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Total eccentricity, adjacent eccentric distance sum and gutman index of certain special molecular graphs

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ABSTRACT

Chemical compounds and drugs are often modeled as graphs where each vertex represents an atom of molecule, and covalent bounds between atoms are represented by edges between the corresponding vertices. This graph derived from a chemical compounds is often called its molecular graph, and can be different structures. In this paper, we determine the total eccentricity, adjacent eccentric distance sum and Gutman index of fan molecular graph, wheel molecular graph, gear fan molecular graph, gear wheel molecular graph, and their r-corona molecular graphs.

KEYWORDS

Chemical graph theory; Total eccentricity; Adjacent eccentric distance sum; Gutman index; *r*-Corona molecular graph.

INTRODUCTION

Wiener index, edge Wiener index, Hyper-wiener index, Geometric-arithmetic index and Zagreb indices are introduced to reflect certain structural features of organic molecules. Several papers contributed to determine the distance-based index of special molecular graphs (See Yan et al.,[1] and[2], Gao and Shi[3] for more detail). Let P_n and C_n be path and cycle with n vertices. The molecular graph $F_n=\{v\}\vee P_n$ is called a fan molecular graph and the molecular graph $W_n=\{v\}\vee C_n$ is called a wheel molecular graph. Molecular graph $I_r(G)$ is called r- crown molecular graph of G which splicing r hang edges for every vertex in G. By adding one vertex in every two adjacent vertices of the fan path P_n of fan molecular graph F_n, the resulting molecular graph is a subdivision molecular graph called gear fan molecular graph, denote as $\tilde{F}_n$. By adding one vertex in every two adjacent vertices of the wheel cycle C_n of wheel molecular graph W_n, The resulting molecular graph is a subdivision molecular graph, called gear wheel molecular graph, denoted as $\tilde{W}_n$.

The molecular graphs considered in this paper are simple and connected. The eccentricity $ec(u)$ of vertex $u\in V(G)$ is the maximum distance between u and any other vertex in G. Then the total eccentricity of the molecular graph G (see[4]), denoted by $\zeta(G)$, is defined as the sum of eccentricities of all vertices of molecular graph G, i.e.,

$$\zeta(G)=\sum_{v\in V(G)} ec(v).$$

Fathalikhani,[5] determined the total Eccentricity of some molecular graph operations.

Sardana and Madan[6] introduced a novel topological descriptor adjacent eccentric distance sum index (AEDS), which is defined to be

$$\xi^{sv}(G)=\frac{ec(v)D(v)}{\deg(v)}.$$

Sardana and Madan[7] investigated the relationship of Wiener index and adjacent eccentric distance sum index with nitroxide free radicals and their precursors. Hua and Yu[8] derived some upper or lower bounds for the adjacent eccentric distance sum in terms of some graph invariants or topological indices such as Wiener index, total eccentricity and minimum degree.

The graphs considered in this paper are simple and connected. Then the Gutman index of G is defined by Gutman[9] as

$$Gut(G)=\sum_{\{u,v\}\subseteq V(G)} d(u)d(v)d(u,v).$$

Andova et al.,[10] showed that among all graphs on n vertices, the star graph S_n has minimal Gutman index. In addition, it presented upper and lower bounds on Gutman index for graphs with minimal and graphs with maximal Gutman index. Mukwembi[11] proved that $Gut(G)\le\frac{2^4}{5^5}n^5+O(n^4)$. Feng[12] characterized n-vertex unicyclic graphs with girth k, having minimal Gutman index. Knor[13] studied the relationship between Gutman index and edge Wiener index of graph.

In this paper, we present the total eccentricity, adjacent eccentric distance sum and Gutman index of $I_r(F_n)$, $I_r(W_n)$, $I_r(\tilde{F}_n)$ and $I_r(\tilde{W}_n)$.

TOTAL ECCENTRICITY

Theorem 1. $\zeta(I_r(F_n)) = r(4n+3)+(3n+2)$.

