The research of grey forecasting model and application on sport problem

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ABSTRACT

Results are the most important factor in competitive sports. If we can scientifically predict race results, it is bound to become a research focus of the sports competitions. In this paper, we use the gray prediction method and neural network theory to establish two prediction results model. Numerical experiments show that the model has high prediction accuracy.

KEYWORDS

Sport performance; Grey system; Forecast; Neural networks.

INTRODUCTION

How much of the human understanding of the real world divides the objective world into three systems: inside the system is completely full of white system; system of internal information to the outside world is ignorant, but only through their contact with the outside world to be observational study black system; the portion of the information within the system is known, another part of the information is unknown, and various factors in the system with uncertain relationship between gray system.

The gray prediction method is a method to predict the system contains both the known information and contains uncertainties. It is widely used to predict the process of randomly ordered gray, so as to find potential law. The gray prediction method distinguishes trends dissimilarity between system factors by correlation analysis method, and uses generation processing method to process raw data and generates strong regularity time series, and then establishes corresponding differential equation model, so features amount prediction things for a certain time in the future or the time of reaching a characteristic quantity.

With the rapid development of high-tech, the overall level of competitive sports has seen an unprecedented improvement. But understanding the laws affecting the sports performance of people is just like our understanding of human beings both familiar and unfamiliar. Obviously, from the existing knowledge of the human, Our awareness of sports law is limited. However, factors that affect sports performance is varied. We already know some, and some are still cognitive. So, from the view of system theory, we can consider the system of influencing sports performance as grey system. Therefore, the gray prediction method is very useful.

Neural network is inspired by the human brain and constitutes a network of information processing systems. It has highly nonlinear dynamic processing power and does not need to know the distribution of the data in the form and variable relationship. When the input and output relationship of some complex system is complex and difficult to use expression of the general formula, the neural network is very easy to implement.
highly nonlinear relationship. The neural network has achieved very good results in the field of applied research, for example pattern recognition, automatic control. In recent years, the neural network model is successfully applied to the economy forecast research. So, we use neural networks to study athletic performance prediction.

GREY FORECASTING MODEL

Typically, in the field of sports, orderly recording the best result of a continuous sport for several years, you can get a set of time series on the athletic performance.

In this paper, we forecast the results trend of the five-year results of female track and field one hundred meters race, and the performance sees TABLE 1.

<table>
<thead>
<tr>
<th>Year</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>Result</td>
<td>11.69</td>
<td>10.92</td>
<td>11.53</td>
<td>11.21</td>
<td>11.13</td>
</tr>
</tbody>
</table>

For the weakening of the randomness of the original time-series, first, the original time series data is processed by the data processing method of accumulated or accumulated save, the new time series is called generate columns.

The calculation method of one time cumulative is

\[ X^{(1)}(k) = X^{(1)}(k-1) + X^{(0)}(k) = \sum_{i=1}^{k} X^{(0)}(i) \]

where \( X^{(1)} = \{ X^{(1)}(1), X^{(1)}(2), \cdots, X^{(1)}(n) \} \) is generate columns and \( X^{(0)} = \{ X^{(0)}(1), X^{(0)}(2), \cdots, X^{(0)}(n) \} \) is original time series.

The calculation method of m time cumulative is

\[ X^{(m)}(k) = \sum_{i=1}^{k} X^{(m-1)}(i) \]

For non-negative data, the accumulating times is more, so the randomness of generated column is weaker. When the accumulate times are enough, the time series is changed from random sequence into non-random sequence. At this point, we generally use available exponential curve to approximate.

The calculation method of repeated reduce is

\[ X^{(1)}(k) = X^{(0)}(k) - X^{(0)}(k-1) \]

It subtracts both before and after data of original time series, so we get the generate columns.

Let \( X^{(0)}(1), X^{(0)}(2), X^{(0)}(3), X^{(0)}(4), X^{(0)}(5) \) be original time series, then we obtain the accumulated generating sequence

\[ X^{(1)} = \{ 11.69, 22.75, 34.33, 45.53, 56.71 \} \]

Establish model

Assume time series \( X^{(0)} \) has n observations,

\[ X^{(0)} = \{ X^{(0)}(1), X^{(0)}(2), \cdots, X^{(0)}(n) \} , \ X^{(i)} \text{ is the new sequence after accumulated generating}, \ X^{(1)} = \{ X^{(1)}(1), X^{(1)}(2), \cdots, X^{(1)}(n) \} \]. Then the differential equation is

\[ \frac{dX^{(1)}}{dt} + \alpha X^{(1)} = \mu \]

where \( \alpha \) is developed grey number and \( \mu \) is endogenous control grey number.

