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# **TOPSIS-based methodology for web service** quality evaluation with intuitionistic fuzzy information

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## ABSTRACT

With respect to intuitionistic fuzzy multiple attribute decision making problems with completely unknown weight information, some operational laws of intuitionistic fuzzy numbers are introduced. To determine the attribute weights, a model based on the information entropy, by which the attribute weights can be determined, is established. Then, based on the TOPSIS method, calculation steps for solving intuitionistic fuzzy multiple attribute decision-making problems with completely unknown weight information are given. The weighted Hamming distances between every alternative and positive ideal solution and negative ideal solution are calculated. Then, according to the weighted Hamming distances, the relative closeness degree to the positive ideal solution is calculated to rank all alternatives. Finally, an illustrative example for evaluating the web service quality is given to verify the developed approach and to demonstrate its practicality and effectiveness.

# KEYWORDS

Multiple attribute decision making; Intuitionistic fuzzy number; Intuitionistic fuzzy weighted averaging (IFWA) operator; Entropy; Web service quality.



### INTRODUCTION

Atanassov<sup>[1,2]</sup> introduced the concept of intuitionistic fuzzy set (IFS), which is a generalization of the concept of fuzzy set<sup>[3]</sup>. The intuitionistic fuzzy set has received more and more attention since its appearance<sup>[4-30]</sup>. Gau and Buehrer<sup>[4]</sup> introduced the concept of vague set. But Bustince and Burillo<sup>[5]</sup> showed that vague sets are intuitionistic fuzzy sets. In<sup>[6]</sup>, Xu developed some geometric aggregation operators, such as the intuitionistic fuzzy weighted geometric (IFWG) operator, the intuitionistic fuzzy ordered weighted geometric (IFOWG) operator to multiple attribute group decision making with intuitionistic fuzzy weighted averaging (IFWA) operator, the intuitionistic fuzzy ordered weighted averaging (IFWA) operator, the intuitionistic fuzzy ordered weighted averaging (IFWA) operator.

In the process of MADM with intuitionistic fuzzy information, sometimes, the attribute values take the form of intuitionistic fuzzy numbers, and the information about attribute weights is incompletely known or completely unknown because of time pressure, lack of knowledge or data, and the expert's limited expertise about the problem domain. All of the above methods, however, will be unsuitable for dealing with such situations. Therefore, it is necessary to pay attention to this issue. Xu<sup>[8]</sup> investigated the intuitionistic fuzzy MADM with the information about attribute weights is completely unknown or completely unknown, a method based on the ideal solution was proposed. The aim of this paper is to develop TOPSIS method, based on the information entropy method, to overcome this limitation. The remainder of this paper is set out as follows. In the next section, we introduce some basic concepts related to intuitionistic fuzzy sets. In Section 3 we introduce the MADM problem with intuitionistic fuzzy information, in which the information about attribute weights is completely unknown, and the attribute values take the form of intuitionistic fuzzy numbers. To determine the attribute weights, a model based on the information entropy method, by which the attribute weights can be determined, is established. Then, based on the TOPSIS method, calculation steps for solving intuitionistic fuzzy multiple attribute decision-making problems with completely unknown weight information are given. In Section 4, an illustrative example for evaluating the web service quality is pointed out. In Section 5 we conclude the paper and give some remarks.

### PRELIMINARIES

In the following, we introduce some basic concepts related to intuitionistic fuzzy sets. **Definition 1.** Let X be a universe of discourse, then a fuzzy set is defined as:

$$A = \left\{ \left\langle x, \mu_A(x) \right\rangle \middle| x \in X \right\}$$
(1)

Which is characterized by a membership function  $\mu_A: X \to [0,1]$ , where  $\mu_A(x)$  denotes the degree of membership of the element *x* to the set  $A^{[3]}$ .

Atanassov extended the fuzzy set to the IFS, shown as follows:

**Definition 2**. An IFS A in X is given by

$$A = \left\{ \left\langle x, \mu_A(x), \nu_A(x) \right\rangle | x \in X \right\}$$
(2)

Where  $\mu_A: X \to [0,1]$  and  $\nu_A: X \to [0,1]$ , with the condition

$$0 \le \mu_A(x) + \nu_A(x) \le 1, \ \forall \ x \in X$$

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**Definition 3.** For each IFS A in X, if

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x), \ \forall \ x \in X.$$
(3)

Then  $\pi_A(x)$  is called the degree of indeterminacy of  $x \text{ to } A^{[1,2]}$ .