Proof. Let $P_n=v_1v_2\ldots v_n$ and the r hanging vertices of v_i be v_i^1, v_i^2,…, v_i^r $(1\le i\le n)$. Let v be a vertex in F_n beside P_n, and the r hanging vertices of v be v^1, v^2, …, v^r. By the definition of total eccentricity, we have

$$\zeta(I_r(F_n)) = ec(v)+\sum_{i=1}^{n} ec(v_i)+\sum_{i=1}^{r} ec(v^i)+\sum_{i=1}^{n}\sum_{j=1}^{r} ec(v_i^j) = 2+3n+3r+4nr = r(4n+3)+(3n+2).\ \square$$

Corollary 1. $\zeta(F_n) = 2n+1$.

Theorem 2. $\zeta(I_r(W_n)) = r(4n+3)+(3n+2)$.

Proof. Let $C_n=v_1v_2\ldots v_n$ and v_i^1, v_i^2,…, v_i^r be the r hanging vertices of v_i $(1\le i\le n)$. Let v be a vertex in W_n beside C_n, and v^1, v^2, …, v^r be the r hanging vertices of v. By the definition of total eccentricity, we have

$$\zeta(I_r(W_n)) = ec(v)+\sum_{i=1}^{n} ec(v_i)+\sum_{i=1}^{r} ec(v^i)+\sum_{i=1}^{n}\sum_{j=1}^{r} ec(v_i^j) = 2+3n+3r+4nr = r(4n+3)+(3n+2).\ \square$$

Corollary 2. $\zeta(W_n) = 2n+1$.

Theorem 3. $\zeta(I_r(\tilde{F}_n)) = r(11n-2)+(9n-2)$.

Proof. Let $P_n=v_1v_2\ldots v_n$ and $v_{i,i+1}$ be the adding vertex between v_i and v_{i+1}. Let v_i^1, v_i^2,…, v_i^r be the r hanging vertices of v_i $(1\le i\le n)$. Let $v_{i,i+1}^1$, $v_{i,i+1}^2$,…, $v_{i,i+1}^r$ be the r hanging vertices of $v_{i,i+1}$ $(1\le i\le n-1)$. Let v be a vertex in F_n beside P_n, and the r hanging vertices of v be v^1, v^2, …, v^r. By virtue of the definition of total eccentricity, we get

$$\zeta(I_r(\tilde{F}_n)) = ec(v)+\sum_{i=1}^{n} ec(v_i)+\sum_{i=1}^{r} ec(v^i)+\sum_{i=1}^{n}\sum_{j=1}^{r} ec(v_i^j)+\sum_{i=1}^{n-1} ec(v_{i,i+1})+\sum_{i=1}^{n-1}\sum_{j=1}^{r} ec(v_{i,i+1}^j)$$

$$=3+4n+4r+5nr+5(n-1)+6r(n-1) = r(11n-2)+(9n-2).\ \square$$

Corollary 3. $\zeta(\tilde{F}_n) = 7n-2$.

Theorem 4. $\zeta(I_r(\tilde{W}_n)) = r(11n+4)+(9n+3)$.

Proof. Let $C_n=v_1v_2\ldots v_n$ and v be a vertex in W_n beside C_n. $v_{i,i+1}$ be the adding vertex between v_i and v_{i+1}. Let v^1, v^2, …, v^r be the r hanging vertices of v and v_i^1, v_i^2,…, v_i^r be the r hanging vertices of v_i $(1\le i\le n)$. Let $v_{n,n+1}=v_{1,n}$ and $v_{i,i+1}^1$, $v_{i,i+1}^2$,…, $v_{i,i+1}^r$ be the r hanging vertices of $v_{i,i+1}$ $(1\le i\le n)$. In view of the definition of total eccentricity, we deduce

$$\zeta(I_r(\tilde{W}_n)) = ec(v) + \sum_{i=1}^{n} ec(v_i) + \sum_{i=1}^{r} ec(v^i) + \sum_{i=1}^{n}\sum_{j=1}^{r} ec(v_i^j) + \sum_{i=1}^{n} ec(v_{i,i+1}) + \sum_{i=1}^{n}\sum_{j=1}^{r} ec(v_{i,i+1}^j)$$

$=3+4n+4r+5nr+5n+6nr = r(11n+4)+(9n+3)$. □

Corollary 4. $\zeta(\tilde{W}_n) = 7n+2$.

