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Total Pathos Edge Semi Entire Block Graph

Venkanagouda M Goudar[1*] and Jagadeesh N[2]

[1]Department of Mathematics, Sri Siddhartha Institute of Technology, Tumkur, Karnataka, India

[2]Research Scholar, Sri Siddhartha Academy of Higher Education, Tumkur, Kalpataru first grade Science College, Tiptur, Karnataka, India

[*]**Corresponding author:** Venkanagouda M Goudar, Department of Mathematics, Sri Siddhartha Institute of Technology, Tumkur, Karnataka, India, Tel: 9448579479; E-mail: vmgouda@gmail.com

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Abstract

In this paper, we introduce the concept of total pathos edge semi entire block graph of a tree. We obtain some properties of this graph. We study the characterization of graphs whose total pathos edge semi entire block graphs are always nonplanar, crossing number one, Eulerian and Hamiltonian.

Keywords:* *Block graph; Edge Semientire graph; Inner vertex number; Line graph

Introduction

Let G (p,q) be a connected graph with vertex set V=p and the edge set E=q. If e=uv is in E (G), then the vertices u and v are adjacent. For the graph theoretical notation, we follow in [1,2].

In [3] Venkanagouda et al. introduced the graph valued function, the pathos edge semi entire block graph of a tree T.

The block graph B (G) of a graph G was introduced in [2]. Further the path graph P (T) of a tree was studied in [3].

The following theorems will be used in the sequel.

Theorem 1 [6]. If G is a (p, q) graph whose vertices have degree d_i then L (G) has q vertices and q_L edges where

$$q_L = -q + \frac{1}{2}\sum d_i^2$$

Theorem 2 [6]. The line graph L (G) of a graph G has crossing number one if and only if G is planar and 1 or 2 holds:

1. The maximum degree d (G) is 4 and there is unique non-cut vertex of degree 4.

2. The maximum degree d (G) is 5, every vertex of degree 4 is a cut vertex and there is a unique vertex of degree 5 and has at most 3 edges in any block.

Theorem 3 [2]. A connected graph G is isomorphic to its line graph if and only if it is a cycle [4].

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Theorem 4 [6]. The line graph L (G) of a graph is planar if and only if G is planar, $\Delta \leq 4$ and if deg v=4 for a vertex v of G, then v is a cut vertex.

Theorem 5 [1]. A graph is planar if and only if it has no subgraph homeomorphic to K_5 or $K_{3,3}$.

Theorem 6 [9]. For any planar graph G, edge semi entire block graph E_b (G) whose vertices have degree d_i has (q+r+b) vertices and $-q+\frac{1}{2}\sum d_i^2+\sum q_i+\sum \frac{b_k\left(b_k-1\right)}{2}+\sum q_r$

edges, where r the number of regions, b be the number of blocks, q_j the number of edges in a block b_j, b_k be the blockdegree of a cut vertex C_k and q_r be the number of edges lies on the region r_1.

Theorem 7 [3]. For any planar graph G, pathos edge semi entire block graph PE_b (G) whose vertices have degree d_i has (2q+k+1) vertices and $\frac{1}{2}\sum d_i^2+\sum q_i+\sum \frac{b_k\left(b_k-1\right)}{2}$ edges, where r be the number of regions, b the number of blocks, q_j the number of edges lies in a block b_j, b_k be the block degree of a cut vertex C_k and q_r be the number of edges lies in the region r_1.

Theorem 8 [1]. A connected graph G is eulerian if and only if each vertex in G has even degree [5].

Theorem 9 [2]. A nontrivial graph is bipartite if and only if all its cycles are even.

Theorem 10 [3]. For any tree T, the pathos edge semi entire block graph PE_b (T) is planar if and only if T is a star graph $K_{1, n}$, for $n \geq 3$.

Total Pathos Edge Semi Entire Block Graph

In this section, we define the total pathos edge semi entire block graph of a tree.

Definition 2.1 The total pathos edge semi entire block graph of a tree T denoted by T_{Pe} (T) is the graph whose vertex set is the collection of edges, blocks, regions and path of pathos of T in which two vertices of T_{Pe} (T) are adjacent if both are the edges of T which are adjacent or both are blocks of T which are adjacent or one is a block and other is an edge e of T and e lies on it, or one is a region and other is an edge e of T and edge e lies on the region or one is a path of pathos of T and other is an edge e of T in which e lies on the path or both are the path of paths p_i and p_j of T and both p_i and p_j have a common cut vertex. In FIG. 1, a graph G and its total pathos edge semi entire block graph are shown [6].

