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Variable structure guaranteed cost control for a class of uncertain networked systems with state delay and communication delay

Hejun Yao[1]*, Xiaoming Dai[2]

[1]School of Mathematics and Statistics, Anyang Normal University, 455000, Anyang, Henan, (CHINA)

[2]School of Mathematics and Physics, Anyang Institute of Technology, 455000, Anyang, Henan, (CHINA)

ABSTRACT

The problem of variable structure guaranteed cost control for a class of uncertain networked systems with state delay and communication delay has been considered in this paper. Based on LMI approach, a new sliding surface that can compensate delay has been designed. Then a variable structure guaranteed cost control controller is designed to make the system state stable. Finally, a numerical example is given to demonstrate the application and effectiveness of the proposed method.

KEYWORDS

Networked control systems;
Delay;
Variable structure control;
Guaranteed cost control.

INTRODUCTION

Feed back control systems in which the control loops are closed through a real time network are called networked control systems (NCSs)[1,2].The defining feature of the NCS is that information is exchanged using a network. Many attractive advantages (for example high system testability and resource utilization, as well as low requirement to weigh, space, power and wiring)of introducing a communication network into a control systems motivate the research on NCSs. NCSs is now widely used in process control, remote control, telemanipulation, robots etc.

Recently a great number of studies reports on analysis and modeling of NCSs have been conduced using continuous and discrete models[3-6]. By choosing a new Lyapunov functional, a new robust H_∞ stabilization criterion for networked control systems with time-varying network-induced delay and data packet dropout is given in[7].Based on the LMI approach, a sufficient condition is obtained using the information of both the lower and upper bounds of the time-varying network-induced delay. Based on Lyapunov stability theory combined with LMI techniques, Yan gives a new delay dependent stability criteria for the system in terms of LMI approach[8]. And they analyze the delay-dependent asymptotic stability and obtain maximum allowable delay bound of NCSs with uncertainties and multiple time-delays. Based on a new time-delay model proposed recently which contains multiple successive delay components in the state[9]. By constructing a sliding manifold that can compensates delay, Fang obtain a novel method for the stabilization of a class of networked control system with communication[10].Then a variable structure control is designed by LMI approach.

The purpose of this paper has been to design a guaranteed variable structure controller to make the uncertain networked control systems with state delay and network-induced delay stable. Based on the Lyapunov stability theorem, a sliding manifold that can compensate delay has been designed by using LMI approach. Finally, a numerical example is given to demonstrate the effectiveness of the proposed method.

PROBLEM FORMULATION

In this paper, a class of real MIMO NCSs with many independent sensors and actuators are considered. Throughout this paper, suppose that all the system's states are available for a state feedback control. The sensor is clock-driven; the controller and actuator are event-driven. In the presence of the control network, data transfers between the controller and the remote systems. Therefore there exist the communication delay between the sensor and the controller and computational delay in the controller. Now we consider the following nominal MIMO networked systems with constant state time delay and networked induced time delay[8]:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-d) + Bu(t-\tau) \\ x(t) &= \phi(t) \qquad -h \le t \le 0 \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ the control input, d, τ constants representing the state delay and the network-induced delay of the system, $h = \max\{d, \tau\}$. $\phi(t)$ is real-valued initial function on $[-h,\ 0]$. A, A_d and B are known real constant matrices with appropriate dimensions.

In practice the NCSs are always inevitably present some uncertainties because it is very difficult to obtain an exact mathematical mode due to environmental noise, all kinds of uncertainties, etc. Therefore, without loss of generality, we consider the following closed-loops MIMO NCSs with structure uncertainties state time delay and network-induced delay:

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A(t))x(t) + (A_d + \Delta A_d(t))x(t-d) \\ &\quad + Bu(t-\tau) \\ x(t) &= \phi(t) \qquad -h \le t \le 0 \end{aligned} \tag{2}$$

where $\varDelta A(t)$ and $\varDelta A_d(t)$ unknown matrices representing time-varying parameter uncertainties assumed to be of the form

$$[\varDelta A(t),\ \varDelta A_d(t)] = DF(t)[E,\ E_d]$$

where D, E and E_d are known real constant matrices of appropriate dimensions. $F(t)$ is unknown time-varying matrix function satisfying

$$F^T(t)F(t) \le I$$

The systems (2) can be rewrite as

$$\dot{x}(t) = Ax(t) + A_d x(t-d) + Bu(t-\tau) + f(t) \tag{3}$$

where

$$\begin{aligned} f(t) &= \Delta A(t)x(t) + \Delta A_d(t)x(t-d) \\ &= DF(t)Ex(t) + DF(t)E_d x(t-d) \end{aligned}$$

and

$$\| f(t) \| \le \| DE \| \| x(t) \| + \| DE_d \| \| x(t-d) \|$$

MAIN RESULTS

The design of guaranteed cost variable structure controller

Now choosing the sliding mode surface such as

$$s(t) = Cx(t) + \int_{t-\tau}^{t} CBu(s)ds + \Gamma \tag{4}$$

where C is proper dimension constant matrix, satisfying that CB is nonsingular, Γ is sliding mode compensator that satisfies

$$\dot{\Gamma} = -C(A - BK)x(t) - CA_d x(t-d)$$

where K is a undetermined constant matrix.

