The research on the horizontal resolution advisory algorithm based on ADS-B system

Zhang Xuehua*, Wei Aiyu
Civil Aviation Flight University of China, Sichuan, (CHINA)
E-mail: 59918031@qq.com

ABSTRACT

Air Traffic Collision Avoidance System (TCAS), as a compulsory equipment by ICAO, can provide two levels of alert, traffic advisory (TA) and vertical resolution advisory (VRA), but there is no horizontal traffic collision avoidance alert provided by TCAS so far. This paper focused on the algorithm of horizontal traffic avoidance. Through the fusion of tracks of ADS-B and TCASII, we can improve the aircraft track accuracy judgment so that realize the horizontal avoidance and alert. There is a new track detection model and algorithm raised by this paper to solve the problem of horizontal turn collision avoidance, and with MATLAB, optimized collision avoidance turn direction and angle can be achieved.

KEYWORDS

TCAS; ADS-B; HRA.
INTRODUCTION

Air Traffic Collision Avoidance System (TCAS), as a compulsory equipment by ICAO, can provide two levels of alert, traffic advisory (TA) and vertical resolution advisory (VRA) so far, but there is no horizontal traffic avoidance alert provided by TCAS so far [3].

ADS-B stands for automatic dependent surveillance-broadcast, which provides “radar-like” surveillance service. The aircraft obtains its accurate position by GPS and continuously broadcasts the information to any aircraft or ground station equipped to receive ADS-B through various data links.

There are two types of services according to ADS-B: ADS-B IN and ADS-B OUT. ADS-B OUT periodically broadcast information about each aircraft, such as identification, current position, velocity and altitude, through an on-board transmitter. It also provides air traffic controllers with real-time position information, which in most case, more accurate than the information available with current radar-based systems. ADS-B IN is the reception by aircraft of FIS-B and TIS-B data and other ADS-B data such as direct communication from nearby aircraft, which makes air traffic collision avoidance possible based on ADS-B system. And furthermore ADS-B system can expand the view detected by TCAS because of more data links available (Figure-1).

METHODS

Aircraft track accuracy judgment

Collision avoidance systems calculate the trajectory of the other aircraft depending on the position of the other aircraft. In order to determine whether there is a conflict between the two aircrafts, Kalman filter may be used in combination of tracks calculated by TCASII system and ADS-B to improve the accuracy of track in ADS-B/TCASII [6].

According to the basic principle of Kalman filtering, kalman filtering equations can be built by the combined track system based on the state variance and two parts of the measurement equation in this system. The corresponding state equation is:

\[
X(k + 1) = F(k)X(k) + G(k)u(k) + V(k)
\]

In the above formula, \(X(k)\) represents state variable, \(F(k)\) represents transition matrix, \(G(k)\) represents the input control matrix, \(u(k)\) represents the given input control signal, \(V(k)\) represents Gaussian noise. By using the combination of ADS-B and TCASII, the mutually exclusive pathway is calculated with the help of Kalman filter algorithm. Here below the measurement equation can be used in both.

\[
z_i(k) = H_i(k)X_i(k) + W_i(k)
\]

\(H_i(k)\) represents measurement matrix, \(W_i(k)\) stands for the measured corresponding noise.

Based on the state equation and measurement equation of the combined system, combined the measurement parameters and the corresponding noise acquired by ADS-B and TCASII independently, the mutually exclusive pathway can be calculated with the help of Kalman filter algorithm. The calculation formula is as follows.

\[
\hat{X}_i(k + 1/k + 1) = \hat{X}_i(k + 1/k) + K_i(k + 1)[z_i(k + 1) - H_i(k + 1)\hat{X}_i(k + 1/k)]
\]

\[
\hat{X}_i(k + 1/k) = F(k)\hat{X}_i(k / k) + G(k)u(k)
\]
\[ K_i(k + 1) = P_i(k + 1/k)H_i^T(k + 1)[H_i(k + 1)P_i(k + 1/k)H_i^T(k + 1) + P_i(k + 1)]^{-1} \tag{5} \]

\[ P_i(k + 1/k) = F(k)P_i(k/k)F^T(k) + Q(k) \tag{6} \]

\[ P_i(k + 1/k + 1) = [I - K_i(k + 1)H_i(k + 1)]P_i(k + 1/k) \tag{7} \]

\( K_i(k) \) represents the gain of Kalman filter, \( P_i(k) \) represents the covariance of aircraft track state estimation, \( I \) represents unitary matrix, the values of \( Q(k) \) can be adjusted by the experience in practice. By using the calculation formula, the valuation of aircraft trajectory was obtained based on the combination of ADS-B and TCASII. To fuse the two values and meet the basic requirement, the Kalman filter algorithm was employed. The relation equations of fusion is as follows.

\[ \hat{X} = W_1\hat{X}_{TCASII} + W_2\hat{X}_{ADS-B} \]

\( W_1, W_2 \) stand for coefficient matrix which still need to be determined, \( \hat{X} \) was proved to be the minimum-variance unbiased estimator when the equations \( W_1 + W_2 = I \) was applied.

