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The new absolute stability of t-s fuzzy lurie control systems with multiple time-delays

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ABSTRACT

The absolute stability of a new class of Takagi-Sugeno (T-S) fuzzy Lurie control systems with multiple time-delays is considered in this paper. By utilizing the Lyapunov stability theory and the linear matrix inequality (LMI) approach, a novel delay-dependent absolutely stable condition is derived. In addition, by using Simulink toolbox in MATLAB, a simulation example is provided to demonstrate effectiveness of the proposed result.

KEYWORDS

Takagi-Sugeno (T-S) fuzzy Lurie systems; Absolute stability; Multiple time-delay; Lyapunov- Krasovskii functional (LKF); Linear matrix inequality (LMI).



INTRODUCTION

Since the notion of absolute stability was introduced by Lur'e^[1], the theory of absolute stability has occupied an important place among exact mathematical methods being used in the design and analysis of control systems. So far, as the time-delay phenomenon is frequently encountered in various of engineering systems such as chemical process, long transmission lines and so on, the stabilization of Lurie systems with time-delay has attracted a large amount of attention over the past years^[2,3]. In addition, several novel conditions for delay-dependent absolute stability of Lurie systems with multiple time-delays have been derived^[4,5] by employing the Linear matrix inequality (LMI) approach. The advantage of this method is that it uses free weighting matrices to express those relationships.

On the other hand, the Takagi-Sugeno (T-S) fuzzy models^[6] can provide an effective representation of complex nonlinear systems. It is known that such models can be used to describe a nonlinear system in the form of a weighted sum of some simple linear subsystems, and then the nonlinear system can be stabilized by a model-based fuzzy controller. So, many researchers have paid great attention to the stability analysis and control synthesis of T-S fuzzy systems with time-delay^[7,8].

However, as far as the authors know, the problem of the delay-dependent condition for absolute stability of T-S fuzzy Lurie systems with multiple delays were seldom studied up to now. We set a new T-S fuzzy Lurie control systems in this paper. Meanwhile, proper Lyapunov functions are defined and a novel delay-dependent absolutely stable condition is obtained by using the method of Lyapunov functional together with Linear matrix inequality (LMI) approach.

PROBLEM FORMULATION

The *i* th rule of the T-S fuzzy model for each $i = 1, 2, \dots, r$ is represented as follows:

Plant Rule *i*: If $s_1(t)$ is M_{i1} , $s_2(t)$ is M_{i2} , \dots , $s_g(t)$ is M_{ig} THEN

$$\begin{cases} \dot{x}(t) = A_i x(t) + \sum_{k=1}^m B_{ik} x(t - \tau_k) + D_i u(t) + b_i f(\sigma(t)), t \geq 0 \\ \sigma(t) = c^T x(t), \\ x(\theta) = \varphi(\theta), \quad \theta \in \left[-\max_{1 \leq k \leq m} \{\tau_k\}, 0\right] \end{cases} \tag{1}$$

Where $s_1(t), s_2(t), \dots, s_g(t)$ are the premise variables, M_{ij} ($j = 1, 2, \dots, g$) is a fuzzy set. $x(t) \in R^n$ denotes the state vector; A_i, B_{ik}, D_i ($i = 1, 2, \dots, r, k = 1, 2, \dots, m$) are the coefficient matrices with appropriate dimensions; $b_i \in R^n$ is the coefficient of the nonlinearities; $c \in R^n; \tau_k \geq 0$ ($k = 1, 2, \dots, m$) is the time-delay; $\varphi(\cdot) \in C\left[-\max_{1 \leq k \leq m} \{\tau_k\}, 0\right]$ is a continuous vector valued initial function; the nonlinearity functions $f(\cdot)$ satisfy the following sector condition:

$$f(\cdot) \in K[0, \infty] = \{f(\cdot) \mid f(0) = 0, 0 < \sigma f(\sigma(t)) < \infty, \sigma \neq 0\} \tag{2}$$

