Thermal instability in anisotropic porous medium saturated by a nanofluid—A realistic approach

Ramesh Chand¹, S.K. Kango², G.C. Rana³

¹Department of Mathematics, Government ARYA College Nurpur, Himachal Pradesh, (INDIA)
²Department of Mathematics, Government College Haripur (Manali), Himachal Pradesh, (INDIA)
³Department of Mathematics, Government College Nadaun, Himachal Pradesh, (INDIA)

MSC: 76E06, 76Exx, 76M40, 76S05
E-mail: rameshnahan@yahoo.com

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ABSTRACT

Thermal instability in a horizontal layer of nanofluid saturated by anisotropic porous medium is investigated for realistic boundary conditions. The flux of volume fraction of nanoparticles is taken to be zero on the isothermal boundaries. The modified Darcy equation that includes the time derivative term used to model the momentum equation. A linear stability analysis based upon normal mode technique is used to study the onset of instabilities of nanofluid saturated by anisotropic porous medium. Rayleigh number on the onset of stationary convection has been derived using Galerkin method and graphs have been plotted for case of stationary convection to study the effects of the thermal anisotropy parameter, mechanical anisotropy parameter, Lewis number, modified diffusivity ratio porosity and nanoparticles Rayleigh number on stationary convection. Oscillatory convection has been ruled under certain condition.

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INTRODUCTION

The subject of thermal instability in porous medium has been studied extensively in recent years. There are many real world applications of thermal instability in porous medium for instance, in geophysics, food processing, oil reservoir modeling, petroleum industry, biomechanics, building of thermal insulations, nuclear reactors and many other areas. Theoretical and experimental results on the stability of cellular convection of a fluid layer in nonporous medium have been given by Chandrasekhar (1961). Lapwood²⁶ has studied the convective flow in a porous medium using linearized stability theory. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding²⁵. A good account of convection problems in a porous medium is given by Vafai and Hadim²⁴ and Nield and Bejan¹⁷. In geothermal system with a ground structure composed of many strata of different permeabilities, the overall horizontal permeability may be up to ten times as large as the vertical component. Processes such as sedimentation, compaction, frost action, and reorientation of the solid matrix are responsible for the creation of aniso-
tropic natural porous media. Anisotropy can also be a characteristic of artificial porous material like pelleting used in chemical engineering process and fiber materials used in insulating purposes.

Nanofluid is the mixture of base fluid such as water or ethylene glycol and other coolants, oil and other lubricants, bio-fluids, polymer solutions etc. with a very small amount of nanoparticles or nanofibres such as metals or metallic oxides (Cu, CuO, Al₂O₃), metal carbides (SiC), nitrides (AlN, SiN) or metals (Al, Cu) etc. having dimensions from 1 to 100 nm. It was Choi [12] who first proposed this term “nanofluid.” The convection of nanofluids based on model of Buongiorno [2] has been studied by Nield and Kuznetsov [18], Nield and Kuznetsov [13,14,19-21], Kuznetsov and Nield [13-15,19,20], Chand and Rana [5-7,10], Chand et al. [3,4,8,9], Chand [3,4] and Rana et al. [10,23] while Agarwal et al. [1] studied the effect of anisotropy on the onset of convection in a porous layer of nanofluid. All these studies based upon Buongiorno model, which incorporates the effects of Brownian motion and thermophoresis. The choice of the boundary conditions imposed in these studies on nanoparticles volume fraction is somewhat arbitrary, it could be argued that zero-flux for nanoparticles volume fraction is more realistic. Recently Nield and Kuznetsov [22] studied the thermal instability of nanofluid in a porous medium by taking normal component of the nanoparticle flux zero at boundary which is more physically realistic. Zero-flux for nanoparticles mean one could control the value of the nanoparticle fraction at the boundary in the same way as the temperature there could be controlled.

In this paper an attempt has been made to study the thermal instability in a horizontal layer of nanofluid in an anisotropic porous medium by imposing nanoparticles flux zero at boundaries.

**MATHEMATICAL FORMULATIONS OF THE PROBLEM**

Consider an infinite horizontal layer of nanofluid of thickness ‘d’ bounded by planes z = 0 and z = d, heated from below in an anisotropic porous medium of medium permeability K and porosity ε as shown in Figure 1. Fluid layer is acted upon by gravity force g (0, 0, -g). The normal component of the nanoparticles flux has to vanish at an impermeable boundary and the temperature T is taken to be T₀ at z = 0 and T₁ at z = d, (T₀ > T₁). The reference scale for temperature and nanoparticles fraction is taken to be T₁ and φ₀ respectively. For simplicity, Darcy’s Law is assumed to be hold and the Oberbeck-Boussinesq is employed. The mathematical equations describing the physical model are based upon the following assumptions.

