



## Full Paper

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## The steady flow cosmological model

### Abstract

The possibility that the cosmological constant is decaying as the universe grows is explored, and we define a cosmological parameter, depending of the vacuum energy and the universe radius, which should be presently ca. 122 orders of magnitude smaller than at the Planck epoch. From it, a new version of the Friedmann equation for a flat universe is obtained, which allows the estimation of the Hubble parameter at any epoch and the reconstruction of the expansion history. The main result is a quasi-linear expansion dynamics in concurrence with a number of previous works. This behavior is compatible with the main features of observational cosmology and avoids the horizon, flatness, cosmological constant, coincidence and age problems without the need of neither inflation nor initial fine-tuning.

### Key Words

Cosmological constant; Hubble flow; Primordial nucleosynthesis; Cosmology: Theory; Early universe.

## INTRODUCTION

During decades the expansion of the universe was assumed to be decelerated by gravitation and the Einstein–de Sitter model without cosmological constant ( $\Lambda$ ) was favored as the Standard cosmology (see e.g. curve  $\Omega_M=0.3$  in Figure 1). However, measurements of type Ia supernovae (SNe Ia) demonstrated that their luminosity distance is larger than expected in such a model<sup>[42,43,45,46,57,62]</sup>. In order to fit these observations within a new Standard model, a positive  $\Lambda$  was reintroduced in the Friedmann equations and a repulsive dark energy, derived from the concept of cosmological constant, has been postulated to drive an slightly accelerated expansion of the universe in our epoch.

The resulting Concordance model, gives a quite elaborate picture for the dynamics of the universe: an inflationary period, right after the initial creation event, with an exponential acceleration, followed by a long lasting deceleration era (including 2 different expansion regimes depending on radiation or matter dominance) and, since redshift  $z \sim 1$ , a new era of tiny acceleration driven by dark energy (see upper curve of Figure 1). In this de-

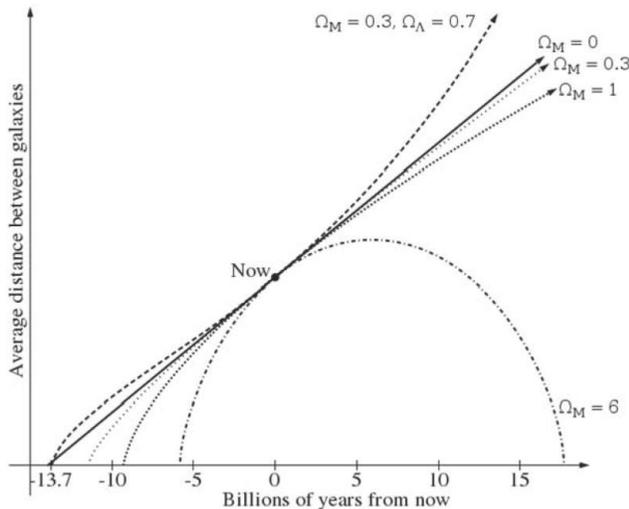
scription the role of  $\Lambda$  is still unclear since it appears and disappears as required. Even so, a possible relationship between a minuscule  $\Lambda$  today and a large cosmological term driving inflation, along with the important number of works on the subject<sup>[47]</sup>, advise considering seriously the case for a  $\Lambda > 0$ .

The current value of  $\Lambda$  obtained from cosmological data is of the order of  $10^{-9} \text{ J m}^{-3}$ . At a theoretical level, dark energy was formerly identified with the vacuum energy, which is expected to arise out of zero-point quantum vacuum fluctuations of several fundamental fields. In Quantum Field theory (QFT), these fluctuations would have Planck energy density, i.e. about  $10^{113} \text{ J m}^{-3}$ . So, the discrepancy between theory and observations is of 122 orders of magnitude. Such a huge gap constitutes the cosmological constant problem<sup>[63]</sup>. The nature and composition of dark energy remains one of the major challenges of modern cosmology.

As a related issue, the cosmic coincidence problem wonders why the density of matter, which decreases as the universe expands, and  $\Lambda$ , which should be constant by definition, are comparable particularly at present times. The radiation energy density and the  $\Lambda$  energy density

should be fine-tuned to an accuracy of one part in  $10^{122}$  at the Planck time in order to ensure this coincidence<sup>[39,41]</sup>.