The dynamic fuzzy model can be represented in the following form:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^r h_i(s(t)) \left[A_i x(t) + \sum_{k=1}^m B_{ik} x(t - \tau_k) + D_i u(t) + b_i f(\sigma(t)) \right] \\ \sigma(t) = c^T x(t), \\ x(\theta) = \varphi(\theta), \quad \theta \in \left[-\max_{1 \leq k \leq m} \{\tau_k\}, 0\right] \end{cases} \tag{3}$$

where

$$h_i(s(t)) = \frac{\omega_i(s(t))}{\sum_{j=1}^r \omega_j(s(t))}, \tag{4}$$

$$\omega_i(s(t)) = \prod_{j=1}^g M_{ij}(s_j(t)), s(t) = [s_1(t), s_2(t), \dots, s_g(t)]^T, i = 1, 2, \dots, r,$$

in which $M_{ij}(s_j(t))$ is the grade of membership of $s_j(t)$ in M_{ij} . In this paper, It is assumed that $\omega_i(s(t)) \geq 0, \sum_{j=1}^r \omega_j(s(t)) > 0, i = 1, 2, \dots, r, \forall t \geq 0$.

Hence, the fuzzy basis functions satisfy $\sum_{i=1}^r h_i(s(t)) = 1$ with $h_i(s(t)) \geq 0, i = 1, \dots, r, \forall t \geq 0$.

Control rule *i* : IF $s_1(t)$ is $M_{i1}(t), s_2(t)$ is $M_{i2}(t), \dots, s_r(t)$ is $M_{ir}(t)$, THEN

$$u(t) = K_i x(t), \quad i = 1, 2, \dots, r$$

where $K_i \in R^{q \times n}$ ($i = 1, 2, \dots, r$) is the local controller gain. Using the fuzzy basis functions defined by (4), the overall fuzzy state feedback controller is represented by :

$$u(t) = \sum_{i=1}^r h_i(s(t)) K_i x(t)$$

Throughout this paper, we shall use the following lemmas:

Lemma 1. (see [9].) For any constant symmetric matrix $R, M = M^T > 0$, scalar $r > 0$, vector function $g : [0, r] \rightarrow R^n$, such that the integrations in the following are well defined, then

$$r \int_0^r g^T(s) M g(s) ds \geq \left[\int_0^r g(s) ds \right]^T M \left[\int_0^r g(s) ds \right]$$

Lemma 2. (see Schur complement [10].) Given constant symmetric matrices $\Sigma_1, \Sigma_2, \Sigma_3$ with appropriate dimensions, where $\Sigma_1 = \Sigma_1^T$ and $\Sigma_2 = \Sigma_2^T$, then $\Sigma_1 + \Sigma_3^T \Sigma_2^{-1} \Sigma_3 < 0$, holds if and only if

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0 \text{ or } \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{bmatrix} < 0$$

MAIN RESULTITS

Theorem1. The system (3) is absolutely stable, if there exist matrix $K_i (i = 1, 2, \dots, r)$ and symmetric positive definite matrices $\tilde{P}, \tilde{Q}_k, \tilde{R}_k (k = 1, 2, \dots, m)$ and scalar $\tilde{\alpha} > 0, \tilde{\beta} > 0$, such that the following LMIs hold:

$$E_{ii} = \begin{bmatrix} W_{1ii} & W_2 \\ * & W_3 \end{bmatrix} < 0, i = j, \tag{5}$$

with

$$W_{1ii} = \begin{bmatrix} 2\Gamma_{1ii} & 2\Gamma_{2i} & 2\Gamma_{3ii} & 0 \\ * & \tilde{Q} & 2\Gamma_{4i} & 0 \\ * & * & \Gamma_{5i} & 0 \\ * & * & * & \tilde{W}_3 \end{bmatrix}, W_2 = \begin{bmatrix} \tilde{Q} & \tau_1 \tilde{R}_1 & \tau_2 \tilde{R}_2 & \dots & \tau_m \tilde{R}_m \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}$$

$$W_3 = \begin{bmatrix} -\tilde{Q} & 0 & 0 & \cdots & 0 \\ * & 0 & 0 & \cdots & 0 \\ * & -\tilde{R}_1 & 0 & \cdots & 0 \\ * & * & -\tilde{R}_2 & \cdots & 0 \\ * & * & * & \ddots & 0 \\ * & * & * & \cdots & -\tilde{R}_m \end{bmatrix},$$

and $E_{ij} = \begin{bmatrix} \Xi_{1ij} & \Xi_2 \\ * & \Xi_3 \end{bmatrix} < 0, \quad 1 \leq i < j \leq r,$ (6)

