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## The self-absorption coefficient of sodium resonance line for the spherical geometry atom cavity

Jian He

School of Science, Henan University of Science and Technology, Luoyang, 471003, (CHINA) E-mail : hejian405@163.com Received: 14<sup>th</sup> May, 2008 ; Accepted: 19<sup>th</sup> May, 2008

### ABSTRACT

In this paper, the concept of the self-absorption coefficient is discussed using the concept of escape factor. The calculation methods of the escape factor for any profile and any atom cavity are discussed. For the spherical geometry, the escape factors  $\Lambda(\mathbf{r})$  at any position r in a spherical geometry plasma (with the radius R) for Lorentzian and Holtsmarkian profiles are calculated for the first time, and a general expression is obtained. As an example, for the sodium 330.3nm resonance line, the total radiance and the real radiance are calculated, and the self-absorption coefficient for the resonance line is discussed. This discussion will be useful for the study of escape factor and the self-absorption of spectral lines. © 2009 Trade Science Inc. - INDIA

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KEYWORDS

Self-absorption coefficient; Resonance line; Escape factor; Spherical geometry atom cavity.

INTRODUCTION

The analysis of spectral line is an important diagnostic tool for physics and chemistry. In order to extract useful spectroscopic information from atom vapors, one has to take in account all the physical phenomena occurring in this medium. Specially, one has to estimate the self-absorption phenomenon, which gives apronounced non-linearity in the calibration function at increasing concentration of the element and is often neglected<sup>[1]</sup>. The self-absorption effect of the atomic spectral lines emitted by the plasma can be used to measure the population density of the metastable and resonant atoms in the discharge. The self-absorption lead to underestimation of the intensity of the resonance lines emitted by no optically thin plasma and to underestimation of the concentration of the species deduced from this analysis<sup>[2]</sup>. For that reason and in order to correct the experimental measure of the intensity (study the reabsorption of resonance lines which are generally more affected by the self-absorption than non-resonances lines), a large number of letters had for subject the problem of self-absorption in optical emission spectroscopy; several methods were proposed to estimate the reduction of the line intensity due to this effect<sup>[3-5]</sup>.

In this paper, we suggest estimating quantitatively the self-absorption phenomenon for the resonance line of sodium resonant line. Theory shows that we can treat the self-absorption of resonance line by means of the escape factor. For the resonance line, the escape factor is defined as the ratio of the radiation, fluxes escaping from the plasma to the fluxes escaping from optically thin plasma. Discussions on the escape factor have been developed greatly in recent years, and many use-

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ful results have been obtained<sup>[6-15]</sup>. In general, escape factors have been used in two similar senses to model the radiation transfer of spectral lines. In one senses, an escape factor multiplies the emission expected from an optically thin plasma to allow for the effect of opacity on the emitted lines. In the other sense, an escape factor is a parameter, which multiplies the radiative transition probability to allow for the effect of photo-excitation on population densities<sup>[16]</sup>.

### Theory of self-absorption

With the assumption of local thermodynamic equilibrium, in the case of an optically thin, homogeneous, and isothermal plasma, the total radiance (W m<sup>-2</sup>sr<sup>-1</sup>) of a particular line is given by<sup>[17]</sup>.

$$I_{\lambda 0}^{\text{thin}}(T) = L_{\lambda 0}(T)R_p \int_0^\infty k_\lambda(T)d\lambda$$
 (1)

where  $L\lambda_0(T)$  is the Planck distribution for the blackbody radiation (W m<sup>-2</sup>sr<sup>-1</sup> m<sup>-1</sup>), R<sub>p</sub> is the length of the emitting plasma region, and k $\lambda'(T)$  is the monochromatic effective absorption coefficient corrected for the effects of stimulated emission<sup>[18]</sup>.

$$k\lambda / (T) = \left(\frac{e}{4\pi\epsilon_0}\right) \left(\frac{\pi}{m_e c}\right) (n_m \frac{g_m}{Q(T)} exp\left(-\frac{Em}{k_B T}\right))$$
$$f_{nm} (1 - exp\left(-\frac{E_m - E_n}{k_B T}\right)) P(\lambda)$$
(2)

where e is the elementary charge, c the light velocity,  $\varepsilon_0$  is the electrical permittivity,  $m_e$  is the electron mass,  $n_m$  is the population number density of the emitting state,  $f_{nm}$  is the oscillator strength of the transition,  $g_m$  is the degeneracy of the upper energy level, En and Em are the energies of the lower and upper energy levels respectively,  $k_B$  is the Boltzmann constant, Q(T) is the internal partition function, and P() the normalized line profile.