We begin with the following direct results**.**

Remark 1. For any tree T, $T \subseteq E_b(T) \subseteq \mathrm{PE}_b(T) \subseteq T_{Pe}(T)$

Remark 2. For any graph G, Tp_e (G)=PE_b (T)U P (T)

Remark 3. For any edge e_i in T with edge degree n, the degree of the vertex e'_i which corresponds to e_i in T_{Pe} (T) is always n+1.

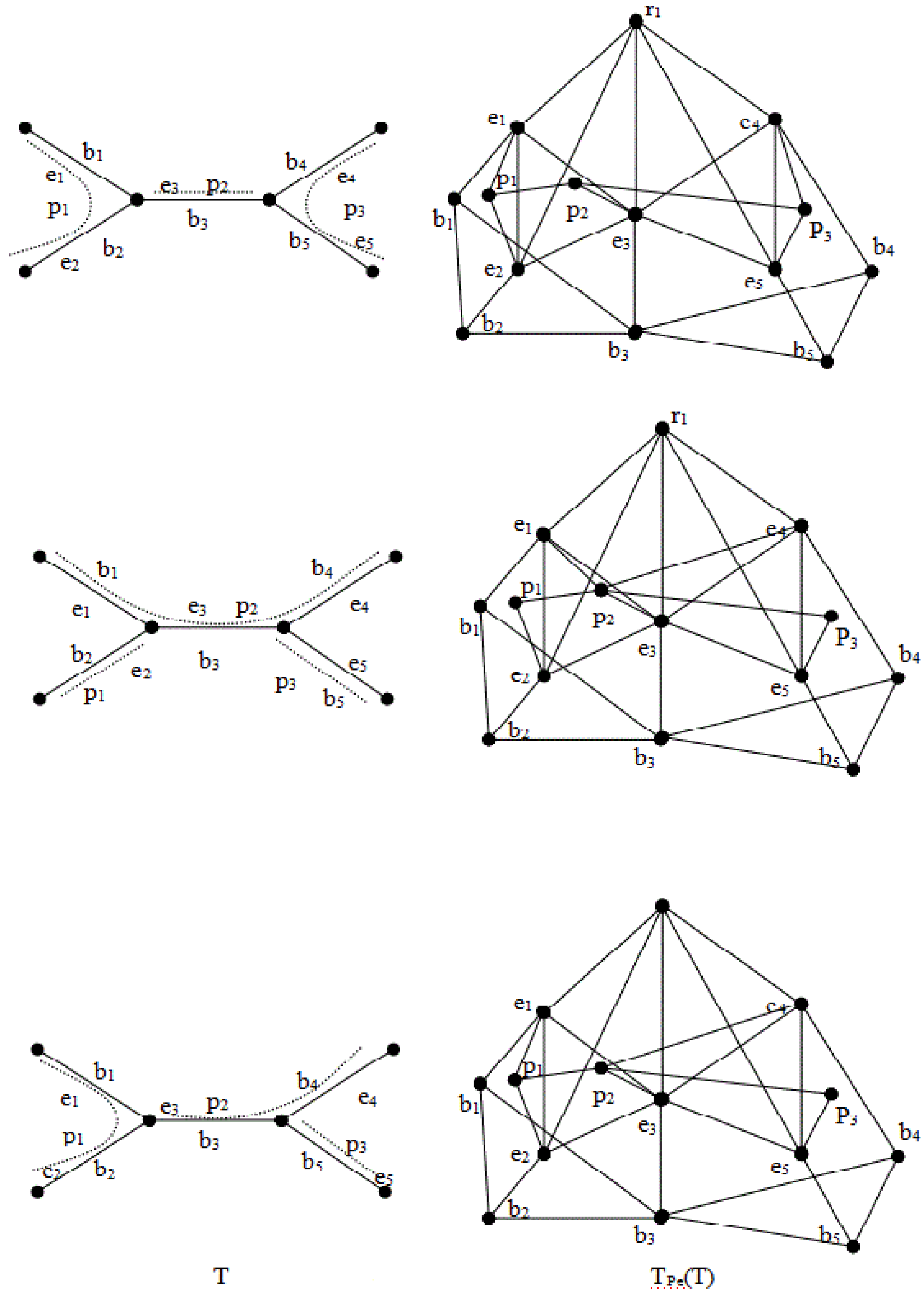

Fig. 1. **Graph G and its total pathos edge semi entire block graph.**

Results

Theorem 11. Let T be a tree, the total pathos edge semi entire block graph T_{Pe} (T) is always non-separable.