By using the algorithm of Kalman filter, the flight course was set into three stages, started by operating uniform flight in 220Kt, then a 10 seconds accelerated flight process in 5 m/s, after that conducted a 20 seconds of uniform flight process, the noise settings in TCASII and ADS-B are 50db and 40db. In addition, Calculated by using MATLAB, the maximum deviation track was 200 meters by using TCASII alone and 180 meters by single using ADS-B, then the input of fusion of final deviations was 100 meters. Therefore, the accuracy that calculating the tracks of target aircrafts based on the collision avoidance system was improved by using the combination of TCASII /ADS-B, which contributes to get the more accurate corresponding judgment of collision avoidance.

**Horizontal position distance obtained by combined ADS-B / TCAS collision avoidance**

The position of other aircrafts can be obtained by the direction and time of response signals by TCASII. Seeing that the aircrafts fly in a Three-dimensional space, the measured distance is a slope distance. If we calculate in a horizontal direction, the slope distance should be converted into horizontal distance, as is shown in figure -2.

![Figure 2: Sketch map of projected horizontal distance](image)

The Formula is:

\[ R = \sqrt{R_i^2 - \Delta z^2} \tag{8} \]

Where:
- \( R \) : Slope distance between each other.
- \( \Delta z \) : Vertical height difference between each other.
- \( R \) : Horizontal distance.

Because of the direction error and time delay of SSR, an enough accuracy of the horizontal distance can never be obtained only rely on Formula (8), so TCASII only can assist avoidance in a vertical direction up and down, not in a horizontal direction right and left. However, when a ADS-B/TCAS combination is used, the distance between the two aircrafts can be obtained from the signals of broadcast by the calculation of latitude and longitude coordinates. Then it can
grasp the position and movement of the other aircraft more exactly, which provide with a necessary condition for the collision avoidance system in horizontal direction.

ALGORITHMS AND SIMULATION MODELS

Horizontal track cross-probe model

The new generation of collision avoidance system will be able to provide a horizontal collision avoidance method. This paper will research horizontal anti-collision algorithm based on ADS-B / TCAS combination. At the horizontal anti-collision calculation, due to the two aircrafts are moving in the air, it should be calculated according to the principle of relative motion. The initial position of the aircraft is set at the origin of coordinates. It can be obtained of the movement direction and the relative speed of the other side aircraft refer to the original aircraft. The relative relationship model of horizontal conflict between the two aircraft is shown in Figure -3.

![Figure 3: The relation model of horizontal conflict](image)

In figure -3, the initial position of original aircraft is marked on A, while the invasive aircraft is marked on B. $C_1$ is the track direction of original aircraft, $C_2$ is the track direction of another aircraft, R is the relative distance between the two aircrafts, is the relative azimuth between original aircraft and invasive aircraft.

The following equations (9) are applied to shown the track direction of original aircraft:

\[
\begin{align*}
    x_1(t) &= v_1 t \sin C_1 \\
    y_1(t) &= v_1 t \cos C_1
\end{align*}
\]

(9)

The equations (10) are applied to shown the track direction of invasive aircraft:

\[
\begin{align*}
    x_2(t) &= R \cos \theta + v_2 t \sin C_2 \\
    y_2(t) &= R \sin \theta + v_2 t \cos C_2
\end{align*}
\]

(10)

Where:
- $x_1$: the value of X axis in the relative Cartesian coordinates for original aircraft.
- $y_1$: The value of y axis for original aircraft.
- $v_1$: the speed of original aircraft.
- $x_2$: The value of X axis in the relative Cartesian coordinates for invasive aircraft.
- $y_2$: The value of y axis for invasive aircraft.
- $v_2$: the speed of invasive aircraft.