ADJACENT ECCENTRIC DISTANCE SUM

Theorem 5. $\xi^{sv}(I_r(F_n)) = \frac{2[r+n+2rn]}{r+n} + 3r[(2r-1)+2n+3nr] + \frac{6[(5r+2)+(2+3r)(n-2)]}{r+2}$ $+\frac{3(n-2)[(7r+3)+(2+3r)(n-3)]}{r+3} + [8r[1+2(r-1)+2(2+3r)+(3+4r)(n-2)]+$ $4(n-2)r[1+2(r-1)+3(2+3r)+(3+4r)(n-3)]]$.

Proof. By the definition of adjacent eccentric distance sum index, we have

$$\xi^{sv}(I_r(F_n)) = \frac{D(v)ec(v)}{\deg(v)} + \sum_{i=1}^{n}\frac{D(v_i)ec(v_i)}{\deg(v_i)} + \sum_{i=1}^{r}\frac{D(v^i)ec(v^i)}{\deg(v^i)} + \sum_{i=1}^{n}\sum_{j=1}^{r}\frac{D(v_i^j)ec(v_i^j)}{\deg(v_i^j)}$$

$=\frac{2[r+n+2rn]}{r+n} + \frac{6[(5r+2)+(2+3r)(n-2)]}{r+2} + \frac{3(n-2)[(7r+3)+(2+3r)(n-3)]}{r+3} + 3r[(2r-1)+2n+3nr]+$ $[2r\times 4[1+2(r-1)+2(2+3r)+(3+4r)(n-2)]+4(n-2)r[1+2(r-1)+3(2+3r)+(3+4r)(n-3)]]$. □

Theorem 6. $\xi^{sv}(I_r(W_n)) = \frac{2[r+n+2rn]}{r+n} + \frac{3n[(7r+3)+(2+3r)(n-3)]}{r+3} +$ $3r[1+2(r-1)+2n+3nr]+4nr[1+2(r-1)+3(2+3r)+(3+4r)(n-3)]$.

Proof. By the definition of adjacent eccentric distance sum index, we have

$$\xi^{sv}(I_r(W_n)) = \frac{D(v)ec(v)}{\deg(v)} + \sum_{i=1}^{n}\frac{D(v_i)ec(v_i)}{\deg(v_i)} + \sum_{i=1}^{r}\frac{D(v^i)ec(v^i)}{\deg(v^i)} + \sum_{i=1}^{n}\sum_{j=1}^{r}\frac{D(v_i^j)ec(v_i^j)}{\deg(v_i^j)}$$

$=\frac{2[r+n+2rn]}{r+n} + \frac{3n[(7r+3)+(2+3r)(n-3)]}{r+3} + [1+2(r-1)+2n+3nr]r\times 3+$ $4nr[1+2(r-1)+3(2+3r)+(3+4r)(n-3)]$. □

Theorem 7. $\xi^{sv}(I_r(\tilde{F}_n)) = \frac{3[r+n+2rn+(n-1)(2+3r)]}{n+r} + \frac{8[(3r+1)+(2+3r)(n-1)+(1+2r)+(n-2)(3+4r)]}{r+2}$ $+\frac{4(n-2)[(3r+1)+(2+3r)(n-1)+2(1+2r)+(n-3)(3+4r)]}{r+3} + [(2r-1)+2n+3nr+(n-1)(4r+3)]4r +$ $[10r[1+2(r-1)+(2+3r)+(3+4r)(n-1)+(2+3r)+(n-2)(4+5r)]+5(n-2)r[1+2(r-1)$

$$+(2+3r)+(3+4r)(n-1)+2(2+3r)+(n-3)(4+5r)]]+\frac{5[r+2(1+2r)+(2+3r)+(n-2)(3+4r)]}{r+2}$$
$$+6(n-1)r[1+2(r-1)+2(2+3r)+(3+4r)+(4+5r)(n-2)].$$