This work shows that the postulate of the same scaling law for  $\Lambda$  and for dark matter density allows circumventing the cosmological constant and the coincidence problems, and leads to an important simplification of the expansion dynamics of the universe. Though this postulate could be seen as artificial, the introduction of a cosmological constant is not less artificial. Thus, it would be not legitimate to set aside this possibility without a previous analysis of it. Anyway, we focus our interest in the consequences of the postulate rather than to justify it *a priori*. Therefore, we will briefly study how our hypothesis influences some of the most salient features of cosmological interest. In section 2 we describe a new large numbers hypothesis (LNH) and define a cosmological parameter depending of the universe scale. In section 3 the Friedmann equation is modified for the case of a flat FRW metrics; the value of the Hubble parameter and a quasi-linear expansion dynamics are so derived. In section 4 the main features of the resulting Steady Flow model are discussed and compared with the Standard model, and in Section 5 we check both models against observations. Finally, section 6 summarizes the conclusions.



**Figure 1 :** Time evolution of the scale factor in arbitrary units for different cosmological models as function of the  $\Omega_M$  and  $\Omega_\Lambda$  contributions.  $\Omega_M$  and  $\Omega_\Lambda$  refer to the ratios between the assumed densities, of mass and  $\Lambda$  respectively, and the critical density for a flat universe.

### A MINIMAL LARGE NUMBERS HYPOTHESIS AND A COSMOLOGICAL PARAMETER $\Lambda_R$

Different authors have recently introduced new sets of huge dimensionless numbers in a similar way to the earliest LNH discussed by Eddington, Dirac and his contemporaries. Various numbers of the order of  $10^{60}$  were obtained by Shemi-Zadeh<sup>[51]</sup> through measuring cosmo-

logical parameters in natural Planck units. The relevance of the Planck scale to LNH was also shown by Marugán and Carneiro<sup>[34]</sup>, who claimed that the relations between large numbers can be explained by the holographic principle assuming that the present energy density is dominated by  $\Lambda$ . Other authors have discussed the significance of yet a bigger number, ca.  $10^{120}$ . Weizsaecker obtained both big numbers from his interpretation of quantum theory in terms of information and identified  $10^{120}$  with the sum of elementary bits of information in the universe<sup>[32]</sup>. Casado<sup>[12]</sup> obtained 3 pure numbers of order  $10^{61}$  and 5 additional numbers of order  $10^{122}$ , all of them derived from relations between the radius,  $R_o$ , the mass,  $M_o$ , and the age,  $t_o$ , of the observable universe at present, and the respective quantum scales given by Planck units:

$$\frac{R_o}{l_p} \approx \frac{t_o}{t_p} \approx \frac{M_o}{m_p} \approx 10^{61}, \tag{1}$$

According to this work, the entropy of the universe is, again,  $10^{122}$  in units of Boltzmann constant. Sidharth<sup>[53]</sup> interpreted the universe as a collection of ca.  $10^{120}$  Planck oscillators. The fact that the number  $10^{122}$  can be represented in such a variety of ways has been considered as a new LNH by Funkhouser<sup>[20]</sup>, who claimed to have resolved these coincidences without departing from the Standard cosmology.

It is tempting to attribute to  $10^{122}$ , probably the largest dimensionless quantity with a physical meaning, a relevant significance in cosmology. For instance, it coincides with the maximum number of elementary quantum logic operations that the universe can have performed, as independently calculated by Lloyd<sup>[31]</sup>. Furthermore, the ratio of the mass-energy in the observable universe to the energy of a photon with a wavelength  $\lambda_o$  can be easily calculated to be ca.  $10^{121}$ . Then, taking into account equation (1), the ratio of Planck mass to the energy of such a photon should be, once more, of the order of  $10^{60}$ .