**Assumptions**

1) Nanoparticles are considered spherical in shape,
2) No chemical reaction in a horizontal layer of fluid,
3) Size of nanoparticles are small as compared to pore matrix,
4) Nanoparticles are spherical;
5) The porous medium is assumed to be possessing isotropy in the horizontal isotropy,
6) The fluid phase and nano particles are in thermal equilibrium state,
7) Radiation heat transfer between the sides of wall is negligible when compared with other modes of the heat transfer,
8) Nanoparticles are being suspended in the nanofluid using either surfactant or surface charge technology, preventing the agglomeration and deposition of these on the porous matrix.

**Governing equations**

According to the works of Chandrasekhar (1961), Nield and Kuznetsov [22] and Agarwal et al. [1], the governing equations in anisotropic porous medium are

\[ \nabla \cdot q = 0, \]  
\[ 0 = -\nabla p + \{\varphi \rho \varphi + (1 - \varphi) \rho (1 - a(T - T_0))\} g - \mu K q, \]  

Figure 1: Geometrical configuration of the problem
where \( \mathbf{q} (u, v, w) \) is the Darcy velocity vector, \( \rho \) the density of nanofluid, \( \rho_p \) density of nanoparticles, \( p \) the hydrostatic pressure, \( \phi \) the volume fraction of the nanoparticles, \( \alpha \) is the coefficient of thermal expansion, \( \mu \) is viscosity and \( \mathbf{K} = K_{ij} \hat{e}^i \hat{e}^j \) the anisotropic permeability tensor; where \( K_x \) denotes the permeability in \( x \)-direction and \( K_z \) permeability in \( z \)-direction.

The energy equation for nanofluid is given by
\[
\frac{\partial T}{\partial t} + \rho c_p \mathbf{q} \cdot \nabla T = \frac{D_p}{\theta} \nabla^2 T + \frac{D_T}{T_i} \nabla^2 T, \tag{3}
\]
where \( (\rho c)_m \) is heat capacity of fluid in porous medium, \( (\rho c)_p \) is heat capacity of nanoparticles and \( k_m \) is thermal conductivity.

The equation of continuity for the nanoparticles is
\[
\nabla \cdot \mathbf{q}_n = 0, \tag{4}
\]
where \( D_B \) is the Brownian diffusion coefficient, given by Einstein-Stokes equation, \( D_T \) is the thermoporetic diffusion coefficient of the nanoparticles and \( \varepsilon \) is porosity in horizontal plane.

We assume that the temperature is constant and nanoparticles flux is zero on the boundaries. Thus boundary conditions\(^{[22]}\) are
\[
w = 0, \quad T = T_k, \quad D_n \frac{\partial \phi}{\partial z} + \frac{D_T}{T_i} \frac{\partial T}{\partial z} = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad w = 0, \quad T = T_k, \quad D_n \frac{\partial \phi}{\partial z} + \frac{D_T}{T_i} \frac{\partial T}{\partial z} = 0 \quad \text{at} \quad z = d. \tag{5}
\]

We introduce non-dimensional variables as
\[
(x', y', z') = \left( \frac{x, y, z}{d} \right), \quad (u', v', w') = \left( \frac{u, v, w}{\kappa x} \right), \quad t' = \frac{t \kappa x}{\sigma d^2}, \quad p' = \frac{p K_{xx}}{\mu K_x}, \quad \phi' = \frac{(\phi - \phi_0)}{\phi_s}, \quad T' = \frac{(T - T_i)}{(T_f - T_i)},
\]
where \( \kappa = \frac{k_m}{(\rho c)_n} \) is effective thermal diffusivity of the fluid.

There after dropping the dashes ('') for simplicity.

