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An Inverse Problem for a Single Orbit

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Introduction

The inverse problem of Dynamics for monoparametric families of orbits F(x,y)= c, produced in a potential field, has been studied in the past for various versions (e.g. for one or two or three parametric families, for two or three dimensional potentials in Cartesian or curvilinear coordinates, in inertial or rotating frames etc) [1].

In this report we refer to the planar motion of a unit mass material point P in an inertial frame Oxy, in a two-dimensional potential V= V(x,y), tracing one orbit (or a branch of the whole orbit) given by the equation y= f(x). We answer two questions: (a): The two functions V=V(x,y) and y=f(x) are given arbitrarily. Are they consistent? (b): An equation y= f(x) is given arbitrarily. For which potentials this equation can stand for an orbit?

Notation

We denote by $V_f = V_f(x) =$ V(x,y=f(x)). (It stands to reason to call it orbital potential). Similarly e.g. $V_{xf} = V_x(x, y = f(x))$ and $V_{yf} = V_y(x, y = f(x))$. For an arbitrary function A(x,y) we have: A_f = A(x,y=f(x)). We consider A(x,y) as adequate if its orbital value is not infinity, i.e. if $A_f \neq \pm\infty$. The total energy of the point P is E_f.

Two Theorems-Examples

<u>Theorem A</u>

An observed branch of an orbit y= f(x) (with f'(x) $\neq$ 0) can be produced by a given potential V= V(x,y) if and only if

$$V_f + \frac{(1+f'^2)}{2f''}(f'V_{xf} - V_{yf}) = \text{constant} = E_f \quad (1)$$

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As a hint for the proof, we refer to Szebehely's formula for monoparametric families (as modified by Bozis (1995)) with an appropriate interpretation of *the slope function* $\gamma(x,y)$ *of the family* and the help of both *the orbital functions* $V_f(x)$ and the orbital function $\gamma_f(x) = \left[\frac{F_y}{F_x}\right]_{y=f(x)} = -\frac{1}{f'}$, introduced here.

Example: Let us verify that the upper branch of the Newtonian ellipse $r = \frac{3}{2(2+\cos\theta)}$ i.e., $y = f(x) = \frac{1}{4}\sqrt{3(-4x^2-4x+3)}$ with $-\frac{3}{2} \le x \le \frac{1}{2}$ and Newton's potential $V(x,y) = -\frac{1}{\sqrt{x^2+y^2}}$ are compatible. Indeed, with e.g. $V_f = \frac{4}{(2x-3)}$,$V_{xf} = \frac{64x}{(3-2x)^3}$ etc., the left hand side of equation (1) gives $E_f = -\frac{1}{2}$, as expected.

Theorem B

All potentials V(x,y) which (for adequate initial conditions, consistent with E_f) can produce an *observed branch* of orbit y= f(x) are given by the formula:

$$V(x,y) = E_f + (1+\frac{1}{f'^2})L(x) + (y-f)A(x,y) \quad (2)$$

where $L(x) = \int f'A_f dx$. The function A(x,y) is an *adequate arbitrary* function and (with the constant of the indefinite integral L(x) taken equal to zero) E_f is the energy of P.

To prove the theorem B, we only need verify that, indeed, the orbit y= f(x) and the potential (2) are consistent, i.e. that they satisfy the equation (1).

Example: For $f = x^3, A = xy, E_f = 1$ we have $A_f = x^4, L = \frac{3x^7}{7}$. So, formula (2) gives *one out of the infinitely many* potentials which can produce the orbit $y = x^3$ with $E_f = 1$. For $-\infty \prec x \le 0$ (so that $E_f \ge V_f$) this potential is

$$V(x,y) = xy^2 - x^4 y + \frac{3x^7}{7} + \frac{x^3}{21} + 1$$

Comments

i. The function y=f(x) is real if $E_f \ge V_f$ and is defined in an interval of the x-axis which can be found from the inequality $L(x) \le 0$.

ii. If the observed (given) orbit is a straight line $f(x) = \lambda x + \mu$ (in which case f'(x)=0), the above formulae (1) and (2) are not applicable. Then, an affirmative answer to the question **1(a)** is given if $V_{yf} = \lambda V_{xf}$. As to the formula for the question **(1b)** (giving the totality of potentials producing the straight line $f(x) = \lambda x + \mu$) now it reads:

$$V(x,y) = a(x+\lambda y) + (y - \lambda x - \mu)(b_y - \lambda b_x) - (1+\lambda^2) b(x,y) \quad (3)$$

where $a(x+\lambda y), b(x,y)$ are arbitrary functions of their respective arguments.

It is directly shown that all the potentials (3) satisfy the relation $V_{yf} = \lambda V_{xf}$.

REFERENCES

1) Bozis G. The inverse problem of dynamics: basic facts. Inverse problems. 1995;11:687-708.