Although some of these large numbers of order 122 are not independent, they all do resemble each other to an extent which allows us to conjecture a deeper underlying principle or connection. If we assume that this pure numbers coincidence is not a mere accident, it seems judicious to explore whether these numbers are providing some significant information on our universe. The present LNH only assumes a direct connection between quantum physics, related to Planck units, and cosmology. This cannot be shown a priori from any existing theory, but its formal elegance invites to pay attention to its possible consequences. Notice that the present discussion does not deal with particle masses, charges, radius or the fine structure constant, which have been frequently involved in previous works on different LNH. That's why the present conjecture can be considered as a *minimal* LNH.

Particularly, we analyse the possibility that  $\Lambda$  could be a scale-dependent quantity in the light of such a LNH. In fact, QFT and some quantum gravity theories have been considering  $\Lambda$  as a dynamical quantity for decades (e.g. Bergmann 1968, Bertolami<sup>[9]</sup>). According to eq. (1),  $10^{61}$  is the order-of-magnitude ratio between the radius of the universe  $R_0$  -both the apparent and the Hubble horizons coincide for flat FRW models<sup>[34]</sup> and the Planck length  $l_p$ , usually identified with the size of the observable universe at the Planck epoch. Taking into account the dimensions of  $\Lambda$ , which is the inverse of the square of a length, it seems natural that  $10^{122} = (10^{61})^2 = (R_0/l_p)^2$  could be a scaling factor of the vacuum energy density. Thus, postulating that the cosmological *constant* is decreasing as universe grows, we can define a *cosmological parameter*  $\Lambda_R$ :

$$\Lambda_R = \frac{\Lambda_P^2}{R^2}. \quad (2)$$

The possibility that  $\Lambda$  varies as  $a^2$  ( $a$  is the scale factor.  $R \propto a$  in our model, as we will see) has been already discussed by different authors<sup>[2,4,11,13,35,38]</sup>. For instance, Chen and Wu<sup>[13]</sup> developed one of these models on the grounds of dimensional considerations in line with quantum cosmology. Pavón<sup>[40]</sup> performed a thermodynamic analysis of non-equilibrium fluctuations of different possible  $\Lambda$  decays, concluding that if  $\Lambda$  diminishes with cosmic expansion, its dependence on scale factor should take the form  $\Lambda \propto a^2$  to avoid conflict with the high degree of isotropy of the cosmic background radiation (CBR). This dependence is also consistent with a dark energy satisfying the thermodynamic equation  $p = -(\rho/3)$ , which could rise, for instance, in a model based on cosmic string loops<sup>[27]</sup>.

In addition, there are studies on empirical models in which the value of the exponent following the law  $\Lambda \propto a^n$  is not fixed *a priori*. Ages of these universes have been calculated and are consistent with observation if  $n < 3$ <sup>[36]</sup>. The power spectrum of matter density perturbations does not appear to be greatly modified by a decaying  $\Lambda$ , at least for  $0 \leq n \leq 2$ <sup>[54]</sup>. Lensing statistics combined with other tests involving CBR anisotropies and the magnitude redshift relation for SNe Ia, favour models with  $n \geq 1.6$ <sup>[55]</sup>. Therefore,  $n=2$  is the only natural exponent unconstrained by all these observations.

This kind of dependence suggests that the holographic principle could be applicable not only to the entropy of the observable universe, but also to its energy density and, in particular, to the evolution of  $\Lambda$ <sup>[23,48,61]</sup>. If  $M \propto R$ , as equation (1) suggests<sup>[12]</sup>, and thus the total density of matter follows the same scale law,  $\rho_M \propto R^{-2}$ , by any reason (for instance, following the holographic principle or perhaps by formation of new matter at a very tiny rate), we can similarly define:

$$\rho_M = \frac{\Sigma_P^2}{R^2}, \quad (3)$$

where  $\Sigma$  is a constant that could be regarded as a primeval matter density. Baryonic matter seems not to obey the above relationship since its density evolves as  $a^{-3}$  (unless new ordinary matter appears, but this should imply some conflicting changes in primordial nucleosynthesis ratios), so that we propose that mass associated to dark matter is the major responsible of equation (1). Anyway, ordinary matter only accounts for ca. 4% of the universe content, so that we can disregard it as a first approximation.