According to the principle of plane geometry, two straight lines in the same plane either parallel or intersect. However, due to the movement of the aircraft track line is the vector direction, equations (9) and (10) have to be combined for horizontal track cross-probe model to determine whether there exists an intersection of the two lines. Then, determine the directions of movement vector for both aircrafts by detecting the change of their location. Finally determine whether this flight is gathering flight. Once determined, we can obtain the distance extreme of the two aircrafts by using the horizontal distance formula (11), thereby determine the closest meeting point (CPA) of them. Then it can obtain the time (Tau) and distance (HMD) when the aircraft arrive at point CPA which maintains current flight status according to the location of point CPA. More detailed judgment process were shown in Figure -4.
The research on the horizontal resolution advisory algorithm based on ADS-B system  

\[ R = \sqrt{(x_1(t) - x_2(t))^2 + (y_1(t) - y_2(t))^2} \]  

Figure 4: Probing process of horizontal track cross

**Horizontal steering anti-collision algorithm model**

If the two aircrafts cross track, at the same time, HMD ≤ 3000 or the value of Tau is less than the standard of table1, then we should take the calculation of collision avoidance. Horizontal avoidance and vertical avoidance up and down are both available. Horizontal avoidance is the main research direction which is consist of changing speed avoidance and turn avoidance.

This article imagines that the original aircraft avoid another aircraft by turning left or turning right. Before the RA warning of turning avoidance, the possibilities and risks of turning left or turning right to avoid collision must be calculated by ADS-B/TCAS collision avoidance system, then a better method will be selected to avoid collision\(^7\). In fact, both of the aircrafts will take the same anti-collision algorithm software, and the avoidance measures must be coordinate in the condition that they have obtained the location message of each other. Before they determine the turning direction, they will take a peer-to-peer data link communications by SSR to coordinate the direction and insure the safety as well as effectiveness. \(^1\).

For ease of calculation, the paper supposes the original aircraft in a relative motivation coordinate system. The original aircraft takes a turning avoidance, the invasive aircraft have a relative motivation. The detailed conflict avoidance detection model of horizontal avoidance is shown in figure-5.

Figure 5: Horizontal turning conflict avoidance detection model

Imagining the situation after the original aircraft turned at \( t_2 \), the speed is \( V_0 \), the heading is \( C_0 \). The absolute speed of another aircraft is \( V_i \), the heading is \( C_i \). All of the parameters can be obtained from the broadcast of ADS-B. \( R_1 \) is the
relative distance between the two aircrafts at t₁, while θ₁ is the relative azimuth; \(R₂\) is the relative distance between the two aircrafts at t₂, while θ₂ is the relative azimuth. All of the parameters can be obtained from the respond signals of TCAS or from the calculation of latitude and longitude coordinates. \(t₂-t₁=1s\), which is the observation time. The location of the original aircraft at t₁ is (0, 0), the invasive aircraft is \((Xₗ, Yₗ)\) at t₂. Abscissa difference is \(Xₗ\), ordinate difference is \(Yₗ\), as is shown in (12),(13).

\[Xₗ = R \times \sin(C₀ + \theta₁)\]
\[Yₗ = R \times \cos(C₀ + \theta₁)\]

Imagining that the original aircraft avoid collision in a uniform speed by turning left or turning right \(t\), thus turning angle is \(\theta\), which ranges from 0° to 180°, the right turning is + value, the left turning is - value. In the progress, the relationship of HMD, Tau, \(Xₗ\), \(Yₗ\) can be seen, and the value of HMD and Tau in CPA point can be obtained, as is shown in (14), (15).

\[
HMD = \frac{V₀(Xₗ \cos C₀ - Yₗ \sin C₀) + Vᵢ(-Xᵢ \cos Cᵢ + Yᵢ \sin Cᵢ)}{Vᵢ}
\]
\[
\tau = \frac{V₀(Xₗ \sin C₀ + Yᵢ \cos C₀) - Vᵢ(Xᵢ \sin Cᵢ + Yᵢ \cos Cᵢ)}{Vᵢ^2}
\]

Examples of horizontal turn collision avoidance calculation analysis

The basic process of horizontal turn collision avoidance: After determining the two aircrafts may have a flight conflict, calculating and determining the curve horizontal distance between the two aircrafts versus time, then determining CPA point by the method of calculating the extreme value point, thereby getting the value of Tau and HMD of CPA. If the value of HMD of CPA is less than the predetermined 3000 meters, then start turning the collision avoidance algorithm. The algorithm is according to the formula (14) and (15). When calculating the HMD and Tau trends of two different angles, turning left or turning right must be calculated. Finally determining the best direction and angle of turn. Flow of collision avoidance algorithm is shown in Figure-6.