Equations (1) - (5) in non-dimensional form can be written as
\[
\nabla \cdot \mathbf{q} = 0, \tag{6}
\]
\[
0 = -\nabla p - \mathbf{q}_n - \nabla \phi, \quad \nabla \phi = \nabla T - \nabla \phi_0, \tag{7}
\]
\[
\frac{\partial T}{\partial t} + \mathbf{q} \cdot \nabla T = \left( \eta \nabla^2 + \frac{\beta^2}{\alpha^2} \right) T + \frac{N_a}{\alpha \theta} \nabla \phi \cdot \nabla T + \frac{N_a N_z}{\alpha \theta} \nabla T \cdot \nabla T, \tag{8}
\]
\[
\frac{1}{\sigma} \frac{\partial \phi}{\partial t} + \frac{\mathbf{q} \cdot \nabla \phi}{\phi_0} = \frac{1}{\alpha \theta} \nabla^2 \phi + \frac{N_a}{\alpha \theta} \nabla^2 T, \tag{9}
\]
where \( \alpha = \frac{K_x}{K_z} \) is Lewis number; \( \gamma = \left( \frac{1}{\xi} u_1, v_1, w_1 \right) \) is anisotropic modified velocity vector; \( \xi = \frac{K_y}{K_x} \) is mechanical anisotropy parameter; \( \eta = \frac{K_{yy}}{K_x} \) is thermal anisotropy parameter; \( \text{Ra} = \frac{\rho g \alpha d K_x (T_f - T_i)}{\mu K_x} \) is Rayleigh Number; \( \text{Rm} = \frac{(\rho_p - \rho_f) \phi_s g d K_x}{\mu K_x} \) is density Rayleigh number; \( \text{Rn} = \frac{\phi_0 (\rho c)_p}{(\rho c)_n} \) is nanoparticles Rayleigh number; \( N_a = \frac{D_p (T_f - T_i)}{D_T T_i \phi_s} \) is modified diffusivity ratio, \( N_a = \frac{\phi_0 (\rho c)_p}{(\rho c)_n} \) is modified particle-density increment.

The dimensionless boundary conditions are
\[
w = 0, \quad T = 1, \quad \frac{\partial \phi}{\partial z} + N_a \frac{\partial T}{\partial z} = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad w = 0, \quad T = 0, \quad \frac{\partial \phi}{\partial z} + N_a \frac{\partial T}{\partial z} = 0 \quad \text{at} \quad z = 1. \tag{10}
\]

Basic solutions

The basic state was assumed to be quiescent and is given by
\[
\mathbf{u} = \mathbf{v} = \mathbf{w} = 0, \quad p = p(z), \quad T = T_z(z) \quad \phi = \phi_z(z).
\]

Equations (6) – (9) using boundary condition (10) give
\[
T_z = 1 - z, \quad \phi_z = \phi_s + N_a z, \tag{11}
\]
where \( \phi_0 \) is reference value for nanoparticles volume fraction.

The basic is identical with solutions obtained by Nield and Kuznetsov\(^{[22]}\) while basic solution for the nanoparticles volume fraction is different than Agarwal et al.\(^{[1]}\).
Perturbation solutions

To study the stability of the system, we superimposed infinitesimal perturbations on the basic state, which are of the forms

\[ q(u,v,w) = 0 + q'(u,v,w), \quad T = T_0 + T', \quad \phi = \phi_0 + \phi', \quad p = p_0 + p', \]

with \( T_0 = 1 - z, \phi_0 = \phi_0 + N \gamma Z \).

Thereafter dropping the dashes ('\) for simplicity.

Using the equation (11) in the equations (6) - (9), we obtain the linearized perturbation (neglecting the product of the prime quantities) equations as

\[ \nabla \cdot q = 0, \]

\[ 0 = -\nabla \cdot q + Ra \tilde{T} - Rn \tilde{\phi}, \]

\[ \frac{\partial T}{\partial z} - w = \left( \frac{\eta \nabla^2 + \frac{\partial^2}{\partial z^2}}{\frac{1}{\left(1 + \frac{\partial^2}{\partial z^2}\right)}} \right) T + N \phi, \]

\[ \frac{1}{\sigma} \frac{\partial \phi}{\partial t} + \frac{1}{\varepsilon} = \frac{1}{\varepsilon} \nabla^2 \phi + \frac{N}{\varepsilon} \nabla^2 T, \]

where \( \nabla^2 \) is two-dimensional Laplacian operator on horizontal plane.

NORMAL MODES AND STABILITY ANALYSIS

Analyzing the disturbances into the normal modes and assuming that the perturbed quantities are of the form

\[ w(z, T, \phi) = [W(z), \Theta(z), \Phi(z) \exp(ik_x x + ik_y y + n t)], \]

where \( k_x, k_y \) are wave numbers in \( x \) and \( y \) directions respectively and \( n \) is growth rate of disturbances.

