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TWO-FLUID COSMOLOGICAL MODELS IN A BIANCHI TYPE I SPACE-TIMES WITH VARIABLE G AND Λ

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ABSTRACT

This paper presents anisotropic, homogeneous two-fluid cosmological models in a Bianchi type I space-time with a gravitational constant G and cosmological constant Λ. In the two-fluid model, one fluid represents the matter content of the universe and another fluid is chosen to model the CMB radiation. The radiation and matter content of the universe are in an interactive phase. We also discuss the behaviour of associated fluid parameters and kinematical parameters.

Key words: Bianchi type I, Two-fluid, Variable G and Λ.

INTRODUCTION

Cosmological models with a cosmological constant are currently serious candidates to describe the dynamics of the Universe. Recent observations of Type-I_a supernovae with the red-shift up to about $z \leq 1$ provided evidence that we may beliving in a low mass-density universe, with the contribution of the non-relativistic matter to the total energy density of the universe of order of $\Omega_m \approx 0.3$ (Riess et al., 1998; Perlmutter et al., 1998, 1999). The value of Ω_m is significantly less than unity (Ostriker and Steinhardt 1995). Thus, a major art of matter content in the universe remains unobserved. This leads to the assumption that there is some additional energy sufficient to reach the value $\Omega_{total} = 1$, predicted by inflationary theory. Several physical models have been proposed to give a consistent physical interpretation of these observational facts. The observational and theoretical features suggest that the most natural candidate for the missing energy is the vacuum energy density or the cosmological constant Λ (Weinberg 1989; Gasperini 1988). But selection of cosmological constant as a vacuum energy faces a serious fine-tuning problem, which demands that the value of Λ must be 120 orders of magnitude greater than its presently observed value.

A number of authors studied cosmological models with a variable cosmological constant. Bertolami (1986) was the first to consider cosmological models in the variable cosmological constant as the form $\Lambda \propto t^{-2}$. Chen and Wu (1990) studied Friedmann-Robertson-Walker (FRW) model with variable

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cosmological constant as the form $\Lambda \propto R^{-2}$, where R is the average scale factor of the universe. Carvalho et al., (1992), Berman (1990), Waga (1993), Silviera and Waga (1994), and Vishwakarma (2000) also investigated cosmological models with a variable cosmological constant by considering a more general Λ term. Al-Rawaf and Taha (1996), Al-Rawaf (1998), Overdin and Cooperstock (1998), and Arbab (2003) investigated cosmological models with the cosmological constant of the form $\Lambda = \beta \frac{\ddot{R}}{R}$, where β is a constant. Pradhan and Otarod (2006) presented exact solutions of Einstein's field equations with perfect fluid for a locally rotationally symmetric (LRS) Bianchi type I space-time. They used a time-dependent declaration parameter and a variable cosmological term.

The idea of a variable gravitational constant G in the frame work of general relativity was first proposed by Dirac (1937). Lau (1985), working in the frame work of general relativity, proposed modification linking the variation of G with that of Λ. This modification allows us to use Einstein's field equations formally unchanged since a variation in Λ is accompanied by a variation of G. A number of authors investigated FRW models using this approach (Abdel-Rahman 1990; Berman 1991; Sisterio 1991; Kalligas et al., 1992; Abdulssathar and Vishwakarma 1997).

Recently, Debnah and Paul (2006) investigated FRW cosmological models with varying G and Λ in the framework of R^2 theory. Singh (2006) studied FRW cosmological models with variable G and Λ in general relativity by using the state equation $p = (\gamma - 1)\rho$, where γ varies continuously as the universe expands. Following the same approach, Singh et al. (2007) also obtained solutions of Einstein's field equations with the varying G and Λ in the presence of bulk viscosity for a FRW universe. Ibotombi Singh and Sorokhaiban (2007) obtained exact solutions for Zeldovich fluid satisfying $G = G_0\left(R / R_0\right)^n$ with variable G and Λ for an FRW metric.

Bianchi space-times provide spatially homogeneous and isotropic models of the universe as compared to the homogeneous and isotropic FRW models. Beesham (1994) and Chakraborty and Roy (1997) studied the Bianchi type cosmological models for perfect fluids, assuming the power law form for G and Λ.

Vishwakarma (2001) has studied the magnitude red-shift relation for the type I_a supernovae data and the angular size red-shift relation for the updated compact radio sources data Gurvits (1999) by considering four variable Λ -models: $\Lambda \sim R^{-2}, \Lambda \sim H^{-2}, \Lambda \sim \rho$ and $\Lambda \sim t^{-2}$. Ray and Mukhopadhayay (2004) have solved Einstein's equations for specific dynamical models of the cosmological terms Λ in the form $\Lambda \sim \frac{\dot{R}^2}{R^2}, \Lambda \sim \frac{\ddot{R}}{R}$ and $\Lambda \sim \rho$ shown that the models are equivalent in the framework of flat RW space time. In this context, the aim of the present work is based on recent available observational information. In this paper the implication of cosmological models with cosmological terms of two different forms: $\Lambda = \beta \frac{\dot{R}^2}{R^2}$, $\Lambda = \alpha \frac{\ddot{R}}{R}$ are analyzed in the two-fluid model, one fluid represents the matter content of the universe and anther fluid is chosen to model the CMB radiation.