Let \( \hat{\alpha} = \left( \frac{\alpha}{\mu} \right) \) be estimated parameters vector, we use the method of least squares to get

\[ \hat{\alpha} = (B^TB)^{-1}B^TY_n \]

where

\[ B = \begin{bmatrix} -\frac{1}{2} [X^{(0)}(1) + X^{(0)}(2)] & 1 \\ -\frac{1}{2} [X^{(0)}(2) + X^{(0)}(3)] & 1 \\ \vdots & \vdots \\ -\frac{1}{2} [X^{(0)}(n-1) + X^{(0)}(n)] & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} = \begin{bmatrix} X^{(0)}(2) \\ X^{(0)}(3) \end{bmatrix} \]

We solve the differential equation and then get the prediction model:

\[ \hat{X}^{(1)}(k+1) = \left( X^{(0)}(1) - \frac{\mu}{\alpha} \right) e^{-\alpha k} + \frac{\mu}{\alpha} \]

, \( k = 0, 1, \cdots, n \).

In the example,

\[ B = \begin{bmatrix} -\frac{1}{2} [X^{(0)}(1) + X^{(0)}(2)] & 1 \\ -\frac{1}{2} [X^{(0)}(2) + X^{(0)}(3)] & 1 \\ -\frac{1}{2} [X^{(0)}(3) + X^{(0)}(4)] & 1 \\ -\frac{1}{2} [X^{(0)}(4) + X^{(0)}(5)] & 1 \end{bmatrix} \]

\[ Y_n = \begin{bmatrix} -17.25 \\ -28.56 \\ -39.92 \\ -51.09 \end{bmatrix} \]

, then

\[ \hat{\alpha} = \left( \frac{\alpha}{\mu} \right) = \left( \frac{-17.25}{-17.25} \right) \begin{bmatrix} 10.92 \\ 11.53 \\ 11.21 \\ 11.13 \end{bmatrix} \]
The research of grey forecasting model and application on sport problem

\[ B^T B = \begin{pmatrix} 5320.1 & -136.85 \\ -136.85 & 4 \end{pmatrix} \]

\[ (B^T B)^{-1} = \begin{pmatrix} 0.00157 & 0.05362 \\ 0.05362 & 2.08429 \end{pmatrix} \]

\[ B^T Y_n = \begin{pmatrix} -1537.36 \\ 44.8 \end{pmatrix}. \]

From \( \hat{\alpha} = (B^T B)^{-1} B^T Y_n \), we get

\[ \hat{\alpha} = \begin{pmatrix} -0.00175 \\ 11.15812 \end{pmatrix}. \]

So, we have \( \alpha = -0.00175, \mu = 11.15812 \). Then,

\[ \frac{dX^{(1)}}{dt} - 0.00175X^{(1)} = 11.15812 \]

\[ X^{(0)}(1) = 11.79 - 6359.37 \]

\[ X^{(0)}(1) - \mu = 11.79 + 6359.37 = 6371.16 \]

We get the prediction model

\[ X^{(1)}(k + 1) = 6371.16 e^{0.00175k} - 6359.37. \]

**Model check**

Grey prediction test usually contains residual test, association test and posterior test.

**Residual test**

We calculate \( \hat{X}^{(1)}(i) \) by prediction model, and repeated reduce \( \hat{X}^{(1)}(i) \) to generate \( \hat{X}^{(0)}(i) \), and then calculate absolute error sequence and relative error sequence of original sequence \( X^{(0)}(i) \) and \( \hat{X}^{(0)}(i) \), i.e.

\[ \Delta^{(0)}(i) = |X^{(0)}(i) - \hat{X}^{(0)}(i)| \]

\[ \Phi(i) = \frac{\Delta^{(0)}(i)}{X^{(0)}(i)}, \quad i = 1,2,\cdots,n. \]

In the example, we put \( k = 0,1,2,3,4 \) into the prediction model and then we get

\[ \hat{X}^{(1)} = \{11.79, 22.98, 34.19, 45.42, 56.67\} \]

\[ \hat{X}^{(0)} = \{11.79, 11.19, 11.21, 11.23, 11.24\} \]

\[ \Delta^{(0)} = \{0.024, 0.37, 0.01, 0.11\} \]

\[ \Phi = \{0.2\%, 3.2\%, 0.01\%, 0.97\%\}. \]

The relative error is less than 3.2\%, so, accuracy of the model is higher.