Using equation (18), equations (17), (15) – (16) become

\[ \left( \frac{1}{\xi} \frac{D^2}{Dz^2} \right) W + a^2 Ra \Theta - a^2 Rn \Phi = 0, \]

\[ W + \left( D^2 - \eta a^2 - n + \frac{N \phi}{\varepsilon} D - \frac{2N \phi}{\varepsilon} D \right) \Theta - \frac{N \phi}{\varepsilon} D \Phi = 0, \]

\[ \frac{W}{\varepsilon} \left( D^2 - \frac{N \phi}{\varepsilon} D \right) \Theta - \left( \frac{1}{\varepsilon} \frac{D^2}{Dz^2} - \frac{n}{\sigma} \right) \Phi = 0. \]

Boundary conditions are

\[ w = 0, \quad T = 0, \quad \frac{\partial \phi}{\partial z} + N \phi = 0 \quad \text{at} \quad z = 0, 1. \]

Where \( D = \frac{d}{dz} \) and \( a = \sqrt{k_x^2 + k_y^2} \) is dimensionless horizontal resultant wave number.

METHOD OF SOLUTION

The Galerkin weighted residuals method is used to obtain an approximate solution to the system of equations (19) – (21) with boundary conditions (22). In this method, the test functions are the same as the base (trial) functions. Accordingly \( W, \Theta \) and \( \Phi \) are taken as

\[ W = \sum_{p=1}^{N} A_p \Theta_p, \quad \Theta = \sum_{p=1}^{N} B_p \Theta_p, \quad \Phi = \sum_{p=1}^{N} C_p \Phi_p. \]

where \( A_p, B_p \) and \( C_p \) are unknown coefficients, \( p = 1, 2, 3, \ldots, N \) and the base functions \( W_p, \Theta_p \) and \( \Phi_p \) are assumed in the following form

\[ \Theta_p = \Theta(z), \quad \Phi(z) = z^p - z^{p+1}, \quad \phi_p = N(z^p - z^{p+1}) \]

\[ \Theta_p = \frac{1}{2} N_z z^p, \quad p = 2, 3, 4, \ldots \]

such that \( W_p, \Theta_p \) and \( \Phi_p \) satisfy the corresponding boundary conditions. Using expression for \( W, \Theta \) and \( \Phi \) in equations (19) – (21) and multiplying first equation by \( W_p \), second equation by \( \Theta_p \) and third by \( \Phi_p \) and integrating in the limits from zero to unity, we obtain a set of \( 3N \) linear homogeneous equations in \( 3N \) unknown \( A_p, B_p \) and \( C_p \); \( p = 1, 2, 3, \ldots, N \). For existing of non trivial solution, the vanishing of the determinant of coefficients produces the characteristics equation of the system in term of Rayleigh number \( Ra \).

STABILITY ANALYSIS

We confine ourselves to the one- term Galerkin approximation. Thus eigenvalue equation is given by

\[ a^2 Ra \left( a^2 + 10 + \frac{n \sigma}{\varepsilon} \right) + a^2 Rn \left( N \right) \left( a^2 + 10 + \frac{N \phi}{\varepsilon} \right) + \left( a^2 + 10 \right) \left( a^2 + 10 + \frac{n \sigma}{\varepsilon} \right) = 0. \]

For neutral stability the real parts of the \( n \) is zero. Hence on putting \( n = i \omega \) (where \( \omega \) is real and dimensionless quantity) in equation (25), we have

\[ \text{...} \]
Stationary convection

For stationary convection $\omega = 0$ with one-term Galerkin approximation, equation (25) reduces to

$$Ra_{\text{c}}^{*} = 10 \left( \frac{\sqrt{\eta_{\xi}} + 1 + \eta_{\xi} + \sqrt{\eta_{\xi}}}{\xi_{\eta}} \right).$$

For isotropic porous medium i.e. if $\xi = 1$, the critical wave number $a_{\text{c}} = \sqrt{10}$ and corresponding critical Rayleigh number $Ra_{\text{c}} = 40$.

Thus in the absence of nanoparticles [$Rn = Le = N_{A} = 0$] for the case of isotropic porous medium [$\xi = 1$] the critical Rayleigh number is given by $Ra_{\text{c}} = 40$, which slightly greater than critical Rayleigh number $Ra_{\text{c}} = 4\pi^{2}$, result obtained by Lapwood (1948) for regular fluid.