**Definition 4.** Let  $\tilde{a} = (\mu, \nu)$  be an intuitionistic fuzzy number, a score function *S* of an intuitionistic fuzzy value can be represented as follows<sup>[9]</sup>:

$$S(\tilde{a}) = \mu - \nu, \quad S(\tilde{a}) \in [-1,1].$$
(4)

**Definition 5.** Let  $\tilde{a} = (\mu, \nu)$  be an intuitionistic fuzzy number, a accuracy function *H* of an intuitionistic fuzzy value can be represented as follows<sup>[10]</sup>:

$$H\left(\tilde{a}\right) = \mu + \nu, \quad H\left(\tilde{a}\right) \in [0,1].$$
(5)

to evaluate the degree of accuracy of the intuitionistic fuzzy value  $\tilde{a} = (\mu, \nu)$ , where  $H(\tilde{a}) \in [0,1]$ . The larger the value of  $H(\tilde{a})$ , the more the degree of accuracy of the intuitionistic fuzzy value  $\tilde{a}$ .

**Definition 6.** Let  $\tilde{a}_1 = (\mu_1, \nu_1)$  and  $\tilde{a}_2 = (\mu_2, \nu_2)$  be two intuitionistic fuzzy numbers, then the Hamming distance between  $\tilde{a}_1 = (\mu_1, \nu_1)$  and  $\tilde{a}_2 = (\mu_2, \nu_2)$  is defined as follows:

$$d(\tilde{a}_1, \tilde{a}_2) = \frac{|\mu_1 - \mu_2| + |\nu_1 - \nu_2|}{2}$$
(6)

## TOPSIS-BASED NUMERICAL COMPUTATION METHODOLOGY FOR INTUITIONISTIC FUZZY MULTIPLE ATTRIBUTE DECISION MAKING

The following assumptions or notations are used to represent the intuitionistic fuzzy MADM problems with entropy weight information:

(1) The alternatives are known. Let  $A = \{A_1, A_2, \dots, A_m\}$  be a discrete set of alternatives;

(2) The attributes are known. Let  $G = \{G_1, G_2, \dots, G_n\}$  be a set of attributes;

(3)The information about attribute weights is incompletely known. Let  $w = (w_1, w_2, \dots, w_n) \in H$  be the weight vector of attributes, where  $w_j \ge 0$ ,  $j = 1, 2, \dots, n$ ,  $\sum_{j=1}^n w_j = 1$ .

Suppose that  $\tilde{R} = (\tilde{r}_{ij})_{m \times n} = (\mu_{ij}, \nu_{ij})_{m \times n}$  is the intuitionistic fuzzy decision matrix, where  $\mu_{ij}$  indicates the degree that the alternative  $A_i$  satisfies the attribute  $G_j$  given by the decision maker,  $\nu_{ij}$  indicates the degree that the alternative  $A_i$  doesn't satisfy the attribute  $G_j$  given by the decision maker,  $\mu_{ij} \subset [0,1], \nu_{ij} \subset [0,1], \mu_{ij} + \nu_{ij} \leq 1, i = 1, 2, \dots, m, j = 1, 2, \dots, n$ .

Entropy<sup>[11]</sup> was one of the concepts in thermodynamics originally and then Shannon first introduced the concept of information entropy in connection with communication theory. He considered entropy was an equivalent to uncertainty. It made a pervasive impact to many other disciplines in extending his work to other fields, ranging from management science, engineering technology and the sociological economic field. In these disciplines entropy is applied as a measure of disorder, unevenness of distribution and the degree of dependency or complexity of a system. Information entropy is an ideal measure of uncertainty and it can measure the quality of effective information. In the intuitionistic fuzzy MADM problems which have m alternatives and n attributes, the *jth* attribute's entropy is defined as follows:

$$H_j = -k \sum_{i=1}^m r_{ij} \ln r_{ij}, i = 1, 2, \dots, m, j = 1, 2, \dots, n.$$

Where

$$r_{ij} = \frac{H\left(\tilde{r}_{ij}\right)}{\sum_{i=1}^{m} H\left(\tilde{r}_{ij}\right)}, \quad k = \frac{1}{\ln m}.$$
(7)

Assume that if  $f_{ii} = 0$ , then  $f_{ii} \ln f_{ii} = 0$ .