$$\Xi_{1ij} = \begin{bmatrix} \Gamma_{1ij} + \Gamma_{1ij}^T & \Gamma_{2i} + \Gamma_{2i}^T & \Gamma_{3ij} + \Gamma_{3ij}^T & 0 \\ * & 2\tilde{Q} & \Gamma_{4i} + \Gamma_{4i}^T & 0 \\ * & * & \Gamma_{5i} & 0 \\ * & * & * & 2\tilde{W}_3 \end{bmatrix},$$

$$\Xi_2 = \begin{bmatrix} \tilde{Q} & \tilde{Q} & \tau_1 \tilde{R}_1 & \tau_1 \tilde{R}_1 & \tau_2 \tilde{R}_2 & \tau_2 \tilde{R}_2 & \cdots & \tau_m \tilde{R}_m & \tau_m \tilde{R}_m \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix},$$

$$\Xi_3 = \begin{bmatrix} -\tilde{Q} & 0 & 0 & 0 & \cdots & 0 & 0 \\ * & -\tilde{Q} & 0 & 0 & \cdots & 0 & 0 \\ * & * & -\tilde{R}_1 & 0 & \cdots & 0 & 0 \\ * & * & * & -\tilde{R}_1 & \cdots & 0 & 0 \\ * & * & * & * & \ddots & 0 & 0 \\ * & * & * & * & \cdots & -\tilde{R}_m & 0 \\ * & * & * & * & \cdots & * & -\tilde{R}_m \end{bmatrix},$$

Let * denotes the elements below the main diagonal of a symmetric block matrix,

$$\Gamma_{1ij} = \tilde{P}A_i + \tilde{P}D_iK_j, \Gamma_{2i} = [\tilde{P}B_{i1}, \tilde{P}B_{i2}, \dots, \tilde{P}B_{im}], \Gamma_{3ij} = \tilde{P}b_i + \tilde{\beta}A_i^Tc + \tilde{\beta}K_j^T D_i^T + \tilde{\alpha}c,$$

$$\Gamma_{4i} = [\tilde{\beta}B_{i1}^Tc, \tilde{\beta}B_{i2}^Tc, \dots, \tilde{\beta}B_{im}^Tc], \Gamma_{5i} = 2\tilde{\beta}b_i^Tc, \tilde{W}_3 = \text{diag}[-\tilde{R}_1, -\tilde{R}_2, \dots, -\tilde{R}_m],$$

$$\tilde{Q} = \text{diag}[-\tilde{Q}_1, -\tilde{Q}_2, \dots, -\tilde{Q}_m], \tilde{Q} = \sum_{k=1}^m \tilde{Q}_k. \quad (1 \leq i < j \leq r).$$

Proof. We define the Lyapunov-Krasovskii functional candidate as follows :

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \tag{7}$$

where

$$V_1(t) = x^T(t) \tilde{P}x(t), \quad V_2(t) = \sum_{k=1}^m \int_{t-\tau_k}^t x^T(s) \tilde{Q}_k x(s) ds,$$

$$V_3(t) = \sum_{k=1}^m \tau_k \int_{t-\tau_k}^0 d\xi \int_{t+\xi}^t x^T(s) \tilde{R}_k x(s) ds, \quad V_4(t) = 2\tilde{\beta} \int_0^{\sigma(t)} f(\sigma) d\sigma.$$

Then, the time derivative of $V(t)$ along the trajectory of system (3) is given by

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t), \tag{8}$$

with

$$\dot{V}_1(t) = 2x^T(t) \tilde{P}\dot{x}(t)$$

$$= 2 \sum_{i=1}^r h_i(s(t)) x^T(t) \tilde{P} \left[A_i x(t) + \sum_{k=1}^m B_{ik} x(t-\tau_k) + D_i u(t) + b_i f(\sigma(t)) \right], \tag{9}$$

$$\dot{V}_2(t) = \sum_{k=1}^m x^T(t) \tilde{Q}_k x(t) - \sum_{k=1}^m x^T(t-\tau_k) \tilde{Q}_k x(t-\tau_k), \tag{10}$$

$$\dot{V}_3(t) = \sum_{k=1}^m \tau_k^2 x^T(t) \tilde{R}_k x(t) - \sum_{k=1}^m \int_{t-\tau_k}^t x^T(s) \tau_k \tilde{R}_k x(s) ds.$$

By Lemma1, we have

$$\dot{V}_3(t) \leq \sum_{k=1}^m \tau_k^2 x^T(t) \tilde{R}_k x(t) - \sum_{k=1}^m \left[\int_{t-\tau_k}^t x(s) ds \right]^T \tilde{R}_k \left[\int_{t-\tau_k}^t x(s) ds \right]. \tag{11}$$