With the nominal system ofÿ3 ÿÿwe have

$$\begin{aligned} \dot{s}(t) &= C\dot{x}(t) + CBu(t) - CBu(t-\tau) + \dot{\Gamma} \\ &= CAx(t) + CA_d x(t-d) + CBu(t-\tau) \\ &\quad + CBu(t) - CBu(t-\tau) - CAx(t) \\ &\quad + CBKx(t) - CA_d x(t-d) \\ &= CBu(t) + CBKx(t) \end{aligned}$$

If $\dot{s}(t) = 0$ ÿwe can obtain the equivalent controller

$$u_{eq}(t) = -Kx(t) \tag{5}$$

Theorem1

For the uncertain network control systems with delay (3), the state will hit the sliding mode surface (4) in finite time if we choose the following controller

$$u(t) = u_{eq}(t) + u_N(t) \tag{6}$$

where

$$u_N(t) = -(CB)^{-1}(Ks(t) + (\| DE \| \| x(t) \| \| C \| + \| DE_d \| \| x(t-d) \| \| C \| + \varepsilon)\operatorname{sgn}(s(t)))$$

Proof Selecting the Lyapunov functional such as

$$V_1(t) = \frac{1}{2}s^T(t)s(t)$$

With the controller (6) and Eqs.(3-4), we obtain that

$$\begin{aligned}
\dot{V}_1(t) &= s^T(t)\dot{s}(t) \\
&= s^T(t)[CAx(t) + CA_d x(t-d) + CBu(t-\tau) \\
&\quad + Cf(t) + CBu(t) - CBu(t-\tau) + \dot{\Gamma}] \\
&= s^T(t)[CAx(t) + CA_d x(t-d) + CBu(t-\tau) \\
&\quad + Cf(t) + CBu(t) - CBu(t-\tau) - CAx(t) \\
&\quad + CBKx(t) - CA_d x(t-d)] \\
&= s^T(t)[Cf(t) + CBu_{eq}(t) + CBu_N(t) + CBKx(t)] \\
&= s^T(t)[Cf(t) - Ks(t) - (\| DE \| \| x(t) \| \| C \| \\
&\quad + \| DE_d \| \| x(t-d) \| \| C \| + \varepsilon)\operatorname{sgn}(s(t))] \\
&\le -K \| s(t) \|^2 - \varepsilon \| s(t) \| \\
&< 0
\end{aligned}$$

From the above inequality, we know that the hitting condition is satisfied.

The stability of the sliding mode

Lemma 1[8]

The LMI

$$\begin{bmatrix} Y(x) & W(x) \\ * & R(x) \end{bmatrix} > 0$$

is equivalent to

$R(x) > 0$ ÿ $Y(x) - W(x)R^{-1}(x)W^T(x) > 0$ ÿ

Where $Y(x) = Y^T(x)$ ÿ $R(x) = R^T(x)$ and $W(x)$ depend on x.

Lemma 2[4]

For given scalar $\varepsilon > 0$ and matrices D, E, F with $F^T F \le I$ ÿthe inequality

$$DEF + E^T F^T D^T \le \varepsilon DD^T + \varepsilon^{-1} E^T E$$

is always satisfied.

Substituting Eq.(5) into (2), the sliding-mode equation is obtained

$$\dot{x}(t) = \bar{A}x(t) + \bar{A}_d x(t-d) + \bar{B}x(t-\tau) \tag{7}$$

where

$$\bar{A} = A + DF(t)E$$
$$\bar{A}_d = A_d + DF(t)E_d$$
$$\bar{B} = -BK$$

Selecting the guaranteed cost functional

$$J = \int_0^\infty x^T(t)Qx(t)dt \tag{8}$$

where $Q \in R^{n \times n}$ is a positive-definite matrix undetermined.