Then, the *jth* attribute's entropy is defined as follows:

$$w_{j} = \frac{1 - H_{j}}{n - \sum_{j=1}^{n} H_{j}}.$$
(8)

Based on the above models, we develop a practical method for solving the MADM problems, in which the information about attribute weights is completely unknown, and the attribute values take the form of intuitionistic fuzzy information. The method involves the following steps:

**Step 1.** Let  $\tilde{R} = (\tilde{r}_{ij})_{m \times n}$  be the intuitionistic fuzzy decision matrix, where  $\tilde{r}_{ij} = (\mu_{ij}, \nu_{ij})$ , which is an attribute value, given by an expert, for the alternative  $A_i \in A$  with respect to the attribute  $G_j \in G$ ,  $w = (w_1, w_2, \dots, w_n)$  be the weight vector of attributes, where  $w_j \in [0,1]$ ,  $j = 1, 2, \dots, n$ ,  $\sum_{j=1}^n w_j = 1$ .

Step 2. Determine the entropy weight of each attribute according to equations (7) and (8).

**Step 3.** Determine the positive ideal and negative ideal solution based on intuitionistic fuzzy numbers.

$$\tilde{r}^{+} = \left( \left( \mu_{1}^{+}, \nu_{1}^{+} \right), \left( \mu_{2}^{+}, \nu_{2}^{+} \right), \cdots, \left( \mu_{n}^{+}, \nu_{n}^{+} \right) \right)$$
(9)

$$\tilde{r}^{-} = \left( \left( \mu_{1}^{-}, \nu_{1}^{-} \right), \left( \mu_{2}^{-}, \nu_{2}^{-} \right), \cdots, \left( \mu_{n}^{-}, \nu_{n}^{-} \right) \right)$$
(10)

where

$$r_{j}^{+} = (\mu_{j}^{+}, \nu_{j}^{+}) = (\max_{i} \mu_{ij}, \min_{i} \nu_{ij}) \ j \in 1, 2, \cdots, n.$$

$$r_j^- = (\mu_j^-, \nu_j^-) = (\min_i \mu_{ij}, \max_i \nu_{ij}), j \in 1, 2, \cdots, n.$$

**Step 4.** Calculate weight information and the weighted hamming distances.

The weighted hamming distances of each alternative from the ideal solution is given as

$$d\left(\tilde{r}_{i},\tilde{r}^{+}\right) = \sum_{j=1}^{n} d\left(\tilde{r}_{ij},\tilde{r}_{j}^{+}\right) w_{j}, i = 1, 2, \cdots, m$$
  
$$= \sum_{j=1}^{n} w_{j} \left[\frac{\left|\mu_{ij}-\mu_{j}^{+}\right| + \left|\nu_{ij}-\nu_{j}^{+}\right|}{2}\right], i = 1, 2, \cdots, m.$$
 (11)

Similarly, the weighted hamming distances from the negative ideal solution is given as

$$d\left(\tilde{r}_{i},\tilde{r}^{-}\right) = \sum_{j=1}^{n} d\left(\tilde{r}_{ij},\tilde{r}_{j}^{-}\right) w_{j} = \sum_{j=1}^{n} w_{j} \left[\frac{\left|\mu_{ij}-\mu_{j}^{-}\right| + \left|v_{ij}-v_{j}^{-}\right|}{2}\right], i = 1, 2, \cdots, m.$$
(12)

**Step 5.** Calculate the relative closeness to the ideal solution. The relative closeness degree of the alternative  $A_i$  with respect to  $\tilde{r}^+$  is defined as

$$c\left(\tilde{r}_{i},\tilde{r}^{+}\right) = d\left(\tilde{r}_{i},\tilde{r}^{-}\right) / \left(d\left(\tilde{r}_{i},\tilde{r}^{+}\right) + d\left(\tilde{r}_{i},\tilde{r}^{-}\right)\right), i = 1, 2, \cdots, m.$$

$$(13)$$

**Step 6.** Rank all the alternatives  $A_i$  ( $i = 1, 2, \dots, m$ ) and select the best one (s) in accordance with  $c(\tilde{r}_i, \tilde{r}^+)(i = 1, 2, \dots, m)$ .

Step 7. End.