Proof. By virtue of the definition of adjacent eccentric distance sum index, we get

$$\xi^{sv}(I_r(\tilde{F}_n))=\frac{D(v)ec(v)}{\deg(v)}+\sum_{i=1}^{n}\frac{D(v_i)ec(v_i)}{\deg(v_i)}+\sum_{i=1}^{r}\frac{D(v^i)ec(v^i)}{\deg(v^i)}+\sum_{i=1}^{n}\sum_{j=1}^{r}\frac{D(v_i^j)ec(v_i^j)}{\deg(v_i^j)}+\sum_{i=1}^{n-1}\frac{D(v_{i,i+1})ec(v_{i,i+1})}{\deg(v_{i,i+1})}+$$
$$\sum_{i=1}^{n-1}\sum_{j=1}^{r}\frac{D(v_{i,i+1}^j)ec(v_{i,i+1}^j)}{\deg(v_{i,i+1}^j)}$$

$$=\frac{3[r+n+2rn+(n-1)(2+3r)]}{n+r}+\frac{8[(3r+1)+(2+3r)(n-1)+(1+2r)+(n-2)(3+4r)]}{r+2}$$
$$+\frac{4(n-2)[(3r+1)+(2+3r)(n-1)+2(1+2r)+(n-3)(3+4r)]}{r+3}+[(2r-1)+2n+3nr+(n-1)(4r+3)]4r+$$
$$[10r[1+2(r-1)+(2+3r)+(3+4r)(n-1)+(2+3r)+(n-2)(4+5r)]+5(n-2)r[1+2(r-1)$$
$$+(2+3r)+(3+4r)(n-1)+2(2+3r)+(n-3)(4+5r)]]+\frac{5[r+2(1+2r)+(2+3r)+(n-2)(3+4r)]}{r+2}$$
$$+6(n-1)r[1+2(r-1)+2(2+3r)+(3+4r)+(4+5r)(n-2)].\ \square$$

Theorem 8. $\xi^{sv}(I_r(\tilde{W}_n))=\frac{3[r+n+2rn+n(2+3r)]}{r+n}+$

$$\frac{4n[(3r+1)+(2+3r)(n-1)+2(1+2r)+(n-2)(3+4r)]}{r+3}+[1+2(r-1)+2n+3nr+n(4r+3)]4r+$$
$$5nr[1+2(r-1)+(2+3r)+(3+4r)(n-1)+2(2+3r)+(n-2)(4+5r)]]$$
$$+\frac{5[r+2(1+2r)+(2+3r)+(n-1)(3+4r)]}{r+2}+6nr[1+2(r-1)+2(2+3r)+2(3+4r)+(4+5r)(n-2)].$$

Proof. In view of the definition of adjacent eccentric distance sum index, we deduce

$$\xi^{sv}(I_r(\tilde{W}_n))=\frac{D(v)ec(v)}{\deg(v)}+\sum_{i=1}^{n}\frac{D(v_i)ec(v_i)}{\deg(v_i)}+\sum_{i=1}^{r}\frac{D(v^i)ec(v^i)}{\deg(v^i)}+\sum_{i=1}^{n}\sum_{j=1}^{r}\frac{D(v_i^j)ec(v_i^j)}{\deg(v_i^j)}+\sum_{i=1}^{n}\frac{D(v_{i,i+1})ec(v_{i,i+1})}{\deg(v_{i,i+1})}+$$
$$\sum_{i=1}^{n}\sum_{j=1}^{r}\frac{D(v_{i,i+1}^j)ec(v_{i,i+1}^j)}{\deg(v_{i,i+1}^j)}$$

$$=\frac{3[r+n+2rn+n(2+3r)]}{r+n}+\frac{4n[(3r+1)+(2+3r)(n-1)+2(1+2r)+(n-2)(3+4r)]}{r+3}+$$
$$[1+2(r-1)+2n+3nr+n(4r+3)]4r+5nr[1+2(r-1)+(2+3r)+(3+4r)(n-1)$$
$$+2(2+3r)+(n-2)(4+5r)]]+\frac{5[r+2(1+2r)+(2+3r)+(n-1)(3+4r)]}{r+2}$$
$$+6nr[1+2(r-1)+2(2+3r)+2(3+4r)+(4+5r)(n-2)].\ \square$$