Proof. Consider T be a tree and let e_i for all iϵT be the edges of T. In a Tree T, every edge becomes a block. Let b_1=e_1, b_2=e_2. b_m=e_m be the blocks, r_1 be the only one region in T and p_1, p_2. p_k be the pathos of T. By the definition of L (T), the vertices e_1, e_2. e_m of L (T) form a sub graph without cut vertex. Also in T_{Pe} (T), the vertex formed from the region called the region vertex is adjacent to e_i for all i=1,2. m, to form a nonseparable graph. Since there are m blocks which are K_2, we have each bi is adjacent to e_i for all i.

Further T contains some pendent pathos, these correspond to the pathos vertices p_i and each p_i is adjacent to only one vertex e_j such that e_j becomes a cut vertex. Also in a tree, at least two paths are always adjacent then T_{Pe} (T) is non-separable [7].

Theorem 12. If T (p, q) is a connected graph whose vertices have degree d_i and if the number of blocks to which the edge e_i belongs in T is b_i, then the total pathos edge semi entire block graph T_{Pe} (T) has 2q+k+1 vertices and $\frac{1}{2}\sum d_i^2+\sum q_j+\frac{1}{2}\sum b_k(b_k-1)+\frac{1}{2}\sum\frac{p_i(p_i-1)}{2}$ edges, where q_j be the number of edges in each block b_j and b_k be the block degree of a cut vertex c_k.

Proof. By Remark 1, $PE_b(T)\subseteq T_{Pe}(T)$ Thus, the number of vertices in T_{Pe} (T) equals the number of vertices of of PE_b (T). By theorem 7, V[PE_b(T)]=2q+k+1.

Further by Remark 2, the number of edges in T_{pe} (T) is same as the number of edges in PE_b (T) and the number of edges P(T). By theorem 7, the number of edges in PE_b (T) is $\frac{1}{2}\sum d_i^2+\sum q_i+\sum\frac{b_k(b_k-1)}{2}$. Also, the number of edges in a path graph P(T) is $\frac{1}{2}\sum p_i(p_i-1)$. Hence the number of edges T_{Pe} (T) $\frac{1}{2}\sum d_i^2+\sum q_j+\frac{1}{2}\sum b_k(b_k-1)+\frac{1}{2}\sum\frac{p_i(p_i-1)}{2}$.

Theorem 13. For any tree T, T_{Pe} (T) is non-complete graph.

Proof. By definition of T_{Pe} (T), there is no adjacency between the regions and blocks. In T_{Pe}(T) there is no edge between the region vertex and block vertex. Hence T_{Pe} (T) is non-complete graph.

Theorem 14. For any tree T, the total pathos edge semi entire block graph T_{Pe} (T) is not a bipartite graph.
Proof. In a tree other than path contains at least one vertex v of T such that degree of v ≥ 3. Let e_1, e_2, e_3 be the edges incident to v. By the definition of T_{Pe} (T), the edges incident with v, form a cycle C_3 in T_{Pe} (T). By Theorem 9, T_{Pe} (T) is not bipartite graph.

Theorem 15. For any tree T, T_{Pe} (T) ≅ PE_b (T) if and only if T is a path.

Proof. Let T be a path. By definition, T_{Pe} (T), PE_b (T) have the same number of vertices. Since T is a path and the pathos graph of a path has no edges, it implies by definitions that T_{Pe} (T) and PE_b (T) are isomorphic.

Conversely suppose T_{Pe} (T) ≅ PE_b (T) and T is non-trivial connected tree. We now prove that T is a path. We assume that T has at least one vertex v such that degree of v ≥ 3. By remark 2, the number of edges in T_{Pe} (T) is the sum of the number of edges PE_b (T) and the number of edges in P (T). Hence the number of edges in PE_b (T) is less than the number of edges in T_{Pe} (T). Thus T_{Pe} (T) ≅ PE_b (T), a contradiction. Thus, T is a path.