**Association test**

First, we calculate correlation coefficient

\[ \eta(i) = \frac{\min\{\Delta^{(0)}(i) + \rho \max\{\Delta^{(0)}(i)\}\}}{\Delta^{(0)}(i) + \rho \max\{\Delta^{(0)}(i)\}} \]

where \( \rho \) is resolution and generally take \( \rho = 0.5 \).

In this example,

\[ \min\{\Delta^{(0)}(i)\} = \min\{0.024, 0.37, 0.01, 0.11\} = 0 \]

\[ \max\{\Delta^{(0)}(i)\} = \min\{0.024, 0.37, 0.01, 0.11\} = 0.37 \]

So, the correlation coefficient is

\[ \eta(k) = \{1.044, 0.33, 0.99, 0.64\}. \]

Second, we calculate associate degree. Let arithmetic mean be \( \gamma = \frac{1}{n} \sum_{k=1}^{n} \eta(k) \), so \( \gamma \) is the associate degree of \( X^{(0)}(k) \) and \( \hat{X}^{(0)}(k) \).

\[ \gamma = \frac{1}{n} \sum_{k=1}^{n} \eta(k) = \frac{1}{5} (1 + 0.44 + 0.33 + 0.99 + 0.64) \]

\[ = 0.68. \]

So, the model satisfies \( \rho = 0.5 \) and \( \gamma > 0.6 \).

**Posterior test**

First, we calculate The standard deviation of original sequence:

\[ S_1 = \sqrt{\frac{\sum (X^{(0)}(i) - \bar{X}^{(0)})^2}{n - 1}} \]

\[ = \sqrt{\frac{642.33 - 56.71^2}{4}} = 0.35 \cdot \]

Second, we calculate absolute error sequence standard deviation:

\[ S_2 = \sqrt{\frac{\sum [\Delta^{(0)}(i) - \bar{\Delta}^{(0)}]^2}{n - 1}} \]

\[ = \sqrt{\frac{0.289 - 0.718^2}{4}} = 0.162. \]
Third, we calculate variance ratio:
\[ C = \frac{S_2}{S_1} = \frac{0.162}{0.35} = 0.463. \]

Finally, we calculate small error probability:
\[ P = p\left\{ |A^{(0)}(i) - \Delta^{(0)}| < 0.6743S_1 \right\}. \]

Let \( e_i = |A^{(0)}(i) - \Delta^{(0)}|, S_0 = 0.6743S_1 \), so
\[ P = p\{ e_i < S_0 \}, \text{ and we have } S_0 = 0.6743S_1 = 0.229, \text{ and } e_i = (0.145, 0.096, 0.228, 0.144, 0.037). \]

So, we have that \( e_i < S_0 \). Then, the model is qualified.

From the prediction formula
\[ X^{(0)}(k + 1) = X^{(1)}(k + 1) - X^{(1)}(k) = 6371.16e^{0.00175k}(e^{0.00175} - 1) \]
we take \( k = 5 \), so we get the result of 2012 is 11.21 second.

**NEURAL NETWORK PREDICTION MODEL**

**Introduction of neural network**

The neural network is able to simulate the human brain’s receive domain of partial adjustment and overwrite each, and does not have local minimum problems, and have learning fast and high fitting precision. It can change the weight value of the indicators, and to make it more consistent with the actual situation.

RBF neural network is three forward network. The first layer is the input layer and is composed of a signal source node; the second layer is hidden layer: the number of hidden units is determined by the needs of problem, and transformation function of hidden unit is RBF, which is nonlinear function; the third layer is output layer, it makes the role of the response to the input pattern. Since the mapping of input to output is nonlinear and the mapping of hidden layer space to output space is linear, so we can greatly accelerate the learning speed and avoid local minima problems.

The structure of RBF neural network is the following

RBF neural network can approximate any arbitrary precision continuous function, particularly suited to solve classification problems, approximation is shown below.