#### **ILLUSTRATIVE EXAMPLE**

Thus, in this section we shall present a numerical example for evaluating the quality of the web service with the intuitionistic fuzzy information in order to illustrate the method proposed in this paper. There is a panel with five possible web service systems  $A_i$  (i = 1, 2, ..., 5) to select. The experts selects four attribute to evaluate the five possible web service systems:  $(1)G_1$  is the performance of the web service systems;  $(2)G_2$  is the availability of the web service systems;  $(3)G_3$  is the reliability of the web service systems;  $(4)G_4$  is the security of the web service systems. The five possible web service systems  $A_i$  (i = 1, 2, ..., 5) are to be evaluated using the intuitionistic fuzzy information by the decision maker under the above four attributes, as listed in the following matrix.

$$\tilde{R} = \begin{bmatrix} (0.6, 0.3) & (0.7, 0.2) & (0.4, 0.5) & (0.5, 0.3) \\ (0.8, 0.2) & (0.8, 0.1) & (0.8, 0.1) & (0.5, 0.4) \\ (0.7, 0.3) & (0.6, 0.3) & (0.6, 0.2) & (0.7, 0.2) \\ (0.9, 0.1) & (0.7, 0.2) & (0.4, 0.3) & (0.3, 0.5) \\ (0.7, 0.1) & (0.5, 0.2) & (0.8, 0.2) & (0.6, 0.2) \end{bmatrix}$$

Procedure for selection of best web service system the following steps. **Step 1** According to equations (7) and (8), we get the weight vector of attributes:

 $w = (0.2825 \ 0.2152 \ 0.2171 \ 0.2853)^{t}$ 

Step 2. Determine the positive ideal solution and negative ideal solution

$$\tilde{r}^{+} = ((0.9, 0.1) \quad (0.8, 0.1) \quad (0.7, 0.2)) \quad \tilde{r}^{-} = ((0.6, 0.3) \quad (0.5, 0.3) \quad (0.4, 0.5) \quad (0.3, 0.5))$$

**Step 3.** Calculate the weighted hamming distances of each web service systems from the ideal solution and negative ideal solution by utilizing the weight vector w and Eq. (11-12) respectively.

$$d(\tilde{r}_{1}, \tilde{r}^{+}) = 0.2218, d(\tilde{r}_{2}, \tilde{r}^{+}) = 0.0853, d(\tilde{r}_{3}, \tilde{r}^{+}) = 0.1321$$
  

$$d(\tilde{r}_{4}, \tilde{r}^{+}) = 0.1865, d(\tilde{r}_{5}, \tilde{r}^{+}) = 0.0964, d(\tilde{r}_{1}, \tilde{r}^{-}) = 0.0893$$
  

$$d(\tilde{r}_{2}, \tilde{r}^{-}) = 0.2258, d(\tilde{r}_{3}, \tilde{r}^{-}) = 0.1790, d(\tilde{r}_{4}, \tilde{r}^{-}) = 0.1246$$
  

$$d(\tilde{r}_{5}, \tilde{r}^{-}) = 0.2147$$

Step 4. Calculate the relative closeness degree to the ideal solution.

$$c(\tilde{r}_{1}, \tilde{r}^{+}) = 0.2872, c(\tilde{r}_{2}, \tilde{r}^{+}) = 0.7258, c(\tilde{r}_{3}, \tilde{r}^{+}) = 0.5754$$
$$c(\tilde{r}_{4}, \tilde{r}^{+}) = 0.4005, c(\tilde{r}_{5}, \tilde{r}^{+}) = 0.6901$$

**Step 5.** Rank all the web service systems  $A_i$  (i = 1, 2, 3, 4, 5) in accordance with the relative closeness degree  $c(\tilde{r}_i, \tilde{r}^+)(i = 1, 2, 3, 4, 5): A_2 \succ A_5 \succ A_3 \succ A_4 \succ A_1$ , and thus the most desirable web service system is  $A_2$ .

### CONCLUSION

In this paper, we have investigated the problem of MADM with completely unknown information on attribute weights which the attribute values are given in terms of intuitionistic fuzzy numbers. To determine the attribute weights, a model based on the information entropy, by which the attribute weights can be determined, is established. Then, based on the TOPSIS method, calculation steps for solving intuitionistic fuzzy multiple attribute decision-making problems with completely unknown weight information are given. The weighted Hamming distances between every alternative and positive ideal solution and negative ideal solution are calculated. Then, according to the weighted Hamming distances, the relative closeness degree to the positive ideal solution is calculated to rank all alternatives. Finally, an illustrative example for evaluating the web service quality is given to verify the developed approach and to demonstrate its practicality and effectiveness. In the future, we shall continue working in the application of the intuitionistic fuzzy multiple attribute decision-making to other domains.

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