Theorem2

The sliding-mode equation (7) is stable if there exists $m \times n$ matrix M ÿ $n \times n$ positive-definite matrix $X, \bar{R}, \bar{T}, \bar{Q}$ and constants $\varepsilon_1 > 0, \varepsilon_2 > 0$ such that the following linear matrix inequality holds

$$\begin{bmatrix} \Sigma & A_d X & -BM & EX & 0 \\ * & -\bar{R} & 0 & 0 & E_d X \\ * & * & -\bar{T} & 0 & 0 \\ * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & -\varepsilon_2 I \end{bmatrix} < 0 \tag{9}$$

where

$$\Sigma = AX + XA^T + \bar{R} + \bar{T} + \bar{Q} + \varepsilon_1 DD^T + \varepsilon_2 DD^T$$

a stable sliding-mode surface is given by

$$s(t) = Cx(t) + \int_{t-\tau}^{t} CBu(s)ds + \Gamma$$

And with the controller ÿthe sliding mode equation(7) is stable, the guaranteed cost functional satisfying $J \le J^*$, where

$$J^* = x^T(0)Px(0) + \int_{-d}^{0} x^T(s)Rx(s) + \int_{-\tau}^{0} x^T(s)Tx(s)ds$$

Proof Choose a Lyapunov functional candidate for the sliding mode equation (7) as follows:

$$V(t) = x^T(t)Px(t) + \int_{t-d}^{t} x^T(s)Rx(s)ds + \int_{t-\tau}^{t} x^T(s)Tx(s)ds$$

Where are positive-definite matrices of systems (7) is given by

$$\dot{V}(t) = 2x^T(t)P[\bar{A}x(t) + \bar{A}_d x(t-d) + \bar{B}x(t-\tau)] + x^T(t)Rx(t) - x^T(t-d)Rx(t-d) + x^T(t)Tx(t) - x^T(t-\tau)Tx(t-\tau)$$
$$= 2x^T(t)P\bar{A}x(t) + 2x^T(t)P\bar{A}_d x(t-d) + 2x^T(t)P\bar{B}x(t-\tau) + x^T(t)Rx(t) - x^T(t-d)Rx(t-d) + x^T(t)Tx(t) - x^T(t-\tau)Tx(t-\tau)$$
$$= 2x^T(t)P(A+DF(t)E)x(t) + 2x^T(t)P(A_d+DF(t)E_d)x(t-d) - 2x^T(t)PBKx(t-\tau) + x^T(t)Rx(t) - x^T(t-d)Rx(t-d) + x^T(t)Tx(t) - x^T(t-\tau)Tx(t-\tau) + x^T(t)Qx(t) - x^T(t)Qx(t)$$
$$= x^T(t)(PA + A^TP + R + T + Q)x(t) + x^T(t)PA_d x(t-d) + x^T(t-d)A_d^T Px(t) - 2x^T(t)PBKx(t-\tau) - x^T(t-d)Rx(t-d) - x^T(t-\tau)Tx(t-\tau) + 2x^T(t)PDF(t)Ex(t) + 2x^T(t)PDF(t)E_d x(t-d) - x^T(t)Qx(t)$$

With the lemma2, we can obtain

$$\dot{V}(t) \le x^T(t)(PA + A^TP + R + T + Q)x(t) + x^T(t)PA_d x(t-d) + x^T(t-d)A_d^T Px(t) - 2x^T(t)PBKx(t-\tau) - x^T(t-d)Rx(t-d) - x^T(t-\tau)Tx(t-\tau) + x^T(t)\varepsilon_1 PDD^TPx(t) + \frac{1}{\varepsilon_1}x^T(t)E^TEx(t) + x^T(t)\varepsilon_2 PDD^TPx(t) + \frac{1}{\varepsilon_2}x^T(t-d)E_d^TE_d x(t-d) - x^T(t)Qx(t)$$
$$\le \xi^T\Theta\xi - x^T(t)Qx(t)$$

where

$$\xi = [x(t) \quad x(t-d) \quad x(t-\tau)]^T$$

$$\Theta = \begin{bmatrix} \psi_1 & PA_d & -PBK \\ * & -R + \frac{1}{\varepsilon_2}E_d^TE_d & 0 \\ * & * & -T \end{bmatrix} < 0 \qquad (10)$$

$$\psi_1 = PA + A^TP + R + T + Q + \varepsilon_1 PDD^TP + \frac{1}{\varepsilon_1}E^TE + \varepsilon_2 PDD^TP$$

Pre- and Post-multiplying the inequality (10) by $\{P^{-1}, P^{-1}, P^{-1}\}$ using some changes of variables $X=P^{-1}$ $M=KX$, we know that the inequality(10)is equivalent to

$$\begin{bmatrix} \psi_2 & A_d X & -BM \\ * & -XRX + \frac{1}{\varepsilon_2}XE_d^TE_d X & 0 \\ * & * & -XTX \end{bmatrix} < 0 \qquad (11)$$

where

$$\psi_2 = AX + XA^T + XRX + XTX + XQX + \varepsilon_1 DD^T + \frac{1}{\varepsilon_1}XE^TEX + \varepsilon_2 DD^T$$

By Lemma1 and changes of variables $R=XRX$, $\bar{T} = XTX$, $\bar{Q} = XQX$ the inequality(11) is equivalent to inequality(9).