Using (1) and (2),the following equation holds

$$\dot{V}_4(t) = 2\tilde{\beta} \dot{\sigma}^T(t) f(\sigma(t))$$

$$= 2\tilde{\beta} \sum_{i=1}^r h_i(s(t)) \left[A_i x(t) + \sum_{k=1}^m B_{ik} x(t-\tau_k) + D_i u(t) + b_i f(\sigma(t)) \right]^T cf(\sigma(t))$$

$$\leq 2\tilde{\beta} \sum_{i=1}^r h_i(s(t)) \left[A_i x(t) + \sum_{k=1}^m B_{ik} x(t-\tau_k) + D_i u(t) + b_i f(\sigma(t)) \right]^T$$

$$cf(\sigma(t)) + 2\tilde{\alpha} x^T(t) cf(\sigma(t)). \tag{12}$$

From (8)~(12), we obtain

$$\dot{V}(t) \leq \sum_{i=1}^r h_i(s(t)) \left\{ 2x^T(t) \tilde{P} \left[A_i x(t) + \sum_{k=1}^m B_{ik} x(t-\tau_k) \right. \right.$$

$$\left. \left. + D_i \sum_{j=1}^r h_j(s(t)) K_j x(t) + b_i f(\sigma(t)) \right] + \sum_{k=1}^m x^T(t) \tilde{Q}_k x(t) \right.$$

$$\left. - \sum_{k=1}^m x^T(t-\tau_k) \tilde{Q}_k x(t-\tau_k) + \sum_{k=1}^m \tau_k^2 x^T(t) \tilde{R}_k x(t) - \sum_{k=1}^m \left[\int_{t-\tau_k}^t x(s) ds \right]^T \tilde{R}_k \left[\int_{t-\tau_k}^t x(s) ds \right] \right.$$

$$\left. + 2\tilde{\beta} \left[A_i x(t) + \sum_{k=1}^m B_{ik} x(t-\tau_k) + D_i \sum_{j=1}^r h_j(s(t)) K_j x(t) + b_i f(\sigma(t)) \right]^T \right.$$

$$\left. cf(\sigma(t)) + 2\tilde{\alpha} x^T(t) cf(\sigma(t)) \right\}$$

$$= \sum_{i=1}^r \sum_{j=1}^r h_i(s(t)) h_j(s(t)) \eta^T(t) \Omega_{ij} \eta(t),$$

$$\tag{13}$$

here we define :

$$\begin{aligned}
 \eta(t) &= [x^T(t) \quad E_1^T(t) \quad f^T(\sigma(t)) \quad E_2^T(t)]^T, \\
 E_1(t) &= [x^T(t-\tau_1) \quad x^T(t-\tau_2) \cdots x^T(t-\tau_m)]^T, \\
 E_2(t) &= \left[\int_{t-\tau_1}^t x^T(s) ds \quad \int_{t-\tau_2}^t x^T(s) ds \cdots \int_{t-\tau_m}^t x^T(s) ds \right]^T, \\
 \Omega_{ij} &= \begin{bmatrix} \Gamma_{1ij} + \Gamma_{1ij}^T + \sum_{k=1}^m (\tilde{Q}_k + \tau_k^2 \tilde{R}_k) & 2\Gamma_{2i} & \Gamma_{3ij} + \Gamma_{3ij}^T & 0 \\ * & \tilde{Q} & 2\Gamma_{4i} & 0 \\ * & * & 2\tilde{\beta}b_i^T c & 0 \\ * & * & * & 2\tilde{W}_3 \end{bmatrix}, \tag{14}
 \end{aligned}$$

where $\Gamma_{1ij}, \Gamma_{2i}, \Gamma_{3ij}, \Gamma_{4i}, \tilde{W}_3$ ($1 \leq i < j \leq r$) are the same as the corresponding items in inequalities (5) and (6). If $\Omega_{ij} < 0$, then there exists a sufficient small scalar $\varepsilon > 0$, such that $\dot{V}(t) \leq -\varepsilon \|x(t)\|^2$ (for $x(t) \neq 0$), which shows that the T-S fuzzy Lurie system with multiple time-delays described by (3) is absolutely stable. Using the Schur complement formula in Lemma2, we know that $\Omega_{ij} < 0$ is equivalent to (5) and (6). This completes the proof.