If the line is not self-absorbed, its intensity is given by Eq.(1). If the self-absorption phenomenon is significant, Drawin and Emard have shown that the real line radiance can be written as<sup>[1]</sup>.

### $I\lambda_{0}(T) = I^{thin}\lambda_{0}(T)\Lambda_{r}(\lambda 0)$ (3)

Where  $\Lambda_r(\lambda 0)$  is the so called "escape factor." This dimensionless parameter whose value lies between 0 and 1 is defined as the ratio between the real radiation flux escaping from the plasma and the radiation flux in the optically thin case.

### 3. The concept of the escape factor in plasma

According to the effect of the escape factor, we will discuss the escape factor on two aspects. In atom absorption measurement, because of the existence of



Figure 1: The diagram for photon escapes

the photon escape factor, the transmission of the laser will deduce, and we call the effect on transmission; For atom emission, for the photon escape and photon capture, the spontaneous emission coefficient will deduce, and we call the effect on emission. We will discuss the two concepts at follows:

### (1) Effect on transmission

In plasma, we assume that an atom is excited to the higher lever by some reason. When it radiates to the lower lever, a photon will emit. Before traveling to the surface, the photon maybe captured, maybe escaped. If the photon emits at  $\bar{r}$ , it travels to  $\bar{s}$  at the surface and escapes from this point as shown in figure 1.

Then the photon escape probabilities for all over the surface, that is the escape factor is given by<sup>[19]</sup>:

$$\theta(\bar{\mathbf{r}}) = \frac{1}{4\pi} \int_{s} \mathbf{T}(\tau_{0} \left| \bar{\mathbf{r}} - \bar{\mathbf{S}} \right|) \frac{(\bar{\mathbf{r}} - \bar{\mathbf{S}}) \cdot d\bar{\mathbf{S}}}{\left| \bar{\mathbf{r}} - \bar{\mathbf{S}} \right|^{3}}$$
(4)

where  $\tau_0$  is the optical depth in the line center,  $T(\tau_0 | \vec{r} - \vec{S} |)$  denotes the probability that a photon escape from  $\vec{s}$ ,  $d\vec{S}$  denote the integral is all over the surface.

For this effect, the escape factor can be called photon escape probability, and has been discussed in detail in Ref.<sup>[20]</sup>. From figure 1 we can find that the plasma geometry has important effect on the escape factor, so we will discuss this problem in later section.

### (2) Effect on emission

Considering a two lever system as shown in figure 2, we know that a photon in higher lever j will emit to lower lever i . For optical thin plasma, the photon can emit to the lower lever without being captured, and we denote the spontaneous emission coefficient A. But for optical thick plasma, the photon will be captured (or absorbed) before emitting to the lower lever, so we



Figure 2 : The diagram for photon emission

think the spontaneous emission coefficient will deduce to  $A\Lambda(\bar{r})$ , where  $\Lambda(\bar{r})$  is the escape factor. The escape factor can be written as

$$\Lambda = 1 - \Psi \tag{5}$$

where  $\Psi$  is the average capture probability, and it is a function of position  $\bar{r}$  and time t,  $\Psi(\bar{r}, t)$  is given by

$$\psi(\mathbf{\bar{r}},t) = \frac{1}{\rho(\mathbf{\bar{r}},t)} \int_{\mathbf{v}} \rho(\mathbf{\bar{r'}},t) G(\mathbf{\bar{r'}},\mathbf{\bar{r}}) d\mathbf{V'}$$
(6)

where  $\rho(\mathbf{r}, t)$  is the distribution of the number density of the atom,  $G(\mathbf{r}', \mathbf{r})$  denotes the probability that a photon is captured in the volume of  $\mathbf{r}$ .