GUTMAN INDEX

Theorem 9. $Gut(I_r(F_n)) = r^2(4n^2+4n+3) + r(15n^2-14n+3) + (21n^2-53n+39)$.

Proof. By the definition of Gutman index, we have

$$Gut(I_r(F_n)) = \sum_{i=1}^{r-1}\sum_{j=i+1}^{r} d(v^i)d(v^j)d(v^i,v^j) + \sum_{i=1}^{r} d(v)d(v^i)d(v,v^i) + \sum_{i=1}^{n} d(v)d(v_i)d(v,v_i) +$$
$$\sum_{i=1}^{n}\sum_{j=1}^{r} d(v)d(v_i^j)d(v,v_i^j) + \sum_{i=1}^{n}\sum_{j=1}^{r} d(v_i)d(v^j)d(v_i,v^j) + \sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{r} d(v_i^j)d(v^k)d(v_i^j,v^k) +$$
$$\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} d(v_i)d(v_j)d(v_i,v_j) + \sum_{i=1}^{n}\sum_{j=1}^{r} d(v_i)d(v_i^j)d(v_i,v_i^j) + \sum_{i=1}^{n}\sum_{j\in\{1,2,\cdots,n\}-i}\sum_{k=1}^{r} d(v_i)d(v_j^k)d(v_i,v_j^k) +$$
$$\sum_{i=1}^{n}\sum_{j=1}^{r-1}\sum_{k=j+1}^{r} d(v_i^j)d(v_i^k)d(v_i^j,v_i^k) + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1}^{r}\sum_{t=1}^{r} d(v_i^k)d(v_j^t)d(v_i^k,v_j^t)$$

$= (r^2-r) + (r+n)r + (r^2n + r(n^2+3n-2) + (3n^2-2n)) + 2nr(r+n) + 2r(rn+3n-2) + 3nr^2 +$
$(r^2(n^2-2n) + r(6n^2-16n+8) + (9n^2-30n+25)) + nr(rn+3n-2) + (r(3n^2-5n+2) + (9n^2-21n+14)) + nr(r-1) + r^2(n-1)(2n-1)$

$= r^2(4n^2+4n+3) + r(15n^2-14n+3) + (21n^2-53n+39)$. □

Corollary 5. $Gut(F_n) = 21n^2 - 53n + 39$.

Theorem 10. $Gut(I_r(W_n)) = r^2(6n^2+2) + r(18n^2-15n-1) + (12n^2-18n)$.

Proof. By the definition of Gutman index, we have

$$Gut(I_r(W_n)) = \sum_{i=1}^{r-1}\sum_{j=i+1}^{r} d(v^i)d(v^j)d(v^i,v^j) + \sum_{i=1}^{r} d(v)d(v^i)d(v,v^i) + \sum_{i=1}^{n} d(v)d(v_i)d(v,v_i) +$$
$$\sum_{i=1}^{n}\sum_{j=1}^{r} d(v)d(v_i^j)d(v,v_i^j) + \sum_{i=1}^{n}\sum_{j=1}^{r} d(v_i)d(v^j)d(v_i,v^j) + \sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{r} d(v_i^j)d(v^k)d(v_i^j,v^k) +$$
$$\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} d(v_i)d(v_j)d(v_i,v_j) + \sum_{i=1}^{n}\sum_{j=1}^{r} d(v_i)d(v_i^j)d(v_i,v_i^j) + \sum_{i=1}^{n}\sum_{j\in\{1,2,\cdots,n\}-i}\sum_{k=1}^{r} d(v_i)d(v_j^k)d(v_i,v_j^k) +$$
$$\sum_{i=1}^{n}\sum_{j=1}^{r-1}\sum_{k=j+1}^{r} d(v_i^j)d(v_i^k)d(v_i^j,v_i^k) + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1}^{r}\sum_{t=1}^{r} d(v_i^k)d(v_j^t)d(v_i^k,v_j^t)$$