SIMULATION EXAMPLE

In this example, the T-S fuzzy Lurie system with multiple time-delays considered is with $u(t) = 0$ and two rules for $i = 2$, $m = 1$.

Plant Rules.

Rule1: IF $s_1(t)$ is M_{11} Then

$$\begin{cases} \dot{x}(t) = A_1 x(t) + B_{11} x(t - \tau_1) + b_1 f_1(\sigma(t)), & t \geq 0 \\ \sigma(t) = c^T x(t), \\ x(\theta) = \varphi(\theta), & \theta \in [-\tau_1, 0], \end{cases}$$

Rule2: IF $s_1(t)$ is M_{21} Then

$$\begin{cases} \dot{x}(t) = A_2 x(t) + B_{21} x(t - \tau_1) + b_2 f_2(\sigma(t)), & t \geq 0 \\ \sigma(t) = c^T x(t), \\ x(\theta) = \varphi(\theta), & \theta \in [-\tau_1, 0], \end{cases}$$

with $f_i(\sigma(t)) = \tan(\sigma(t))$ ($i = 1, 2$), $\tau_1 = 1.0$. The fuzzy basis functions for Rule 1 and Rule 2 are $h_1(s_1(t)) = \sin^2(\pi s_1(t))$, $h_2(s_1(t)) = \cos^2(\pi s_1(t))$.

We suppose

$$\begin{aligned}
 A_1 &= \begin{bmatrix} -2 & 0 \\ -1 & -2 \end{bmatrix}, A_2 = \begin{bmatrix} -3 & 0 \\ -1 & -3 \end{bmatrix}, B_{11} = \begin{bmatrix} -0.2 & -0.5 \\ 0.5 & -0.2 \end{bmatrix}, B_{21} = \begin{bmatrix} -0.1 & -0.45 \\ 0.4 & -0.1 \end{bmatrix}, \\
 b_1 &= \begin{bmatrix} -0.5 \\ -0.3 \end{bmatrix}, b_2 = \begin{bmatrix} -0.2 \\ -0.3 \end{bmatrix}, c = \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}
 \end{aligned}$$

then

$$\begin{aligned}
 \tilde{P} &= \begin{bmatrix} 2.3199 & -0.7415 \\ -0.7415 & 2.2759 \end{bmatrix}, \tilde{Q}_1 = \begin{bmatrix} 2.3534 & 0.2223 \\ 0.2223 & 2.4531 \end{bmatrix}, \tilde{R}_1 = \begin{bmatrix} 1.9443 & 0.2483 \\ 0.2483 & 2.3020 \end{bmatrix}, \\
 \tilde{\alpha} &= 2.7076, \tilde{\beta} = 0.7563,
 \end{aligned}$$

Using the MATLAB Simulink Toolbox, the state response is shown in Fig.1.

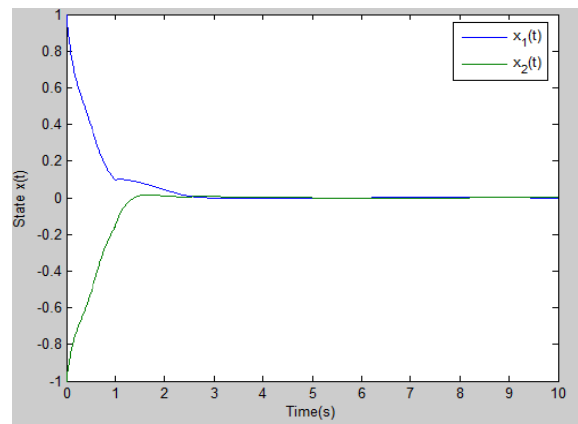


Figure 1: Response of the state $x(t)$

The simulation result in Fig.1 has shown that, under the conditions of Theorem 1, the T-S fuzzy Lurie control system with time-delays is absolutely stabilized.

CONCLUSION

The absolute stability for a class of T-S fuzzy Lurie control systems with multiple time-delays is considered in this paper. By taking the advantage of T-S fuzzy model, a new system is created, which represents a new trend of research for Lurie control system. Using the Lyapunov -Krasovskii functional (LKF) and the linear matrix inequality (LMI) approach, a new delay-dependent condition for such systems is obtained and described in the form of LMI, which is different from existing ones. Finally, the simulation result has shown that the proposed results are effective.

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