Oscillatory convection

For oscillatory convection we have $\omega \neq 0$, thus on equating the real and imaginary parts of equation (26), we have

$$a^{2}Ra(a^{2} + 10) + a^{2}Rn(a^{2} + 10)N_{A} + (\eta a^{2} + 10)\frac{Le}{\xi} \leq 0.$$ 

In order for $\omega$ to be real it is necessary that

Thus oscillatory convection has been ruled out if

$$a^{2}Ra(a^{2} + 10) + a^{2}Rn(a^{2} + 10)N_{A} + (\eta a^{2} + 10)\frac{Le}{\xi} \leq 0.$$ 

As we have noted that for typical nanofluid $Le$ is of order $10^{-2} - 10^{2}$ and $N_{A}$ is not greater than 10, $Rn$ are of order $10^{-2}$ and $\xi$, $\eta$ and $\xi_{\eta}$ are of the order $10^{-3} - 10^{-1}$. Under these approximations inequality (31) does not hold if

$$a^{2}Ra(a^{2} + 10) + a^{2}Rn(a^{2} + 10)N_{A} + (\eta a^{2} + 10)\frac{Le}{\xi} \geq 0.$$ 

and hence $\omega$ is not real.

Thus oscillatory convection has been ruled out if

$$a^{2}Ra(a^{2} + 10) + a^{2}Rn(a^{2} + 10)N_{A} + (\eta a^{2} + 10)\frac{Le}{\xi} \geq 0.$$
To study the influence of thermal anisotropy parameter $\eta$, mechanical anisotropy parameter $\xi$, Lewis number $L$, modified diffusivity ratio $N_A$ and porosity $\varepsilon$ on the stationary convection for the case of bottom-heavy distribution of nanoparticles [negative value of $R_n$], we examine the behaviour of $\frac{\partial Ra}{\partial \eta}$, $\frac{\partial Ra}{\partial \xi}$, $\frac{\partial Ra}{\partial Le}$, $\frac{\partial Ra}{\partial N_A}$ and $\frac{\partial Ra}{\partial \varepsilon}$ analytically.

From equation (27), we have

$$Ra = \frac{1}{a^2} \left( a^2 + \frac{10}{\xi} \right) \left( \eta a^2 + 10 \right) \left( \eta a^2 + 10 \right) \frac{Le}{\varepsilon} R_n.$$ 

1) $\frac{\partial Ra}{\partial \eta} = \left( \frac{a^2 + 10}{\xi} \right) \frac{Le}{\varepsilon} R_n$. As we have noted that for typical nanofluid $L$ is of order $10^2$-$10^3$, $\varepsilon$, $\xi$ and $\eta$ are of the order $10^3$-$10^4$ and $R_n$ < 0 [for a bottom-heavy distribution of nanoparticles]. Under these approximations $\frac{\partial Ra}{\partial \eta} > 0$, thus thermal anisotropy parameter $\eta$ has stabilizing effect on the stationary convection.

2) $\frac{\partial Ra}{\partial \xi} < 0$, thus mechanical anisotropy parameter $\xi$ destabilizes the stationary convection.

3) $\frac{\partial Ra}{\partial \varepsilon} < 0$, thus porosity destabilizes the stationary convection.

4) $\frac{\partial Ra}{\partial Le} > 0$, thus Lewis number stabilizes the stationary convection.

5) $\frac{\partial Ra}{\partial N_A} > 0$, which mean that modified diffusivity ratio stabilizes the stationary convection.

**RESULTS AND DISCUSSION**

The thermal instability in a horizontal layer of nanofluid is investigated. The expression for the stationary Rayleigh number is given by equation (27). The graphical representation of the effects of anisotropic parameters on stationary convection shows the stationary Rayleigh number and different values of the porosity and it is found that the Rayleigh number decreases with increase in the value of porosity thus porosity destabilizes the stationary convection. This is good agreement of the result obtained by Chand and Rana[5].

Figure 5 shows the variation of stationary Rayleigh number $Ra$ with wave number $a$ for for the fixed value $Le = 500$, $\eta = 0.7$, $\xi = 0.4$, $N_A = 5$, $R_n = -1$ and different value of the porosity and it is found that the Rayleigh number decreases with increase in the value of porosity thus porosity destabilizes the stationary convection. This is good agreement of the result obtained by Chand and Rana[5].

Figure 5 shows the variation of stationary Rayleigh number $Ra$ with wave number $a$ for for the fixed value $Le = 500$, $\eta = 0.8$, $\xi = 0.4$, $N_A = 5$, $R_n = -1$ and different value of the porosity and it is found that the Rayleigh number decreases with increase in the value of porosity thus porosity destabilizes the stationary convection. This is good agreement of the result obtained by Chand and Rana[5].