$= (r^2-r) + r(r+n) + n(r+n)(3+r) + 2nr(r+n) + 2nr(3+r) + 3nr^2 + (3+r)^2(n^2-2n) + nr(3+r) +$
$r(3n^2-5n)(3+r) + nr(r-1) + r^2n(2n-3)$

$= r^2(6n^2+2) + r(18n^2-15n-1) + (12n^2-18n)$. □

Corollary 6. $Gut(W_n) = 12n^2 - 18n$.

Theorem 11. $Gut(I_r(\tilde{F}_n)) = (r^2(30n^2 - 45n + 40) + r(63n^2 - 131n + 106) + (50n^2 - 137n + 114))$.

Proof. By virtue of the definition of Gutman index, we get

$$Gut(I_r(\tilde{F}_n)) = \sum_{i=1}^{r-1}\sum_{j=i+1}^{r} d(v^i)d(v^j)d(v^i,v^j) + \sum_{i=1}^{r} d(v)d(v^i)d(v,v^i) + \sum_{i=1}^{n} d(v)d(v_i)d(v,v_i) +$$

$$\sum_{i=1}^{n}\sum_{j=1}^{r} d(v)d(v_i^j)d(v,v_i^j) + \sum_{i=1}^{n}\sum_{j=1}^{r} d(v_i)d(v^j)d(v_i,v^j) + \sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{r} d(v_i^j)d(v^k)d(v_i^j,v^k) +$$

$$\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} d(v_i)d(v_j)d(v_i,v_j) + \sum_{i=1}^{n}\sum_{j=1}^{r} d(v_i)d(v_i^j)d(v_i,v_i^j) + \sum_{i=1}^{n}\sum_{j\in\{1,2,\cdots,n\}-i}\sum_{k=1}^{r} d(v_i)d(v_j^k)d(v_i,v_j^k)$$

$$+ \sum_{i=1}^{n}\sum_{j=1}^{r-1}\sum_{k=j+1}^{r} d(v_i^j)d(v_i^k)d(v_i^j,v_i^k) + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1}^{r}\sum_{t=1}^{r} d(v_i^k)d(v_j^t)d(v_i^k,v_j^t) + \sum_{i=1}^{n-1} d(v)d(v_{i,i+1})d(v,v_{i,i+1}) +$$

$$\sum_{i=1}^{n-1}\sum_{j=1}^{r} d(v)d(v^j_{i,i+1})d(v,v^j_{i,i+1}) + \sum_{i=1}^{r}\sum_{j=1}^{n-1} d(v^i)d(v_{j,j+1})d(v^i,v_{j,j+1}) + \sum_{i=1}^{r}\sum_{j=1}^{n-1}\sum_{k=1}^{r} d(v^i)d(v^k_{j,j+1})d(v^i,v^k_{j,j+1}) +$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n-1} d(v_i)d(v_{j,j+1})d(v_i,v_{j,j+1}) + \sum_{i=1}^{n}\sum_{j=1}^{n-1}\sum_{k=1}^{r} d(v_i)d(v^k_{j,j+1})d(v_i,v^k_{j,j+1}) + \sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{n-1} d(v_i^j)d(v_{k,k+1})d(v_i^j,v_{k,k+1}) +$$

$$\sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{n-1}\sum_{t=1}^{r} d(v_i^j)d(v^r_{k,k+1})d(v_i^j,v^r_{k,k+1}) + \sum_{i=1}^{n-2}\sum_{j=i+1}^{n-1} d(v_{i,i+1})d(v_{j,j+1})d(v_{i,i+1},v_{j,j+1})$$