A solution is the amplitude of  $\eta(\bar{r}, t)$  is  $\rho(\bar{r})$ , but will decrease as the time

$$\mathbf{n}(\mathbf{r},t) = \boldsymbol{\rho}(\mathbf{r})\exp(-\boldsymbol{\beta}t) \tag{7}$$

where  $\rho(\mathbf{r})$  and  $\beta$  satisfy the equation

$$(1 - \frac{\beta}{A})\rho(\bar{\mathbf{r}}) = \int_{\mathbf{V}} \rho(\bar{\mathbf{r}}) G(\bar{\mathbf{r'}}, \bar{\mathbf{r}}) d\mathbf{V'}$$
(8)

# 4. Calculation of the escape factor with different profiles

For different widen profiles, the esape factor can be calculated in different approximation formula.

### **Gaussian profile**

For the thermal motion of the plenty of atoms, according to the Doppler effect, we think the widen is described by the Gaussian profile:

### $\mathbf{f}(\boldsymbol{\sigma}) = \mathbf{f}_0 \exp[-4(\boldsymbol{\sigma} \cdot \boldsymbol{\sigma}_0)^2 \ln 2/\Delta \mathbf{v} \mathbf{D}^2]$ (9)

where  $\sigma$  is the wave number  $\mathfrak{L}^{\neg} f_0$  is the radiative intensity of the line center.

The escape factor for the Gaussian profile is given by

$$G(\tau_0) = \frac{1 + \tau_0 / (2 + \tau_0)^2}{1 + \tau_0 \sqrt{\pi \ln(1 + \tau_0)}}$$
(10)

#### Lorentzian profile

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For the frequently collisions among the atoms, we think the widen is described by the Lorentzian profile:

$$\mathbf{f}(\boldsymbol{\sigma}) = \frac{\mathbf{s}}{\pi} \frac{\Delta \mathbf{v}_{\mathrm{L}}}{\left(\boldsymbol{\sigma} - \boldsymbol{\sigma}_{0}\right)^{2} + \Delta \mathbf{v}_{\mathrm{L}}^{2}}$$
(11)

where s is a constant with the radiation.

The escape factor for the Lorentzian profile is given by

$$L(\tau_0) = \frac{1 + \frac{(1 + \tau_0)}{(2 + \tau_0)}}{1 + \sqrt{\pi \tau_0}}$$
(12)

### Voigt profile

The voigt profile is the convolution of the Gaussian profile and the Lorentzian profile, and it is presented as an integral form:

$$P(v) = \frac{f'y}{\pi} \int_{\infty}^{\infty} \frac{\exp(-t^2)}{y^2 + (x-t)^2} dt$$
 (13)

Where  $y = \Delta v_L / \Delta v_D \times (In2)^{1/2}$ ,  $x = v - v_0 / \Delta_D \times (In2)^{1/2}$ ,  $f' = 1 / \Delta v_D \times (In2)^{1/2}$ . We have done some work on the Voigt profile.

The escape factor for the Voigt profile can be given by the non-linear combination of that of Gaussian profile and Lorentzian profile :

$$\mathbf{V}(\boldsymbol{\tau}\mathbf{0}) = \mathbf{G}(\boldsymbol{\tau}_0) + \mathbf{F}(\boldsymbol{\alpha}, \boldsymbol{\tau}_0) \mathbf{L}(\boldsymbol{\tau}_0)$$
(14)  
where

where

$$F(\alpha,\tau_0)\sqrt{\pi Z} \frac{\alpha\tau_0}{1 + (\tau_0\sqrt{\alpha + \alpha\sqrt{\tau_0}})}$$
(15)

Z has the following form

$$Z = \frac{\alpha \tau_0}{\sqrt{\pi (1 + \pi \alpha_2)}} \left[ 1 - \frac{\alpha}{\sqrt{\pi (1 + \pi \alpha_2)}} \right]$$
(16)

where  $\alpha$  is the damping constant which is a measure of the half-width of Lorentzian profile relative to that of Doppler profile, that is

$$\alpha = \sqrt{\ln 2} \frac{\Delta v_{\rm L}}{\Delta v_{\rm D}} \tag{17}$$

In Eq.(17),  $G(\tau_0)$  and  $L(\tau_0)$  are the escape factor for pure Doppler and Lorentzian line profiles, respectively. The function  $F(\alpha,\tau_0)$  must satisfy the limiting cases: for  $\alpha \rightarrow 0$ : $F(\alpha,\tau_0) \rightarrow 0$ ,  $G(\tau_0) = L(\tau_0)$ , for any  $\tau_0$ , for  $0 < \alpha < 3$ :  $F(\alpha,\tau_0)L(\tau_0) \rightarrow G(\tau_0)$ , large  $\tau_0$ , and for  $\alpha > 3$ :  $V(\tau_0) \rightarrow L(\tau_0)$ .