The metric and the field equations

We consider the plane symmetric metric in the form

$$ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2 dz^2 \qquad \ldots(1)$$

where A and B are functions of time t only.

Einstein's field equations with time-dependent gravitational and cosmological constants are given by

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi G(t) T_{ij} - \Lambda(t) g_{ij} \quad \ldots(2)$$

The energy momentum tensor for two-fluid source is

$$T_{ij} = T_{ij}^{m} + T_{ij}^{r} \quad \ldots(3)$$

where $T_{ij}^{(m)}$ is the energy momentum tensor for matter field and $T_{ij}^{(r)}$ is the energy momentum tensor for the radiation field which are given by

$$T_{ij}^{(m)} = (p_m + \rho_m) u_i^m u_j^m - p_m g_{ij} \quad \ldots(4)$$

$$T_{ij}^{(r)} = \frac{4}{3} \rho_r u_i^r u_j^r - \frac{1}{3} \rho_r g_{ij} \quad \ldots(5)$$

with

$$g^{ij} u_i^m u_i^m = 1 \qquad g^{ij} u_i^r u_i^r = 1 \quad \ldots(6)$$

The off diagonal equations of (2) together with energy conditions imply the matter and radiation are both co-moving:

$$u_i^{(m)} = (0,0,0,1) \qquad u_i^{(r)} = (0,0,0,1) \quad \ldots(7)$$

In Einstein's theory, the principal of equivalence requires that G and Λ not enter the equation of motion of particle and photons; i.e., only g_{ij} must enter the usual conservation law for Einstein's field equation $T_{;j}^{ij} = \left(T^{(m)ij} + T^{(r)ij}{}_{;j} \right) = 0$ (the semicolon denotes covariant divergence) and the vanishing covariant divergence of the Einstein tensor in (2) results in

$$\dot{\Lambda} = -8\pi \dot{G} (\rho_m + \rho_r) \quad \ldots(8)$$

Using (3), (4), (5) and (7) the field equations (2) reduces to

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi G \left(p_m + \frac{\rho_r}{3} \right) + \Lambda \quad \ldots(9)$$

$$\frac{\dot{A}^2}{A^2} + 2\frac{\ddot{A}}{A} = -8\pi G \left(p_m + \frac{\rho_r}{3} \right) + \Lambda \quad \ldots(10)$$

$$\frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} = 8\pi G (\rho_m + \rho_r) + \Lambda \quad \ldots(11)$$

Here, an over dot denotes a derivative with respect to cosmic time t.

Solutions of the field equations

There are four equations (8-11) in seven unknowns. Thus, to get a solution we need three additional relations. These relations may be taken to involve field variables as well as physical variables. We attempt to solve them by choosing an additional relation in the form of some physical condition signifying some particular scenario, viz., the two-fluid are in interactive phase and the matter distribution obeys the γ -law of equation of state. We assume that the expansion scalar θ in the model is proportional to the shear σ . This condition leads to the relation between metric potential and further gravitational constant G assuming (Beesham 1994).

$$p_m = (\gamma - 1)\rho_m \qquad 1 \le \gamma \le 2 \qquad \text{…(12)}$$

$$B = A^n \qquad \text{…(13)}$$

$$G = (n+2)(at+b)^m \qquad \text{…(14)}$$

where n is positive constant.

Equations (9) and (10) lead to

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{A}^2}{A^2} - 2\frac{\ddot{A}}{A} = 0 \qquad \text{…(15)}$$

Using equation (13) in equation (15), we get

$$\frac{\ddot{A}}{A} + (1+n)\frac{\dot{A}^2}{A^2} = 0 \qquad \text{…(16)}$$

The solution of equation (16) is given by

$$A = (n+2)^{\frac{1}{n+2}}(at+b)^{\frac{1}{n+2}} \qquad \text{…(17)}$$

and hence

$$B = (n+2)^{\frac{n}{n+2}}(at+b)^{\frac{n}{n+2}} \qquad \text{…(18)}$$

Using above, the space-time can be written as

$$ds^2 = dt^2 - (n+2)^{\frac{2}{n+2}}(at+b)^{\frac{2}{n+2}}(dx^2 + dy^2) - (n+2)^{\frac{2n}{n+2}}(at+b)^{\frac{2n}{n+2}}dz^2 \qquad \text{…(19)}$$