In this scenario, the coincidence problem could also be avoided, or at least greatly alleviated. A straightforward consequence of equations (2) and (3) is that the present coincidence of mass and dark energy densities is not a mere chance. It appears to be the natural result of some long lasting coupling between dark matter and dark energy, since both of them show the same dependence on the universe scale. Thus, the coincidence problem reduces to explain why the amounts of dark matter and dark energy have been of the same order of magnitude for most of the time since both coexist. Previous works have shown that some coupling between dark matter and dark energy can ease the coincidence problem, and such a coupling is favored by observations<sup>[5,6,16,48,59,66]</sup>. This close connection could even suggest some relationship between the nature of dark matter and dark energy. For instance, dark energy, whose total amount increases as space expands, could be decaying into dark matter<sup>[1,48,61]</sup>.

## A MODIFIED FRIEDMANN EQUATION

Accurate measurements of the angular power spectrum of anisotropies in the CBR have shown that the universe curvature is very close to flatness (e.g. Bernardis et al. 2001). In a flat universe the first Friedmann equation can be simplified since the curvature term disappears ( $k=0$ ), leading in our case to the reduced version:

$$H^2 = \frac{8\pi G\rho_M + \Lambda_R c^2}{3}, \quad (4)$$

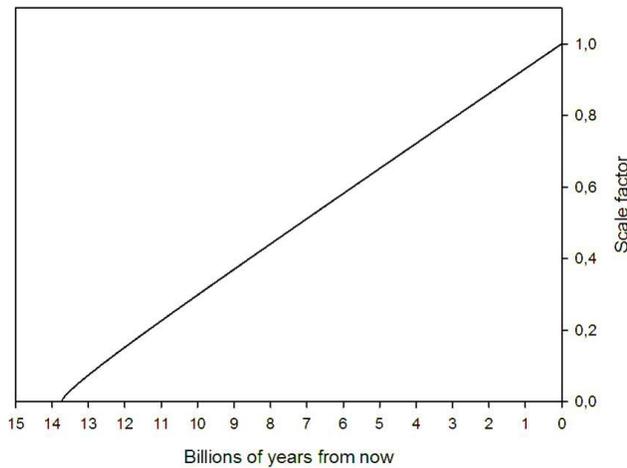
where now  $\Lambda_R$  stands for the cosmological parameter defined in equation (2). We can obtain the approximate value of the cosmological parameter at Planck epoch ( $\Lambda_P$ ) from first principles taking into account that from QFT the density of energy of vacuum was of the order of the Planck density, which in the appropriate units reads:

$$\Lambda_P \approx \rho_P G / c^2 = c^3 / \hbar G = 1/l_p^2 \approx 10^{70} \text{ m}^{-2} \quad (5)$$

Now, we can calculate a semi-empirical value of the Hubble parameter at any epoch from the empirical matter density and the expansion factor of the observable universe since the Planck epoch. For the present time we can use an approximate matter density of  $10^{-27} \text{ kg/m}^3$  and the above mentioned large number  $10^{61}$ , respectively. Using equations (2) and (4), we obtain in round

numbers  $H_0 = 2 \cdot 10^{-18} \text{ s}^{-1}$ , which corresponds to ca.  $60 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , in remarkable agreement (considering the roughness of the input data) with the observational consensus of ca.  $70 \text{ km s}^{-1} \text{ Mpc}^{-1}$  (e.g. Spergel<sup>[56]</sup>, Jarosik<sup>[26]</sup>). This is not trivial, since had  $\Lambda_p$  been either smaller or bigger, a very different value for  $H_0$  would have been obtained. We could equally well live in an epoch where the universe was smaller or larger, with the same consequence. After all, both  $\Lambda_p$  (a time-independent constant) and  $R_0$  are properties that should not depend on the rate at which the universe expands.

Similarly, the Hubble parameter can be calculated for any other scale of the universe. For instance, when the universe was 10 times smaller,  $H$  was 10 times larger, leading to the general result  $H \propto a^{-1}$ . In other words, the simplest solution of equation (4) implies  $da/dt$  is constant. Therefore, according to these results the Hubble flow is steady, neither decelerated nor accelerated. That's the reason why the present hypothesis is called Steady Flow model. Its main feature is a linear expansion with  $a$   $t$ , at least during the last 13 billion years, subsequent to the domination of radiation and baryonic matter in the early universe (Figure 2).