Theorem 16. For any tree T, total pathos edge semi entire block graph T_{Pe} (T) is always nonplanar.

Proof. Let T be a connected tree and be a star K_1, n, for n ≤ 3. By Theorem 10, PE_b (T) is planar. Let $e_1=b_1$, $e_2=b_2$, and $e_3=b_3$ be the edges which are also blocks, r_1 be the region and p_1, p_2 be the path of pathos of T. In T_{Pe} (T), the vertices e_1, e_2, e_3 and r_1

form a complete graph K_4. The block vertices b_1, b_2, b_3 form a complete graph K_3 and the edges between p_1 and p_2 must crosses the edges already drawn and hence T_{Pe} (T) is nonplanar.

If $T=K_{1,n}$, $n \geq 4$, by the Theorem 10, PE_b (T) is nonplanar and hence T_{Pe} (T) is nonplanar.

Theorem 17. For any tree T, the total pathos edge semi entire block graph T_{Pe} (T) has crossing number one if and only if T is $K_{1,3}$.

Proof. Suppose T_{Pe} (T) has crossing number one, clearly T_{Pe} (T) is nonplanar. By Theorem 16, we have $T=K_{1,n}$, $n \geq 4$. Assume that $T=K_{1,n}$ for n=4. By the definition of L (T), L ($K_{1,4}$)=K_4. Since every edge of T lies on only one region r_1 so in T_{Pe} (T), all vertices of K_4 are adjacent to the region vertex r_1 to form K_5, clearly it has crossing number one. Further in a tree T, each edge is a block and all blocks b_1, b_2, b_3 and b_4 are adjacent to each other. By the definition of T_{Pe} (T), all four block vertices form K_4 as sub graph, clearly the inner vertex b_i is adjacent to the edge e_i which corresponds to e'_i in T_{Pe} (T) which form another one crossing number. Hence T_{Pe} (T) has crossing number at least two, a contradiction [9,10].

Proving conversely, show that K_5 is a subgraph which has crossing number one. By the definition of line graph, L ($K_{1,3}$)=K_3 and all edges lies on only one region. In T_{Pe} (T), the region vertex r_1 is adjacent to all vertices e_i^1 which corresponds to the edges of T and it form a complete graph K_4. Further each edge is a block and all blocks form K_3 as a sub graph. Also, the pathos vertices p_1 adjacent is e_1 and e_2, p_2 is adjacent to e_3 and these pathos vertices are adjacent to each other and it form crossing number one. Hence, C_r [T_{Pe} (T)]=1.

Theorem 18. For any tree T, the total pathos edge semientire block graph T_{Pe} (T) is Eulerian if and only if T is a star $K_{1,4n+2}$ for $n \geq 1$.

Proof. Suppose T_{Pe} (T) is Eulerian. We consider the following cases.

Case 1. Assume that T be any non-star tree T, we have the following sub cases.

Sub case 1.1. Let u, v ϵT such that degree (u)=3 and degree (v)=2. Let e_1, e_2, e_3 be the edges incident to u and e_3, e_4 be the edges incident to v. Clearly edge degree of an edge e_1 is 4. By the Remark 3, the corresponding vertex e_i^1 in T_{Pe} (T) have degree 5, which is odd. By Theorem 8, T_{Pe} (T) is noneulerian, a contradiction.

Sub case 1.2. Assume that T contain an edge have edge degree of every edge is even, which is possible only when the total number of edges of T will be odd. By definition of T_{Pe} (T), the deg (r_i) becomes odd. By Theorem 8, T_{Pe} (T) is noneulerian, a contradiction.

Case 2. Assume that $T=K_{1,p}$ where $p \neq 4n+2$. We have the following subcases.

Sub case 2.1. We assume that $T=K_{1,}$ 3n+1 n=1.2. k. By case 1, each edge of $K_{1,3}$ have edge degree 4, by Theorem 2, the corresponding vertices e_i^1 in T_{Pe} (T) have edge degree odd, which is odd. By the Theorem 8, T_{Pe} (T) is noneulerian, a contradiction.

Sub case 2.2. Assume that $T=K_{1,4}$. By case 1, each edge e_i in T have edge degree odd. By the Remark 3, the corresponding vertices e_i^1 in T_{Pe} (T) have even degree. The region vertex r'_i is adjacent to all four vertices v_i^1, i=1, 2, 3, 4 which form a graph with all vertices of even degree. In T, there are two paths of pathos p_1 and p_2 and each pathos contains two edges. In T_{Pe} (T), both pathos vertices adjacent to form graph with both p_1 and p_2 have odd degree. By Theorem 8, T_{Pe} (T) is noneulerian, a contradiction.