In order to investigate the physical behaviour of the fluid parameters and cosmological and gravitational constants we consider two different cases as given in the following

Case I : [illegible]

Using equation (9), (11) and (12), we get energy density of matter, energy density of radiation and total energy density as

$$8\pi\rho_m = \frac{1}{(3\gamma-4)}\left(\frac{2(2n+1)a^2}{(n+2)^3(at+b)^{m+2}} + \frac{4\beta a^2}{9(n+2)(at+b)^{m+2}}\right) \qquad \text{...(20)}$$

$$8\pi\rho_r = \frac{1}{(3\gamma-4)}\left(\frac{(3\gamma-6)(2n+1)a^2}{(n+2)^3(at+b)^{m+2}} - \frac{3\gamma\beta a^2}{9(n+2)(at+b)^{m+2}}\right) \qquad \text{…(21)}$$

$$8\pi\rho = \frac{(2n+1)a^2}{(n+2)^3(at+b)^{m+2}} - \frac{\beta a^2}{9(n+2)(at+b)^{m+2}}$$

Particular Cases

(i) Dust model $(\gamma = 1)$: In order to investigate the physical behaviour of the fluid parameters we can consider the particular case of dust.

The scalar of expansion, shear scalar and declaration parameter are given by

$$\theta = 3H = \frac{a}{at+b}$$

$$\sigma^2 = \frac{a^2(n-1)^2}{3(n+2)^2(at+b)^2}$$

$$q = 2$$

For $\gamma = 1$, (20) and (21) imply density parameters as

$$\Omega_m = -\left(\frac{18(2n+1)+4\beta(n+2)^2}{3(n+2)^2}\right)$$

$$\Omega_r = \frac{9(2n+1)+\beta(n+2)^2}{(n+2)^2}$$

$$\Omega_\Lambda = \frac{\beta}{3}$$

$$\Omega = \Omega_r + \Omega_m + \Omega_\Lambda = 1 - \left(\frac{(n+2)^2 - 3(2n+1)}{(n+2)^2}\right)$$

In this case ρ_m is negative and ρ_r is positive.

(ii) Radiation Universe ($\gamma = \frac{4}{3}$):

The scalar of expansion, shear scalar and declaration parameter are given by

$$\theta = 3H = \frac{a}{at+b}$$

$$\sigma^2 = \frac{a^2(n-1)^2}{3(n+2)^2(at+b)^2}$$

$$q = 2$$

For $\gamma = \frac{4}{3}$, (20) and (21) imply density parameters as

$$\Omega_m = \infty$$

$$\Omega_r = \infty$$

(iii) Hard Universe $\left(\gamma = \frac{5}{3}\right)$:

The scalar of expansion, shear scalar and declaration parameter are given by

$$\theta = 3H = \frac{a}{at+b}$$

$$\sigma^2 = \frac{a^2(n-1)^2}{3(n+2)^2(at+b)^2}$$

$$q = 2$$

For $\gamma = \frac{5}{3}$, (20) and (21) imply density parameters as

$$\Omega_m = \frac{18(2n+1)+4\beta(n+2)^2}{3(n+2)^2}$$

$$\Omega_r = \frac{-9(2n+1)-5\beta(n+2)^2}{3(n+2)^2}$$

$$\Omega_\Lambda = \frac{\beta}{3}$$

$$\Omega = \Omega_r + \Omega_m + \Omega_\Lambda = 1 - \left(\frac{(n+2)^2 - 3(2n+1)}{(n+2)^2}\right)$$

Here ρ_m is positive and ρ_r is negative.

(iv) Zeldovich Universe $(\gamma = 2)$:

The scalar of expansion, shear scalar and declaration parameter are given by

$$\theta = 3H = \frac{a}{at+b}$$

$$\sigma^2 = \frac{a^2(n-1)^2}{3(n+2)^2(at+b)^2}$$

$$q = 2$$

For $\gamma = 2$, (20) and (21) imply density parameters as

$$\Omega_m = \frac{9(2n+1)+2\beta(n+2)^2}{3(n+2)^2}$$

$$\Omega_r = -\beta$$

$$\Omega_\Lambda = \frac{\beta}{3}$$

$$\Omega = \Omega_r + \Omega_m + \Omega_\Lambda = 1 - \left(\frac{(n+2)^2 - 3(2n+1)}{(n+2)^2} \right)$$

Here ρ_m is positive and ρ_r is negative.