**Figure 2 :** Detailed time evolution of the scale factor for the steady flow model. Notice a non-vanishing curvature at early stages of expansion, before dark matter and dark energy became the dominating components.

Until now we have considered the long lasting eras when the dark sector has been dominating the expansion rate. However, given the different dependence of radiation and ordinary matter on the scale factor, these sectors of energy density successively dominated the dynamics of the universe during the early expansion, leading to well-known expansion regimes. Let's now look at the Friedmann equation containing also these terms.

Since the radiation density decays as  $a^{-4}$  and ordinary matter density varies as  $a^{-3}$ , the complete Friedmann equation, after multiplication of all terms by the factor  $a^2$ , reads:

$$\left(\frac{da}{dt}\right)^2 = \frac{8\pi G(\rho_r/a^2 + \rho_m/a + \rho_M) + \Lambda_0 c^2}{3}, \quad (6)$$

where  $\rho_r$  is the radiation density and  $\rho_m$  is the ordinary matter density. Both,  $\rho_M$  and  $\Lambda_0$  refer to their present values. In order to perform a numerical integration of equation 6, we have taken  $\rho_r = 4.64 \cdot 10^{-31} \text{ kg/m}^3$ ,  $\rho_m = 2 \cdot 10^{-28} \text{ kg/m}^3$  (20% of dark matter density), and we have corrected our theoretical calculation of  $H_0$  ( $60 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ) by applying a factor close to 1 in order to match its empirical value. The result of such integration is depicted in Figure 2 and the resulting implications will be discussed in the following section.

Before finishing this section however, let's call attention to the fact that even in case of  $\Lambda=0$ , equation 6 implies a linearly expanding universe at late times, although the density of (dark) matter should be substantially increased in order to obtain a realistic value of  $H_0$ .

### FEATURES OF THE STEADY FLOW MODEL

Let's now discuss the implications of the present model, whose main feature is a linear expansion, except for the very early universe. In this scenario the expansion time follows the well known relationship  $t=1/H$  at almost any epoch. This time should be  $13.7 \cdot 10^9$  years at present (Figure 2). Amazingly, this value coincides with the universe age obtained from the Concordance model<sup>[58]</sup>, as shown by the time axis interceptions of the two upper curves in Figure 1. Notice however that, in contrast to Standard cosmology, the Steady Flow model avoids the use of free parameters or adjustable functions to fit the observations. Note also that the present time is the only time in the Concordance scenario when  $t_0=1/H_0$ . Is this just another fortuitous coincidence? Or perhaps one more hint indicating that  $t=1/H$  may well be right for most of the expansion history?

The horizon problem vanishes in the Steady Flow model, as in any other linear expanding cosmology, since particle horizons only occur in models with  $a(t) \propto t^\alpha$  for  $\alpha < 1$ . A linear evolution of the expansion is also clean of the flatness or the fine-tuning problems<sup>[7,14,15,17]</sup>. The scale factor in power-law models with  $\alpha \geq 1$  does not constrain the density parameter and consequently, they are free from the flatness problem. So, although a still uncertain mechanism, perhaps inflation, is obviously required to trigger the expansion, inflation is not needed anymore to solve the classical problems that motivated its introduction.

This model allows for the smooth formation of first galaxies, clusters and voids during a longer time than the Concordance model. For similar reasons, longer times for the development of structure seeds observed in CBR

are obtained. One can calculate that recombination occurred at an expansion time of ca.  $2 \cdot 10^6$  years, as opposed to  $3.8 \cdot 10^5$  years in the Concordance model. The Hubble radius at recombination is therefore almost one order of magnitude greater for the Steady Flow model. This fact, coupled with the absence of any horizon, could well have falsified the model. Any concordance with observations, which will be checked in next section, is therefore very significant. On the other hand, some results of Big Bang cosmology, such as the evolution of temperature with the scale factor, the cosmic recombination phenomenology or the primordial nucleosynthesis (as we will see in next section), remain essentially unchanged.