In this example, setting the normal cruise flight speed of the original aircraft is 900km/h, then calculate the distance between this unit and intrusive aircraft \(R₁\) is 6.5Nm, relative azimuth angle is 40° by receiving the 1st ADS-B information of intrusive aircraft. The original aircraft receive ADS-B broadcast location information of the second intrusive aircraft to get the distance between the original aircraft and relative intrusive aircraft \(R₂ = 6.4Nm\), relative azimuth angle is 39.8°. The time between the two received ADS-B of intrusive aircraft is 1second. Getting the minimum extremes by using the formula (11). The horizontal encounter distance between two aircrafts in point of CPA is 2633m \(HMD=2633\), \(\tau=61s\). Since this value of HMD is less than the minimum safe level separation distance 3000m, so the aircraft starts turn collision avoidance algorithms.

Figure-7 is curved that HMD and Tau values changing with a variation of aircraft left turning angle \(\theta\), when the aircraft is taking left turning collision avoidance. The conclusions are obtained by the MATLAB simulation.

From figure -7, when the aircraft started to turn left, the estimated horizontal distance \(\text{HMD}\) value decreases. HMD reaches minimum when the left turning angle is 12.2°, then gradually increases. When left turning angle is 48.0°, the estimated value of HMD reaches maximum value 12038m, at the same time, Tau reaches the minimum 0s. At this point the two aircrafts fly in the same direction, and the relative velocity is minimal, HMD is maximum. When left turning angle is 23.1°, HMD between the two aircrafts can achieve a minimum level of security interval which is 3004m, estimated Tau time is 106s. In the process of calculation of the turn collision avoidance, reacting time for pilots manoeuvre usually satisfied \(T≥15s\), which of course does not mean that alarm time threshold of the occurrence of TA or RA changes.

When left turning angle is between 23.1° and 47.2°, the horizontal meeting distance between the two aircrafts \(\text{HMD}\) is 3000 meters or bigger than minimal HMD, the estimated value of the Tau time \(≥15s\), therefore, this turning angle range of the aircraft is safe left turn collision avoidance range.

Figure-8 is curved that HMD and Tau values change with a vary of aircraft right turning angle \(\theta\) when the aircraft takes right turning collision avoidance. The conclusions are obtained by the MATLAB simulation.
Figure 6: The flow diagram of horizontal turn collision avoidance algorithm

Figure 7: The value of Tau and HMD when left turning angle is $\theta$
From figure 8, when the aircraft started to turn right, the estimated horizontal distance (HMD) value increases. Then HMD reaches maximum value at 12030 meters when the right turning angle is 124.7°, while Tau reaches the minimum 0s. At this time, the two aircrafts fly in an opposite direction, the relative velocity is the largest, the preserved time reaches the minimum. When right turning angle is 2.5°, the estimated HMD value is 3010 meters, while Tau (T) is 58s. When right turning angle is 72.2°, HMD=10264m, estimated Tau time is 15s. Therefore, the angle of safe right turning collision avoidance range is 2.5° to 72.2°.

The above calculation results show that aircrafts can avoid the intrusive aircraft by turning left or turning right. But the angle which original aircraft need to turn right is less than turn left, so combined ADS-B/TCAS collision avoidance system will give priority to the right turn as collision avoidance measure to reduce the impact and extent of the damage which turning avoidance make to make the aircraft's flight path reaches the minimum.

CONCLUSIONS

The paper raised an application method of traffic collision avoidance system according to the working principle and characteristics of ADS-B and TCAS by taking the advantages of ADS-B to conduct aircraft position state broadcasting. It improved the aircraft track accuracy judgment during the course of horizontal turn collision avoidance by calculating the ADS-B/TCASII fusion track. Based on the study of implementation methods and performance goals for the achievement of the horizontal turning collision avoidance, this paper analysed the horizontal turn collision avoidance detection model and proposed the algorithm flow of the horizontal turn collision avoidance, getting the related curve between the different turning angle and the corresponding change of horizontal miss distance (HMD) and time-to-go (Tau) by using MATLAB simulation. In addition, the horizontal turning collision avoidance algorithm, by making the accurate evaluation of the feasibility of process of horizontal turning collision avoidance, getting a preferable collision avoidance turning direction and angle. The study of the theories and algorithms provide references for the achievements of horizontal turning collision avoidance and is of practical significance to a combined ADS-B/TCAS collision avoidance systems in the future.

REFERENCES