Case II: $\Lambda = \alpha \frac{\ddot{R}}{R}$

Using equation (9), (11) and (12), we get energy density of matter, energy density of radiation and total energy density as

$$8\pi\rho_m = \frac{1}{(3\gamma - 4)} \left(\frac{2(2n+1)a^2}{(n+2)^3 (at+b)^{m+2}} - \frac{8\alpha a^2}{9(n+2)(at+b)^{m+2}} \right) \quad \ldots(22)$$

$$8\pi\rho_r = \frac{1}{(3\gamma - 4)} \left(\frac{(3\gamma - 6)(2n+1)a^2}{(n+2)^3 (at+b)^{m+2}} + \frac{2\gamma\alpha a^2}{3(n+2)(at+b)^{m+2}} \right) \quad \ldots(23)$$

$$8\pi\rho = \frac{(2n+1)a^2}{(n+2)^3 (at+b)^{m+2}} + \frac{2\alpha a^2}{9(n+2)(at+b)^{m+2}}$$

Particular Cases

(i) Dust model $(\gamma = 1)$:

The scalar of expansion, shear scalar and declaration parameter are given by

$$\theta = 3H = \frac{a}{at+b}$$

$$\sigma^2 = \frac{a^2 (n-1)^2}{3(n+2)^2 (at+b)^2}$$

$$q = 2$$

For $\gamma = 1$, (22) and (23) imply density parameters as

$$\Omega_m = \frac{8\alpha(n+2)^2 - 18(2n+1)}{3(n+2)^2}$$

$$\Omega_r = \frac{9(2n+1) - 2\alpha(n+2)^2}{(n+2)^2}$$

$$\Omega_\Lambda = -\frac{2\alpha}{3}$$

$$\Omega = \Omega_r + \Omega_m + \Omega_\Lambda = 1 - \left(\frac{(n+2)^2 - 3(2n+1)}{(n+2)^2} \right)$$

Here both ρ_m and ρ_r are positive.

(ii) Radiation Universe ($\gamma = \frac{4}{3}$):

The scalar of expansion, shear scalar and declaration parameter are given by

$$\theta = 3H = \frac{a}{at+b}$$

$$\sigma^2 = \frac{a^2(n-1)^2}{3(n+2)^2(at+b)^2}$$

$$q = 2$$

For $\gamma = \frac{4}{3}$, (22) and (23) imply density parameters as

$$\Omega_m = \infty$$

$$\Omega_r = \infty$$

(iii) Hard Universe $\left(\gamma = \frac{5}{3}\right)$:

The scalar of expansion, shear scalar and declaration parameter are given by

$$\theta = 3H = \frac{a}{at+b}$$

$$\sigma^2 = \frac{a^2(n-1)^2}{3(n+2)^2(at+b)^2}$$

$$q = 2$$

For $\gamma = \frac{5}{3}$, (22) and (23) imply density parameters as

$$\Omega_m = \frac{18(2n+1) - 8\alpha(n+2)^2}{3(n+2)^2}$$

$$\Omega_r = \frac{10\alpha(n+2)^2 - 9(2n+1)}{3(n+2)^2}$$

$$\Omega_\Lambda = -\frac{2\alpha}{3}$$

$$\Omega = \Omega_r + \Omega_m + \Omega_\Lambda = 1 - \left(\frac{(n+2)^2 - 3(2n+1)}{(n+2)^2}\right)$$

Here both ρ_m and ρ_r are positive.

(iv) Zeldovich Universe $(\gamma = 2)$:

The scalar of expansion, shear scalar and declaration parameter are given by -

$$\theta = 3H = \frac{a}{at+b}$$

$$\sigma^2 = \frac{a^2(n-1)^2}{3(n+2)^2(at+b)^2}$$

$$q = 2$$

For $\gamma = 2$, (22) and (23) imply density parameters as

$$\Omega_m = \frac{9(2n+1) - 4\alpha(n+2)^2}{3(n+2)^2}$$

$$\Omega_r = 2\alpha$$

$$\Omega_\Lambda = -\frac{2\alpha}{3}$$

$$\Omega = \Omega_r + \Omega_m + \Omega_\Lambda = 1 - \left(\frac{(n+2)^2 - 3(2n+1)}{(n+2)^2}\right)$$

Here both ρ_m and ρ_r are positive.

CONCLUSION

In this paper we have presented two-fluid cosmological models obeying Einstein field equations in plane symmetric universe with gravitational constant G and cosmological constant Λ and considered two different cases comprising $\gamma-$law of equation of state for matter field. In all the cases, we get $q = 2$. This implies that these two fluids models are expanding with constant velocity. Also in all the cases, we get $\frac{\sigma}{\theta}$ = constant. Therefore, these models do not approach isotropy for large value of t. The sign of deceleration parameter q indicates whether the model accelerates or not. The positive sign of $q(\succ 1)$ corresponds to decelerating model whereas the negative sign $(-1 \prec q \prec 0)$ indicates acceleration and $q = 2$ corresponds to expansion with constant velocity. The model comes out to be rotating as well as expanding ones, the rate of expansion decreases with time, which can be thought of as realistic models. In both cases Ω_m and Ω_r depends only on n, α and β.

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