To ascertain the evolution of temperature of the CBR as function of  $a$  we take the classical result of blackbody radiation thermodynamics:

$$\frac{S}{V} = \frac{4}{3} b T^3 \quad (7)$$

where  $S$  is the entropy,  $V$  is the volume and  $b$  is the blackbody constant. Then, since  $V \propto a^3$ , in an adiabatic expansion one has that the product  $aT$  is constant. Therefore, the present model agrees with the standard one in the way that temperature of the CBR decreases as the cosmic scale increases, i.e.:

$$\frac{T_0}{T} = a \quad (8)$$

where  $T_0$  denotes today's temperature. Notice, however, that the evolution of temperature with time is different than in the Standard scenario.

A number of models, pioneered by the well-known Milne universe and developed on different theoretical grounds and assumptions, have also arrived to linear expansion laws (e.g. Dev<sup>[14]</sup>, Gehlaut<sup>[21]</sup>, Petri<sup>[44]</sup>, Benoit-Lévy and Chardin<sup>[8]</sup>). In fact, a linear expanding cosmology, independent of the equation of state of matter, is a generic feature in a class of models that attempt to dynamically solve the cosmological constant problem<sup>[15,17,63]</sup>. For example, John and Joseph<sup>[29]</sup> generalized the Chen-Wu ansatz mentioned in section 2 to the total energy density of the universe. The resulting model has a linear expansion and is devoid of most of the cosmological problems, but it predicts the continuous creation of mass at low rates and a mass density well above the observational limits to avoid serious contradictions with Big Bang nucleosynthesis (BBN). Notice, by the way, that if this creation of mass was in the form of dark matter, as Steady Flow model proposes, the conflict with nucleosynthesis could be avoided.

Sethi *et al.*<sup>[49]</sup> have considered a generic empirical model where the scale factor depends on time with a power law ( $a(t) \propto t^\alpha$ ,  $\alpha$  being a free parameter), concluding that cosmological observations point to  $\alpha=1$  as the best-fit solution. Let's now see how the predictions of

Concordance model and our proposal compare with the main observational facts.

## OBSERVATIONAL CONSTRAINTS

So far, the main direct evidence favoring an apparent acceleration of the expansion in recent times comes from distant supernovae Ia (SNe Ia) observations<sup>[10,42,43,45,46,57,62]</sup>.

The best fit curve of SNe Ia data suggest a tiny acceleration of the expansion at present times, but these data are also compatible with an universe that expands linearly without either deceleration or acceleration of the Hubble flow ( $q=0$ ) (Sethi *et al.*<sup>[49]</sup>, see also Figure 1 in Gehlaut *et al.*<sup>[21]</sup>), i.e. with the prediction of the Friedmann equations for an empty universe (line  $\Omega_M=0$  in Figure 1). This fact was also recognized by Perlmutter *et al.* in their seminal work<sup>[42]</sup>.

Take for instance the data summarized by Tonry *et al.*<sup>[57]</sup> in a residual Hubble diagram with respect to an empty universe (Figure 8 therein). A detailed inspection of that plot evidences the following characteristics:

- Average error bars are larger for data at higher redshift, particularly for  $z > 0.1$ .
- Several error bars are underestimated and should be larger because no single smooth curve can be drawn along all of them.
- Overall data fit to a universe of null density as well as to the Concordance model. While the differences between both models are never higher than 0.2 magnitudes, deviations of at least 0.4 magnitudes from any of them are frequent among plotted data points.