From the inequality (10) or (9), we can obtain that $\dot{V} < 0$ then the sliding-mode equation (7) is stable.

With the inequality (10), we have

$$\dot{V}(t) \le \xi^T\Theta\xi - x^T(t)Qx(t) \le -x^T(t)Qx(t)$$

therefore

$$\int_0^t \dot{V}(t)dt \le -\int_0^t x^T(t)Qx(t)dt$$

If $t \to \infty$, then $x(t) \to \infty$ and $V(\infty) = \infty$

The above inequality is

$$V(\infty) - V(0) \le -\int_0^\infty x^T(t)Qx(t)dt$$

i.e.

$$\int_0^\infty x^T(t)Qx(t)dt \le V(0) = J^*$$

where

$$J^* = x^T(0)Px(0) + \int_{-d}^0 x^T(s)Rx(s) + \int_{-\tau}^0 x^T(s)Tx(s)ds$$

NUMERICAL EXAMPLE

Consider the network control systems in the form of (3), where

$$A=\begin{bmatrix}-6 & 0\\ 0 & -3.8\end{bmatrix}, B=\begin{bmatrix}1\\1\end{bmatrix}, A_d=\begin{bmatrix}-0.5 & 0\\ 0 & -0.3\end{bmatrix},$$

$$E=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix}, E_d=\begin{bmatrix}-0.2 & 0\\ 0 & 0.1\end{bmatrix}, D=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},$$

$$F(t)=\begin{bmatrix}\sin t & 0\\ 0 & \cos t\end{bmatrix}, \text{ and select } C=[1 \quad 0]$$

$d=0.1, \quad \tau=0.2$.

Solving the linear matrix inequality (9), we can obtain that

$$X=\begin{bmatrix}0.8053 & 0.2144\\ 0.2144 & 0.9726\end{bmatrix}, Q=\begin{bmatrix}2.5991 & -0.8815\\ -0.8815 & 0.7561\end{bmatrix}$$

$$P=\begin{bmatrix}1.3192 & -0.2909\\ -0.2909 & 1.0923\end{bmatrix}, R=\begin{bmatrix}2.6488 & -0.9094\\ -0.9094 & 1.0317\end{bmatrix},$$

$$T=\begin{bmatrix}2.5241 & -0.8226\\ -0.8226 & 2.4682\end{bmatrix}$$

The initial conditions are given by

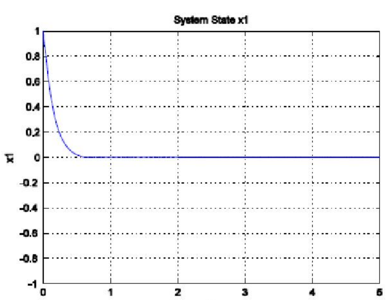
$$\phi(t)=[1 \quad -1]^T$$

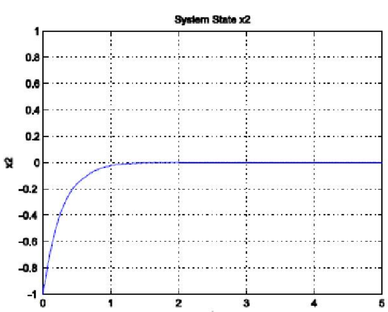
The gain matrix K of the stabilizing controller $u_{eq}(t)$ and the guaranteed cost functional are obtained

$$K=[-0.4789 \quad 2.2648] \quad ,$$

$$J^*=x^T(0)Px(0)+\int_{-d}^{0}x^T(s)Rx(s)+\int_{-\tau}^{0}x^T(s)Tx(s)ds$$

The simulation results are shown in the following two figs.





In the above figures, one can see that the system is well stabilized with respect to the admissible uncertainties.

CONCLUSION

In this paper, we have presented a new approach to stabilize the uncertain network control systems with delay. Based on the Lyapunov stability theory, and with the linear matrix inequality approach, a sliding mode surface is designed. Then in terms of linear matrix inequality, a sufficient condition is expressed to design the guaranteed cost variable structure controller. An example is provided to demonstrate the effectiveness of the proposed approach.

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