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### Holtsmarkian profile

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For the frequently collisions between the absorption atoms and other atoms, we think the widen is described by the Holtsmarkian profile:

$$f(\sigma) = \frac{5\sin(\frac{2\pi}{5})}{4\pi\Delta\nu 1 + \left(\frac{\sigma}{\Delta\nu}\right)^{5/2}}$$
(18)

where  $\Delta v$  is the half width of the Holtsmarkian profile.

The escape factor for the Holtsmarkian profile is given by:

$$\mathbf{H}(\tau_0) = \mathbf{0.451}/\tau_0^{3/5} \tag{19}$$

Where  $\tau_0$  is the optical depth in the line center. For the absorbing volume of width L, the optical depth in the line center 0 is given by:

#### $\tau_0 = \sigma NLP(0)$ (20)

where  $\sigma$  is the Ladenburg cross-section given by  $\sigma = (\pi e^2/mc) \cdot f_{ij}$ , where  $f_{ii}$  is the oscillator strength of the resonance transition, and it is given by<sup>[21]</sup>

$$\mathbf{f}_{ij} = \frac{\varepsilon_0 \mathbf{mc}}{2\pi e^2} \frac{\mathbf{g}_2}{\mathbf{g}_1} \frac{\lambda_0^2}{\tau_{21}}$$
(21)

where  $\varepsilon_0$  is permittivity of free space,  $\varepsilon$  is charge of electron, mis mass of electron, c is speed of light, g is statistical weight,  $\tau_{21}$  is lifetime of excited state, and  $\lambda_0$  is the resonance wavelength of the atom transition. In Eq.(20), Nis the number density of the absorbing atoms in the ground state. The relation between N and the maximum absorption coefficient k for Voigt distribution is

$$N = k_{m} \frac{(\Delta \lambda_{N} + \Delta \lambda_{L}) \Delta \lambda_{D}}{4\lambda^{2} f_{ij}} \sqrt{\frac{\pi}{\ln 2}} \frac{mc^{3}}{e^{2}}$$
(22)

where  $\Delta v_{\rm N}$ ,  $\Delta v_{\rm I}$  and  $\Delta v_{\rm D}$  are the natural, Lorentzian and Doppler half-widths respectively.

The Doppler half-widths is given by:

$$\Delta v_{\rm p} = \sigma 0 (7.1623 \times 10^{-7}) (T/M)^{1/2}$$
(23)

where T is the absolute temperature of the gas, M is the mass of lithium atom.

The natural half-widths is given by:

$$\Delta v_{\rm N} = 1/2\pi \tau_{21} {\rm C} \tag{24}$$

where  $\tau_{21}$  is the lifetime of excited state, c is speed of light.

The Lorentzian half-widths is given by:

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$$\Delta v L = 2r_{air} (296/T)^{n} P$$
(25)

where n is the temperature coefficient, and in general gas, n=0.75,  $r_{air}$  is the widen coefficient in atmosphere,  $r_{air} = 3.34 \times$  $10^{-2}$  cm<sup>-1</sup>/atm



### 5. The escape factor for spherical geometry atom cavity

There are three geometry atom cavities in atom absorption measurement, slab geometry, cylindrical geometry and spherical geometry. The escape for slab and cylindrical geometries has been done<sup>[22]</sup>. Here we will give the breif description of the brivation.

#### Infinite slab geometry

Consider a source of infinite plane-parallel slab geometry as shown in figure 3, in which the z axis is normal to the two surfaces (which correspond to z=0 and z=D) and  $\alpha$  denotes inclination to the z axis. Such a geometry corresponding approximately to a plasma where the emissivity and absorption coefcient drops rapidly with distance from the target surface, but is typically approximately constant in the plane of the target over the focal area of the plasma.