$$+ \sum_{i=1}^{n-1}\sum_{j=1}^{r} d(v_{i,i+1})d(v^j_{i,i+1})d(v_{i,i+1},v^j_{i,i+1}) + \sum_{i=1}^{n-1}\sum_{j\in\{1,2,\cdots,n-1\}-i}\sum_{k=1}^{r} d(v_{i,i+1})d(v^k_{j,j+1})d(v_{i,i+1},v^k_{j,j+1}) +$$

$$\sum_{i=1}^{n-1}\sum_{j=1}^{r-1}\sum_{k=j+1}^{r} d(v^j_{i,i+1})d(v^k_{i,i+1})d(v^j_{i,i+1},v^k_{i,i+1}) + \sum_{i=1}^{n-2}\sum_{j=i+1}^{n-1}\sum_{k=1}^{r}\sum_{t=1}^{r} d(v^k_{i,i+1})d(v^t_{j,j+1})d(v^k_{i,i+1},v^t_{j,j+1})$$

$$= (r^2 - r) + r(n + r) + (r^2 n + r(n^2 + 3n - 2) + (3n^2 - 2n)) + (2n^2 r + 2r^2 n) + (2r^2 n^2 + r(6n^2 - 4n)) + 3nr^2 +$$
$$(r^2(n^2 - n) + r(6n^2 - 10n + 4) + (9n^2 - 21n + 14)) + (r^2 n + r(3n - 2)) + (3r^2 n^2 + r(9n^2 - 6n)) + nr(r-$$
$$1) + 2r^2 n(n-1) + (2r^2 + r(2n + 4) + 4n) + (3r^2(n-1) + 3nr) + (3r^2(n-1) + r(6n - 6)) + 4r^2(n-1) +$$
$$(r^2(3n^2 - 7n + 4) + r(15n^2 - 41n + 30) + (18n^2 - 54n + 44)) + (r(4n^2 - 8n + 4) + (12n^2 - 32n + 24)) +$$
$$(r^2(4n^2 - 8n + 4) + r(8n^2 - 16n + 8)) + r^2(5n-4)(n-1) + (r^2(2n^2 - 8n + 8) + r(8n^2 - 32n + 32) + (8n^2 - 32n + 32)) +$$
$$(r^2(n-1) + r(2n - 2)) + (r^2(5n^2 - 19n + 18) + r(10n^2 - 38n + 36)) + r(n-1)(r-1) + (n-2)(3n-5)r^2$$

$$= (r^2(30n^2 - 45n + 40) + r(63n^2 - 131n + 106) + (50n^2 - 137n + 114)).$$ □

Corollary 7. $Gut(\tilde{F}_n) = 50n^2 - 137n + 114$.

Theorem 12. $Gut(I_r(\tilde{W}_n)) = (r^2(32n^2 - 16n + 2) + r(3n^3 + 63n^2 - 54n - 1) + (6n^3 + 16n^2 - 25n))$.

Proof. In view of the definition of Gutman index, we deduce

$$Gut(I_r(\tilde{W}_n)) = \sum_{i=1}^{r-1}\sum_{j=i+1}^{r} d(v^i)d(v^j)d(v^i,v^j) + \sum_{i=1}^{r} d(v)d(v^i)d(v,v^i) + \sum_{i=1}^{n} d(v)d(v_i)d(v,v_i) +$$

$$\sum_{i=1}^{n}\sum_{j=1}^{r} d(v)d(v_i^j)d(v,v_i^j) + \sum_{i=1}^{n}\sum_{j=1}^{r} d(v_i)d(v^j)d(v_i,v^j) + \sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{r} d(v_i^j)d(v^k)d(v_i^j,v^k) +$$

$$\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} d(v_i)d(v_j)d(v_i,v_j) + \sum_{i=1}^{n}\sum_{j=1}^{r} d(v_i)d(v_i^j)d(v_i,v_i^j) + \sum_{i=1}^{n}\sum_{j\in\{1,2,\cdots,n\}-i}\sum_{k=1}^{r} d(v_i)d(v_j^k)d(v_i,v_j^k) +$$