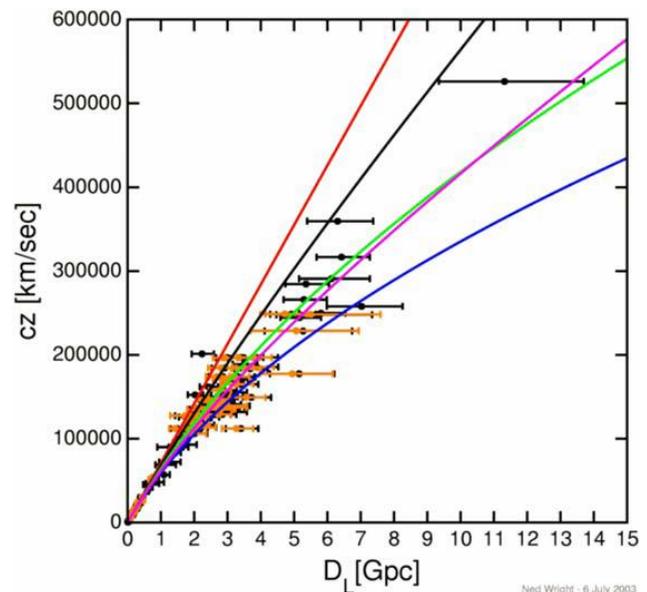


Figure 3 : Recession velocities vs. luminosity distance for SNe Ia. The curves show a closed universe ( $\Omega = 2$ ) in red, the critical density universe ( $\Omega = 1$ ) in black, the empty universe ( $\Omega = 0$ ) in green, the Steady State model in blue, and the Concordance model with  $\Omega_M = 0.27$  and  $\Omega_\Lambda = 0.73$  in purple.

Another plot of similar data, shown in figure 3, shows two data sets: Wang et al.<sup>[62]</sup> in orange and Tonry et al.<sup>[57]</sup> in black, adapted by Wright<sup>[64]</sup>. Once again, the scattering of data and the error bars are much larger than the differences between the Concordance model (purple curve) and the empty universe (green curve). In fact, the scattering of these data only allows empirically ruling out the closed universe (red curve) and the Steady State model (blue curve). Besides, the case for an Einstein De Sitter model (black curve) could not be reconciled with SNe Ia data either<sup>[10]</sup>. On the other hand, a linearly expanding universe has not been conclusively discarded so far by these results and it can be consistent with supernovae observations up to  $z=1$  if we take into account a reasonable size of systematic errors<sup>[8]</sup>.

Obviously the universe has a non-zero density, so that the curve for an empty universe shown by Perlmutter, Tonry and other authors has been considered so far as a mere approximation or a reference. However this curve could be indeed a good description of the actual expansion kinematics.

The measurements of peak brightness of these remote supernovae explosions are extremely difficult and require several corrections. Moreover, there are systematic differences in the corrections made for the same objects by different groups of observers<sup>[30]</sup>. Considering these troubles, the self-consistency of the data is remarkable. Nevertheless, especially the decelerated expansion at  $z > 1$  is still based in too few observations to be considered as conclusively demonstrated.

In addition, Schwarz and Weinhorst<sup>[52]</sup> have found an unexpected anisotropy of the Hubble diagram between both galactic hemispheres, which suggests a systematic error in the SNe Ia reduced data. Their model independent test failed to detect acceleration of the universe at high statistical significance (see Figure 2 therein), and concluded that it is too early to take accelerated expansion for granted, as the evidence relies on the *a priori* assumption of the Concordance model.

The present Steady Flow model can be falsified through further SNe Ia data at different redshifts, and explicitly predicts that supernovae of  $z > 1$  should not show any deceleration in the past expansion of the universe, at least for the last 13 billion years.

Although measurements of WMAP agree with the Standard model, a linear expansion surprisingly clears preliminary constraints on structure formation and CBR anisotropy<sup>[8,21]</sup>. In spite of a significantly different evolution with time, the recombination history of a linearly expanding cosmology gives the location of the primary acoustic peaks in the same range of angles as that given in Standard cosmology.

Previously, it has been reported that a linear expan-

sion model would be consistent with the observed H/He ratio, although producing primordial metallicity much higher than that produced in BBN<sup>[7]</sup> or requiring additional mechanisms for the production of deuterium<sup>[8]</sup>. However, the Steady Flow model does not modify the successful results of BBN because the expansion rates during nucleosynthesis epoch were practically the same as in the Standard model. This was due to the overwhelming dominance of radiation on expansion at that time, according to equation (6).

It is well known that Baryon Acoustic Oscillations (BAO) are consistent with cosmic acceleration in recent times, so that BAO signals are accounted as evidences in favor of the Standard model. However, Shafieloo and col.<sup>[50]</sup> have found that, allowing dark energy to vary, a linear expansion model ( $q_0=0$ ) fits the data of SN Ia + BAO + CBR at nearly the same level of confidence as the Standard model. In fact their model provides an excellent fit to the assembly of data and also leads to a decay of dark energy with time.