Figure 2 indicates the effect of thermal anisotropy parameter $\eta$ on the stationary convection for the fixed value of $Le = 500$, $\eta = 0.6$, $\varepsilon = 0.4$, $N_A = 5$, $R_n = -1$ and it is found that the stationary Rayleigh number increases with increase in the value of thermal anisotropy parameter $\eta$, indicating that the effect of thermal anisotropy parameter $\eta$ is to inhibit the onset of convection.

Figure 3 indicates the effect of mechanical anisotropy parameter $\xi$ on the stationary convection for the fixed value of $Le = 500$, $\eta = 0.8$, $\varepsilon = 0.4$, $N_A = 5$, $R_n = -1$ and it is found that the stationary Rayleigh number decreases with increase in the value of mechanical anisotropy parameter $\xi$, thus mechanical anisotropy parameter $\xi$ is to advance the onset of stationary convection.

Figure 4 shows the variation of Rayleigh number with wave number for the fixed value of $Le = 500$, $\eta = 0.8$, $\xi = 0.4$, $N_A = 5$, $R_n = -1$ and different value of the porosity and it is found that the Rayleigh number decreases with increase in the value of porosity thus porosity destabilizes the stationary convection. This is good agreement of the result obtained by Chand and Rana[5].
Figure 3: Variation of stationary Rayleigh number with wavenumber for different values of mechanical anisotropy parameter $\xi$.

Figure 4: Variation of stationary Rayleigh number with wavenumber for different values of porosity $\epsilon$.

Figure 5: Variation of stationary Rayleigh number with wavenumber for different values of Lewis number.

Figure 6: Variation of stationary Rayleigh number with wavenumber for different values of modified diffusivity ratio $N_A$.

Figure 4 shows the variation of stationary Rayleigh number $Ra$ with wave number $a$ for fixed values of $\eta = 0.7$, $\xi = 0.4$, $\epsilon = 0.5$, $N_A = 5$, $Rn = -1$ and different values of Lewis number. It is found that the Rayleigh number $Ra$ increases as values of Lewis number increases. Thus, Lewis number stabilizes the stationary convection. This is good agreement of the result obtained by Chand and Rana (2012a).

Figure 6 shows the variation of stationary Rayleigh number $Ra$ with wave number $a$ for fixed value of $\eta = 0.7$, $\xi = 0.4$, $\epsilon = 0.5$, $Le = 500$, $Rn = -1$ and different values of modified diffusivity ratio $N_A$. It is found that the Rayleigh number $Ra$ increases as values of modified diffusivity ratio increases. Thus, modified diffusivity ratio stabilizes the stationary convection. This is good agreement of the result obtained by Chand and Rana [5].

NOMENCLATURE

- \(a\): wave number
- \(d\): depth of fluid layer
- \(D_B\): diffusion coefficient
- \(D_T\): thermophoretic diffusivity coefficient
- \(g\): gravity force
- \(K\): Permeability tensor
- \(K_{x^*}\): permeability in $x$-direction
- \(K_{z^*}\): permeability in $z$-direction
- \(Le\): Lewis number
- \(n\): growth rate of disturbances
- \(N_A\): modified diffusivity ratio
- \(N_B\): modified particle-density increment
CONCLUSIONS

Thermal instability in a horizontal layer of nanofluid an in anisotropic porous medium is investigated. The influences of anisotropic parameters and other parameters on the stationary have been investigated both analytically and graphically.

The main conclusions are:

1) Basic solution for the nanoparticles volume fraction is changed with zero-flux of volume fraction of nanoparticles.

2) Presence of nanoparticles decreases the stability of system.

3) Oscillatory convection has been ruled out if

\[ a^2Ra(a^2 + 10) + a^2Rn(\rho + 10) + \frac{\rho a^2 + 10}{\rho_N} \geq \]

4) The presence of the nanoparticles lowers the value of the critical Rayleigh number by usually by substantial amount. Thus nanofluid is less stable as compared to regular fluid.

5) Thermal anisotropy parameter, Lewis number and modified diffusivity ratio stabilize the stationary convection for the case of bottom-heavy distribution of nanoparticles [negative value of Rn].

6) Porosity and mechanical anisotropy parameter destabilizes the stationary convection for the case of bottom-heavy distribution of nanoparticles [negative value of Rn].

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