$$\sum_{i=1}^{n}\sum_{j=1}^{r-1}\sum_{k=j+1}^{r} d(v_i^j)d(v_i^k)d(v_i^j,v_i^k) + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1}^{r}\sum_{t=1}^{r} d(v_i^k)d(v_j^t)d(v_i^k,v_j^t) + \sum_{i=1}^{n} d(v)d(v_{i,i+1})d(v,v_{i,i+1}) +$$

$$\sum_{i=1}^{n}\sum_{j=1}^{r} d(v)d(v^j_{i,i+1})d(v,v^j_{i,i+1}) + \sum_{i=1}^{r}\sum_{j=1}^{n} d(v^i)d(v_{j,j+1})d(v^i,v_{j,j+1}) + \sum_{i=1}^{r}\sum_{j=1}^{n}\sum_{k=1}^{r} d(v^i)d(v^k_{j,j+1})d(v^i,v^k_{j,j+1}) +$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} d(v_i)d(v_{j,j+1})d(v_i,v_{j,j+1}) + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{r} d(v_i)d(v^k_{j,j+1})d(v_i,v^k_{j,j+1}) + \sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{n} d(v_i^j)d(v_{k,k+1})d(v_i^j,v_{k,k+1}) +$$

$$\sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{n}\sum_{t=1}^{r} d(v_i^j)d(v^r_{k,k+1})d(v_i^j,v^r_{k,k+1}) + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} d(v_{i,i+1})d(v_{j,j+1})d(v_{i,i+1},v_{j,j+1})$$

$$+ \sum_{i=1}^{n}\sum_{j=1}^{r} d(v_{i,i+1})d(v^j_{i,i+1})d(v_{i,i+1},v^j_{i,i+1}) + \sum_{i=1}^{n}\sum_{j\in\{1,2,\cdots,n\}-i}\sum_{k=1}^{r} d(v_{i,i+1})d(v^k_{j,j+1})d(v_{i,i+1},v^k_{j,j+1}) +$$

$$\sum_{i=1}^{n}\sum_{j=1}^{r-1}\sum_{k=j+1}^{r} d(v^j_{i,i+1})d(v^k_{i,i+1})d(v^j_{i,i+1},v^k_{i,i+1}) + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1}^{r}\sum_{t=1}^{r} d(v^k_{i,i+1})d(v^t_{j,j+1})d(v^k_{i,i+1},v^t_{j,j+1})$$

$$= (r^2-r) + (r^2+nr) + (r^2n + r(n^2+3n) + 3n^2) + (2r^2n + 2rn^2) + (2r^2n + 6rn) + 3nr^2 +$$
$$(r^2(n^2-n) + r(6n^2-6n) + (9n^2-9n)) + \quad (r^2n + 3rn) + (r^2(3n^2-3n) + r(9n^2-9n)) \quad + nr(r-1) + 2n(n-1)r^2 +$$
$$(2r^2n + r(2n^2+4n) + 4n^2) + (3r^2n + 3rn^2) + (3r^2n + 6rn) + 4r^2n +$$
$$(r^2(3n^2-4n) + r(3n^3+2n^2-8n) + (6n^3-8n^2)) + (r^2(4n^2-4n) + r(12n^2-12n)) + (r^2(4n^2-4n) + r(8n^2-8n))$$
$$+ r^2n(5n-4) + (r^2(2n^2-4n) + r(8n^2-16n) + (8n^2-16n)) + (r^2n + 2rn) + (r^2(5n^2-9n) + r(10n^2-18n)) + nr(r-1) + (3n^2-5n)r^2$$

$$= (r^2(32n^2-16n+2) + r(3n^3+63n^2-54n-1) + (6n^3+16n^2-25n)) \quad \square$$

Corollary 8. $Gut(\tilde{W}_n) = 6n^3 + 16n^2 - 25n$.

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