Jain *et al.*<sup>[24]</sup> have studied the angular size–redshift relation in power-law cosmologies by using measurements for a large sample of compact radio sources. They found as a best-fit exponent  $\dot{a}=1 \pm 0.3$  at 68% confidence level. The agreement of this kind of data with a linear expansion has been confirmed by Abdel-Rahman and Riad<sup>[3]</sup>. Besides, the X-ray mass fraction data of galaxy clusters agree with a flat universe following a power-law expansion of exponent very close to 1<sup>[65]</sup>. What's more, a linear expansion is also consistent with gamma ray burst data<sup>[44]</sup>.

High quality observations of radio sources gravitational lensing and SNe Ia also favour a time-evolving dark energy instead of a cosmological constant<sup>[25]</sup>. Moreover, a linear expansion model is also consistent with gravitational lensing statistics within  $1\sigma$ <sup>[14,49]</sup>. In contrast, the observed quasar lensing fraction appears to be lower than expected in a Standard flat cosmology with  $\Omega_\Lambda = 0.7$ <sup>[33]</sup>.

The Steady Flow model has not any age problem. Its calculated expansion time accommodates the ages of the oldest stars and globular clusters, including the age of the oldest known star:  $13.2 \cdot 10^9$  years<sup>[18]</sup>. Moreover, a linear expansion model can easily accommodate old high-redshift galaxies and quasars (e.g. Sethi *et al.*<sup>[49]</sup>) and can help to understand the observations of early galaxies, which appear to be more fully formed and mature than Concordance model would expect (e.g. Krauss<sup>[28]</sup>). This so-called high redshift ‘age crisis’ appears to be more restrictive than the total age as a cosmological test. The most striking case corresponds to the old, high-redshift quasar APM 08279+5255 ( $z=3.91$ ,  $t=2.1$  Gyr)<sup>[19,22]</sup>, which Standard flat FRW models with cosmological constant fail to accommodate. The growth of dark matter perturbations can be enhanced due to the above mentioned coupling

between dark matter and dark energy, which has been used to explain the age of this old quasar<sup>[60]</sup>. In any case, according to the Steady Flow model such a redshift would correspond to an expansion time  $>3$  Gyr, which can accommodate the age of this old object without difficulty.

## CONCLUDING REMARKS

SNe Ia observations demonstrate that the Hubble flow is not decelerating and suggest a slight acceleration of it, but these results are also compatible with a linear expansion of the universe. The introduced cosmological parameter,  $\Lambda_R$ , along with the minimal LNH, leads to such a Steady Flow model of the expansion. Therefore, the possible existence of a decaying dark energy, derived from the vacuum energy, does not necessarily imply an accelerated expansion.  $\Lambda_R$  allows the derivation of  $H$  at any time. If this plain model was correct, it would imply that vacuum energy is driving the space expansion and, concurrently, is ‘diluted’ by such expansion. This is not as surprising as it might seem at first glance: since the vacuum energy affects the large scale structure and the expansion of the universe, but should originate from effective local vacuum fluctuations, it may well provide a natural connection between macro and microphysics. Thus, the large numbers *coincidences* can be regarded as a natural connection of quantum and cosmic scales, i.e. quantum microphysics can determine some properties of the whole cosmos. In particular, the dimensionless number  $10^{122}$  provides an explanation to the vast difference between vacuum energy and dark energy. The cosmological constant problem and the coincidence problem can be avoided in this scenario. The first one vanishes because  $\Lambda_R$  is not constant anymore, and the second one is critically alleviated if both mass and dark energy densities evolve with the universal expansion following the same scaling law, namely  $\rho_M \propto \Lambda_R \propto R^{-2}$ .

The Steady Flow model is only one among the plethora of alternatives proposed so far to the Standard cosmology and, as most of them, it is probably flawed and yet incomplete, but it has the elegance of simplicity (no adjustable parameters or functions are needed to obtain a linear expansion rate that essentially depends on vacuum energy) and, above all, it works (avoids the main cosmological problems, agrees so far with the available observational data and is predictive, and thus falsifiable). In any case, the growing evidence here reported indicates that perhaps this linear expansion paradigm deserves some attention as a feasible alternative to the Concordance model.

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