Because the slab is infinite, we can choose a circle element of surface area. So

$$d\vec{S} = 2\pi s ds = 2\pi s |\vec{r} - \vec{s}| d\alpha$$
 (26)  
and

a

$$\frac{\left|\vec{\mathbf{r}}-\vec{\mathbf{s}}\right|.\mathbf{d}\vec{\mathbf{S}}}{\left|\vec{\mathbf{r}}-\vec{\mathbf{s}}\right|^{3}} = \frac{\left|\vec{\mathbf{r}}-\vec{\mathbf{s}}\right| \left|\mathbf{d}\vec{\mathbf{S}}\right|\cos(\pi-\alpha)}{\left|\vec{\mathbf{r}}-\vec{\mathbf{s}}\right|^{3}} = -2\pi\sin\alpha\cos\alpha d\ \alpha$$

$$=2\pi\cos\alpha d\,\cos\alpha = 2\pi\mu d\mu \tag{27}$$

where  $\mu = \cos \alpha \mathfrak{t}$  the following relation has been used

$$\sin \alpha = \frac{\left| \vec{s} \right|}{\left| \vec{r} - \vec{s} \right|}$$
(28)

Bringing Eq.(27) to Eq.(4), noticing that  $\bar{r} - \bar{s} = \frac{z}{\cos \alpha}$  and

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the integral is over the whole surface area, we have

$$\theta(z) = \frac{1}{2} \int_0^1 T(\frac{\tau_0 z}{\mu}) \mu d\mu + \frac{1}{2} \int_0^1 T(\frac{\tau(D-z)}{\mu}) \mu d\mu$$
(29)

At the centre of the slab

$$\theta(\frac{D}{2})\int_{0}^{1}T(\frac{\tau_{0}D}{2\mu})\mu d\mu$$
(30)

### Infinite cylindrical geometry

Consider the infinite cylindrical geometry as shown in figure 4, we can find that

$$d\overline{S} = 2|\overline{r} - \overline{S}|d\alpha \sec \alpha\xi d\phi / \cos\beta$$
(31)

then

$$(\mathbf{r} - \mathbf{s}) \cdot \mathbf{dS} / |\mathbf{r} - \mathbf{s}|^2 = \cos \alpha \cos \beta \mathbf{dS} / |\mathbf{r} - \mathbf{s}|^2 = \cos \alpha \mathbf{d} \quad \alpha \mathbf{d} \phi \quad (32)$$

so Eq.(4) can be written as

$$\theta(\mathbf{r}) = \frac{1}{\pi} \int_0^{\pi} d\phi \int_0^{\pi/2} \mathbf{T}(\tau_0 \xi \sec \alpha) \cos \alpha d\alpha$$
(33)

where the relation of  $\xi$  and  $\phi$  is

$$\xi^2+2r\xi \cos \phi$$
-(R<sup>2</sup>-r<sup>2</sup>)=0 (34)  
At the center, r=0,  $\xi$ =R, Eq.15 can be written as

$$\theta(\mathbf{r}_{c}) = \int_{0}^{\pi/2} T(\tau_{0} \mathbf{R} \sec \alpha) \cos \alpha d\alpha$$
(35)

### Spherical geometry

In this section, we will discuss the escape factor for spherical geometry using this method.

In plasma, we assume that an atom is excited to the higher lever by some reason. When it radiates to the lower lever, a photon will emit. Before traveling to the surface, the photon maybe captured, maybe escaped. If the photon emits at  $\vec{r}$ , it travels to  $\vec{s}$  at the surface and escapes from this point. For the spherical geometry plasma, a figure is shown in figure 5.

Then the photon escape probabilities for all over the surface, that is the escape factor is given by<sup>[23]</sup>:

$$\Lambda(\mathbf{\bar{r}}) = \frac{1}{4\pi} \int_{s} T(\mathbf{k}_{0} \left| \mathbf{\bar{r}} - \mathbf{\bar{s}} \right|) \frac{(\mathbf{\bar{r}} - \mathbf{\bar{s}}) \cdot \mathbf{dS}}{\left| \mathbf{\bar{r}} - \mathbf{\bar{s}} \right|^{3}}$$
(36)

where  $k_0$  is the peak absorption coefficient,  $T(k_0 | \bar{r} - \bar{s} |)$  denotes the probability that a photon escape from  $\bar{s}$ ,  $d\bar{s}$  denote the integral is all over the surface.