Conversely suppose $T=K_{1,\ 4p+2}$ for p=1,2. n. In a tree T, every edge e_i have odd degree, by Remark 3, the vertices e_i^1, corresponds to e_i of T in T_{Pe} (T) becomes even degree. Further the total number of edges of T is even, then the region vertex in T_{Pe} (T) becomes even degree. Also, each edge is a block $e_i=b_i$ for all i, each block b_i adjacent to remaining blocks and e_i adjacent to b_i to form even degree. Lastly the pathos vertices which are adjacent to even number of edges and each pathos are adjacent to remaining 2n path of pathos. Then in T_{Pe} (T), every vertex is of even degree. Hence T_{Pe} (T) is Eulerian.

Theorem 19. For any tree T, T_{Pe} (T) is Hamiltonian.

Proof. Suppose T be a tree. We consider the following cases.

Case 1. Let T be a star $K_{1,\ n}$, n is odd with vertices u_1, u_2u_3.... u_nu_{n+k} such that degree of u_1=n and let b_1, b_2 . . .b_m be the number of blocks, r_i be the regions, the number of paths of pathos are $\frac{n+1}{2}$ and T contains only one region. By the definition of T_{Pe} (T), $V[T_{Pe}(T)]=\{e_1, e_2, e_3 \ldots\ldots e_n, b_1, b_2, \ldots\ldots b_{n-1}\} \cup \{p_1, p_2, \ldots., p_{(n+1)/2}\} \cup r_1$. Then there exist a cycle containing the vertices of T_{Pe} (T) as p_1,p_2, p_3,…p_k,e_5,e_k,b_kb_{k-1},. . . b_1,e_1,e_3… r.e_2 p_1 and it includes all the vertices of T_{Pe} (T). Hence it is a Hamiltonian cycle.

Case 2. Let $T=K_{1,n,}$ n>2 and is even. Then the number of path of pathos is $\frac{n}{2}$. Let $V[T_{Pe}(T)]=\{e_1,e_2 \ldots.e_n, b_1,b_2 \ldots b_{n-1}\}$ U $\{p_1, p_2 \ldots\ldots p_{\frac{n}{2}}\}$ U r_1 then there exist a cycle which contains the vertices of T_{Pe} (T) as p_1, p_2,…e_k b_k, b_{k-1}, . . . b_1,e_1,…r.,e_2 p_1 and is a Hamilton cycle. Hence T_{Pe} (T) is Hamiltonian.

Case 3. Suppose T is neither a path nor a star, then T contains at least two vertices of degree >2. Let e_1, e_2. . . . .e_n be the edges of T such that $e_1=b_1$, $e_2=b_2$. . . . . $e_n=b_n$ be the blocks. Then $V[T_{Pe}(T)]=\{e_1, e_2 \ldots e_n, b_1, b_2 \ldots b_n\} \cup \{p_1, p_2 \ldots p_i\} \cup r_1$ where T has p_i pathos vertices for i > 1 and each pathos vertex is adjacent to the edge of T, where the corresponding pathos lie on the edges of T. Then there exist a cycle C which contains all the vertices of T_{Pe} (T) as p_1, $e_1b_1e_2p_2e_3$. . . . . r_1 e_n p_1. Hence T_{Pe} (T) is a Hamiltonian. Clearly T_{Pe} (T) is a Hamiltonian.

Case 4. Suppose T is a path. Let u_1, u_2u_3. . . . . u_n be a path. The vertex set $V[T_{Pe}(T)]=\{e_1,e_2 \ldots.e_n, b_1,b_2 \ldots b_{n-1}\}$ U $\{p_1\}$ U r_1. Clearly there exist a cycle which contains the vertices of T_{Pe} (T) as p_1,…e_k b_k, b_{k-1}, . . . b_1,e_1,…r.,e_2 p_1 and is a Hamilton cycle. Hence T_{Pe} (T) is Hamiltonian.

Conclusion

In this paper, we defined the total pathos edge semi entire block graph of a tree. We characterized the graphs whose total pathos edge semi entire block graphs are planar, Hamiltonian and have crossing number one.

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