![Diagram of neural network](image1)

**Model establish**

Assume \( X = (x_1, x_2, \ldots, x_m)^T \) is output of network, \( H = (h_1, h_2, \ldots, h_m)^T \) be radial basis vector, where \( h_j \) is Gaussian basis functions. \( C_j = (c_{1j}, c_{2j}, \ldots, c_{nj})^T \) is center vector of network j-th node, \( B = (b_1, b_2, \ldots, b_m)^T \) is wide base vector, where \( b_j \) is base width parameter. The weight vector is \( W = (w_1, w_2, \ldots, w_m) \). So, the output of network at time k is
\[ y_m(k) = w_n h_1 + w_2 h_2 + \cdots + w_m h_m. \]

Assume \( y(k) \) is ideal output, we get the performance index function is
\[ E(k) = \frac{1}{2} (y(k) - y_m(k))^2. \]

**First layer: input layer**

From Figure 1, we have that the input layer has m
neurons. The input and output is
\[ I_i^1 = x_i \]
\[ O_j^1 = x_j, \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n. \]

**Second layer: hidden layer**

RBF neural network has n evaluation ranks. From Figure 1, we know that hidden layer has \( m \times n \) neurons. In this paper, we divide the evaluation rank into four kinds: \( \{A_y\} = \{ \text{excellent, good, qualified, unqualified} \} \), so we take \( n = 4 \), that is four fuzzy subset, and then we need four parameters \( a_1, a_2, a_3, a_4 \).

We use trigonometric function to represent the membership function \( \mu(x) \), see Figure 3.

When the output is \( I_i^2 = O_j^1, \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n \), the hidden layer will output all levels of membership value \( O_j^2 = A_y(x_j) \).

**Third layer: output layer**

Output layer mainly completes comprehensive evaluation of the input indicators, then we get the evaluation rank and evaluation vector:
\[ I_i^3 = O_j^2 \]
\[ O_1^3 = \sum_{j=1}^{m} w_j I_j^3 \]
where \( i = 1, 2, \ldots, m \) and \( j = 1, 2, \ldots, n \).

However, RBF neural network has shortcomings, for example, slow convergence, local minimum of the energy value and so on. In order to solving this problem, we use modified network to perfect network connection weight.

In the paper, we use the reverse of network to calculate the output value, and then get the error of output value and actual value, and then use the forward of network to test the obtained value and modify the connection weight value, so to achieve the purpose of reducing network error.

Let \( d_p = t_p - y_p \) be output error, so the error function is
\[ e_p = \frac{1}{2}(t_p - y_p)^2. \]
Then, we use the gradient descent method of RBF neural network learning algorithm to modify positive weight vector \( W \), so to achieve the purpose of reducing \( d_p \) and increasing the calculation accuracy. The gradient descent method is following:

Let
\[ w_j(k) = w_j(k-1) + \eta h_j (y(k) - y_m(k)) + \alpha w_j(k-1) - w_j(k-2) \]
and
\[ \Delta b_j = (y(k) - y_m(k)) w_j h_j \left\| X - C_j \right\|_{b_j} \]
so
\[ b_j(k) = b_j(k-1) + \eta \Delta b_j + \alpha (b_j(k-1) - b_j(k-2)) \].

Let
\[ \Delta c_{ij} = (y(k) - y_m(k)) w_j \frac{x_j - c_{ij}}{b_j^2} \],
so
\[ c_{ij}(k) = c_{ij}(k-1) + \eta \Delta c_{ij} + \alpha (c_{ij}(k-1) - c_{ij}(k-2)) \]
where \( \eta \) is learning rate, \( \alpha \) is momentum factor. Finally, using Jacobian array, we get the result
\[ \frac{\partial y(k)}{\partial u(k)} \approx \frac{\partial y_m(k)}{\partial u(k)} = \sum_{j=1}^{m} w_j h_j \frac{c_{ij} - x_i}{b_j^2}. \]

Assume \( \Delta W \) is the adjusted value of \( W \), using gradient descent method, we get the iterative algorithm formula
\[ \Delta W^{(n)} = -\eta \frac{\partial e_p}{\partial W} + \alpha \Delta W^{(n-1)}. \]
We use the formula of \( \Delta W \) to iterate. When the error is satisfied, we end the network training.

**Application**

In this paper, we use the male one hundred contest of Olympic game to train network. The data of male one hundred contest see TABLE 2.

From TABLE 2, we get that the calculated value obtained by neural network prediction model and the actual value have higher fitting accuracy.
CONCLUSION

In this paper, we use the gray prediction method and neural network theory to establish two prediction results model. Since the impact of competitive sports results can be attributed to the gray system, the gray prediction method can be used to forecast the future of sport achievements. We need to note that: calculation should be retained long enough significant digits and reduce calculation error.

Competitive sport is a multi-factor, multi-level, multi-target linkages and mutual restraint system. Its prediction of the results is very difficult. Traditional forecasting methods is based on the discrete recursive model and has obvious limitation. However, neural network identification is not subject to the limitations of the non-linear model. The neural network forecasting model comprehensively observes, analyzes and forecasts the development and changes in the system, so that the accuracy of the prediction is greatly improved.

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