For the spherical geometry plasma, we can choose a circle plane as shown in figure 6. We can find



Figure 4 : The infinite cylindrical geometry



Figure 5 : The spherical geometry plasma

$$\left| \mathbf{dS} \right| = \mathbf{R}\pi\mathbf{R} \quad \sin \alpha \mathbf{dI} = 2\pi\mathbf{R}^2 \sin \alpha \mathbf{d} \alpha$$
 (37)  
So in Eq.(36), we have

$$\frac{\left|\overline{\mathbf{r}}-\overline{\mathbf{s}}\right|.d\overline{\mathbf{S}}}{\left|\overline{\mathbf{r}}-\overline{\mathbf{s}}\right|^{3}} = \frac{\left|d\overline{\mathbf{S}}\right|\cos\beta}{\left|\overline{\mathbf{r}}-\overline{\mathbf{s}}\right|^{2}} = -\frac{2\pi R^{2}\sin\alpha\cos\beta}{\left|\overline{\mathbf{r}}-\overline{\mathbf{s}}\right|^{2}}d\alpha$$
(38)

From figure 6 we can find that

$$\left|\vec{\mathbf{r}} - \vec{\mathbf{s}}\right|^2 = \mathbf{R}^2 + \mathbf{r}^2 + 2\mathbf{R}\mathbf{r}\cos\alpha$$
(39)  
So we can rewrite Eq. (38) as

So we can rewrite Eq.(38) as

$$\frac{\left|\bar{\mathbf{r}}-\bar{\mathbf{s}}\right|\cdot \mathbf{d}\bar{\mathbf{S}}}{\left|\bar{\mathbf{r}}-\bar{\mathbf{s}}\right|^{3}} = \frac{2\pi R^{2} \sin\alpha\cos\beta}{\left|\bar{\mathbf{r}}-\bar{\mathbf{s}}\right|^{2}} d\alpha = -\frac{2\pi R^{2} \cos\beta}{R^{2} + r^{2} + 2Rr\cos\alpha} d\cos\alpha$$
(40)

According to Eq.(36), the escape factor can be rewritten as

$$\Lambda(\bar{\mathbf{r}}) = -\frac{1}{4\pi} \int_{s} T(\mathbf{k}_{0} | \bar{\mathbf{r}} - \bar{\mathbf{s}} |) \frac{2\pi R^{2} \cos\beta}{R^{2} + r^{2} + 2Rr \cos\alpha} d\cos\alpha$$
$$T(\tau_{0}) \int_{-\infty}^{\infty} \exp(-\tau_{0}) \phi(\mathbf{u}) d\mathbf{u} \qquad (41)$$
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where  $T(k_0)$  is the transmission factor, which denotes the average probability that a photon propagates at least an optical depth through the source without being captured, i.e.

$$T(\tau_0 \int_{-\infty}^{\infty} \exp(-\tau_0) \phi(u) du$$
 (42)

which has the following asymptotic forms: Lorentzian profile,  $T(\tau_0) = /(\pi \tau_0)^{1/2}$ Holtsmarkian profile,  $T(\tau_0) = 0.451/\tau_0^{3/5}$ 

In figure 6 we can find that  $\beta$  is little and varies little, so we can choose  $\cos \beta = 1$  as an approximation. In Eq.(41), we can obtain

$$\Lambda(\bar{\mathbf{r}}) = \frac{R}{4r} \ln \frac{R^2 + r^2 + 2Rr}{R^2 + r^2 - 2Rr}) T(\mathbf{k}_0) = \frac{R}{4r} \ln(1 + \frac{4Rr}{R^2 + r^2 - 2Rr}) T(\mathbf{k}_0)$$
(43)

Using the Taylor series

$$In(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \dots + (-1)^{n-1} \frac{x^n}{n} + \dots$$
(44)

Remaining the first order, we have

$$\bar{\Lambda(\mathbf{r})} = \frac{R}{4r} In(1 + \frac{4Rr}{R^2 + r^2 - 2Rr}) T(\mathbf{k}_0) \approx \frac{4Rr}{R^2 + r^2 - 2Rr} T(\mathbf{k}_0) = \frac{4Rr}{1 + \frac{r^2}{R^2} + -2\frac{r}{R}} T(\mathbf{k}_0)$$
(45)

Because r<R, we can neglect  $r^2/R^2$  in Eq.(45), so we have

$$\bar{\Lambda(\mathbf{r})} = \frac{1}{1 - 2\frac{\mathbf{r}}{R}} T(\tau_0)$$
(46)

Eq.(46) is the escape factor at any position r for the spherical geometry. The calculation fomula of optical depth in the line center can be find in Ref.<sup>[24]</sup>. If we choose the optical depth in the line center  $\tau_0=10^3$  and the radius of the absorbing sphere R=2m, the escape factor at any position r for Lorentzian and Holtsmarkian profiles are shown in figure 7.

