170.10^{-5} \times R_s^{-1}$

An Equation to Calculate the Curvature of Space-Time according to the Schwarzschild Model

We are going to study the data of the Gaussian curvature by means of an equation obtained by fitting.

We have used an Excel program to determine a regression equation that turns out to be a potential function and we have used the 20 data obtained in the adjustment. The result is as follows:

Fit equation between 1 and 1400 Schwarzschild radii

Gaussian curvature: $k=0.5268 (r/R_s)^{-3.054} \times R_s^{-2}$

Fit quality: $R^2=0.9999$

Rounding decimals and considering that:

$$R_s=2GM/c^2$$

we can express the above equation as:

$$k=GM/c^2r^3$$

where k is the Gaussian curvature of space-time.

Conclusion

Using the Schwarzschild model, we have calculated values of the curvature of space-time at different points of it. These points cover a distance to the point mass between 1 and 1400 Schwarzschild radii. The calculation of the curvature at each point is laborious; therefore, an equation has been attached that allows calculating, in a more comfortable way, the curvature values in that range of distances. The equation is expected to have a high degree of accuracy according to the characteristics of the adjustment made. Our calculations allow us to obtain absolute values of curvature where the Schwarzschild model is valid.

REFERENCES

1. Schwarzschild K. On the Gravitational Field of a Point Mass according to the Einsteinian Theory. In Source Book Astron. Astrophys., Harvard University Press. 1900-1975. 1979:451-55.
2. Differential geometry of surfaces
3. Iza F.S. Calculation of the space-time curvature in the vicinity of Schwarzschild black holes. Application to Sagittarius A*. J. Phys. Astron. 2023;11(6):350