From figure 7 we can find that the escape factor for the Holtsmarkian profile is larger than for the Lorentzian profile with any r; as the increasing ofr, the escape factor will increase, which shows that the photon near the surface is easy to escape from the surface.

There are other widen profile in the discussion of the escape factor, such as the Voigt profile, and we





Figure 6 : A circle plane we choosed



Figure 7 : The escape factor with different r for Lorentzian and Holtsmarkian profiles

have done many works on the Voigt profile<sup>[25-26]</sup>. But here we only discuss the Lorentzian profile and the Holtsmarkian profile for the complexity of the Voigt profile.

### 6. The self-absorption of Na 330.3nm

According to Eq.(1) and Eq.(2), because  $P(\lambda)$  is normalized, we have

$$\int_{0}^{\infty} P(\lambda) d\lambda = 1$$
(47)

The Planck distribution for the blackbody radiation  $L\lambda_0(T)$  is<sup>[27]</sup>.

$$L_{\lambda_0}(T) = \frac{2\pi h c^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1}$$
(48)

where h is the Planck constant, T is the temperature of the atoms.

The population number density of the emitting state  $n_m$  can be given by

$$\mathbf{n}_{\mathrm{m}} = \mathbf{e}^{-\frac{\mathbf{E}_{\mathrm{m}}}{\mathbf{k}_{\mathrm{B}}^{\mathrm{T}}}} \tag{49}$$



Figure 8 : The total radiance of the sodium 330.3nm resonance line



Figure 9 : The real line radiance of sodium 330.3nm resonance line

TABLE 1 : The transition characteristics of the sodium330.3nm resonance line

λ(nm)	g <sub>m</sub>	gn	$E_m(cm^{-1})$	$E_n(cm^{-1})$	f <sub>nm</sub>
330.3	4	2	30272.6	0	9×10-3

Where  $E_m$  is the energy,  $K_B$  is the Boltamann constant. The internal partition function Q(T) is<sup>[28]</sup>

$$Q(T) = \sum_{i} g_{i} e^{\frac{E_{i}}{kT}}$$
(50)

For the sodium 330.3nm resonance line, it is from the election transition of  $2p_63s-2p_64p$ , and the transition characteristics are shown in TABLE 1<sup>[29]</sup>.

When the temperature varies from 1000~2000K, the total radiance of the Na 330.3nm we calculated is shown in figure 8.

Let us Consider any temperature, such as T=1500K, the total radiance we calculated is  $I^{\text{thin}}\lambda_0(T)=3.8\times10^{-25}$ . According to Eq.(3), the real line radiance at any position r in a spherical geometry is given by figure 9.

From figure 9 we can find that for the sodium 330.3nm resonance line, the real line radiance is less than the total radiance, which we call self-absorption. As the increasing of the position r, the escape factor will increase, and the self-absorption of the resonance will decrease.

### 7.CONCLUSIONS

In this paper, the self-absorption coefficient of sodium 330.3nm for the spherical geometry atom cavity is discussed using the escape factor, for Lorentzian and Holtsmarkian profiles. From discussion, the following conclussions can be drawn:

- 1. The escape factor can be denoted by the ratio of the real radiance with the total radiance, which can be used to explain the self-absorption for a spectral line.
- 2. The escape factor for the spherical geometry is calculated, and a simple expression is obtained. As the increasing of the position r, the escape factor will increase, which shows that the photon near the surface is easy to escape from the surface.
- 3. For the sodium 330.3nm resonance line, the real line radiance of is less than the total radiance, which we call self-absorption. As the increasing of the position r, the escape factor will increase, and the self-absorption of the resonance will decrease.

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