The best weight loss effect study based on stride frequency and energy consumption – taking obese youth as study object

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ABSTRACT
Along with social progress and the development of science and technology, people’s material life has been greatly improved, so there appears a growing number of obese youth; the incidence of this group is higher, the impact on families and community is larger, and thus many experts put forward their own views for weight loss exercise. Through the study on the relationship between the walking and running sport - energy consumption - fat reduction, this paper proposes a weight loss strategy by prolonged low-intensity exercise from a optimal mathematical point of view; in order to study the relationship walking and running and energy, the paper first studies the relationship between the heart rate and the two respectively; in the study course it uses a non-equidistant Logistic model, and obtains reasonable results through the model; in addition this paper also establishes the optimization model of fat consumption and in constraint condition of stride frequency in order to verify the reliability of Logistic model; moreover, it establishes the mechanism model of fat consumption, the results validate the error between the two is within 10%, obtains the optimal exercise strategy with exercise time 120 minutes and stride frequency 140/min; in this mode the exercise can reach a theory effect of weekly weight loss 49g; the experiment proves that the result is a safe range for weight loss, which provides a theoretical basis and methods for the weight loss exercise of majority of obese youth. © 2013 Trade Science Inc. - INDIA

KEYWORDS
Logistic model; Optimization model; Fat consumption; Weight loss strategies.

INTRODUCTION
The walking and running sport is one of the simplest movements with highly feasibility. In the eyes of the majority of obese young people it is a good weight loss tool. In the end it is questionable how to achieve the best weight loss result. Since people will appear the energy consumption phenomenon in the walking and running course, and the energy can establish a relationship between the sugar and fat in the human body, the safety maximized fat consumption is the purpose that obese people are trying to achieve. Thus this paper studies the mathematical model of energy consumption and fat consumption during the walking and running exercise, in order to provide the best weight loss strategy for the majority of dieters from a mathematical point of
view in conditions of security.

Many people have made efforts for the weight loss during walking and running exercise and the Logistic model and mechanism model of fat consumption used in this article. It is these people’s efforts that make the weight loss exercise and various model have been greatly developed. For the research of weight loss exercise, some people have made a reasonable point. On the basis of previous efforts this article establishes the function relationship of stride frequency and heart rate using Logistic gray model, uses the model to study the function relationship of the fat consumption ratio; on the determination of the fat consumption ratio, it establishes an optimization model of maximized fat consumption, and through the mechanism model of fat consumption test the gray models, this study expects to provide weight loss strategy with high feasibility for obese youth.

ENERGY CONSUMPTION MODEL

The linear regression equation of per unit energy consumption

The speed of young people aged 15-25 in the walking and running process is usually 2-10km/h, according to gender the linear regression equation between per unit energy consumption and stride frequency is shown as the formula (1) and formula (2):

\[ EE = a_1 F + b_1 \text{ (male)} \] (1)
\[ EE = a_2 F + b_2 \text{ (female)} \] (2)

In Formula (1) and (2) EE indicates the unit energy consumption (kcal/min/kg), F means the frequency step (l/min); in order to more accurately describe the model, this paper introduces real-time heart rate when studying the object’s movement, and at this time the linear regression equation of the energy consumption per unit is shown in formula (3) and (4) below:

\[ EE = \alpha_1 F + \beta_1 H + \gamma_1 \text{ (male)} \] (3)
\[ EE = \alpha_2 F + \beta_2 H + \gamma_2 \text{ (female)} \] (4)

The real time heart rate during the movement is represented by H in the formula (3) and (4), its unit is l/min, which is consistent with the step frequency dimensionally.

The calculation method of the energy consumption

According to the linear regression equation obtained from formula (3) and (4), the function of the gender, stride frequency, and heart rate and energy consumption can be derived as \( f(F_i, H_i) \), then conduct the integral of this function over time \( t \), and multiplied by the body weight \( w \), the internal energy consumption value \( E \) of the study object can be drawn within a period of time, as shown in the formula (5) below:

\[ E = w \times \int_0^T f(F_i, H_i)dt = \sum_{i=1}^{n} f(F_i, H_i) \times w \times t_i \] (5)

In Formula (5) \( T \) means the total time of walking and running, \( t \) means the time of duration for the \( i \)-th segment stride frequency and its unit is min; the accumulation part will divide the different stride frequencies of a period of time into \( n \) segments when walking and running.

Usually it is more difficult to collect the real time heart rate of the study object in the movement; in order to facilitate the study, we can take the regression equation (1) and (2) instead without the heart rate, then the expression of the internal energy consumption \( E \) is in the formula (6):

\[ E = w \times \int_0^T f(F_i)dt = \sum_{i=1}^{n} f(F_i) \times w \times t_i \] (6)

The maximization model of fat consumption

For the effect of weight loss by exercise, we often focus on the exercise intensity and exercise time these two key factors, the two determine the body fat consumption ratio and consumption value in the motion course. From a biological point of view, during the exercise of certain intensity, the lengthening of movement time can achieve the effect of an increase in fat consumption ratio. The key question of lose weight by exercise is to achieve the optimal exercise weight loss under what kind of exercise intensity. Moreover the personalized exercise capacity has a direct relationship with the relative maximal oxygen uptake of the human body, the determination for maximal oxygen uptake and oxygen uptake function has a vital role to determine the ratio of fat consumption and exercise intensity function.
The maximal oxygen uptake function can be obtained by using statistical regression method, we use the regression model in the literature [7] (Li Qiang, 2007), as shown in the formula (7) below:

\[ VO_{2\text{max}} = (34.777 + 11.685 \times s - 0.808 \times \text{BMI} + 0.159 \times h) \times W \]  

(7)

In Formula (7) \( VO_{2\text{max}} \) indicates the maximal oxygen uptake, \( s \) means the sex, it is \( s = 1 \) for male and \( s = 0 \) for female, \( h_{\text{estimated}} \) indicates the estimated heart rate reserve (maximum heart rate minus basal HR), and \( W \) means the weight.

Because the same volume of oxygen is consumed, the sugar and fat release energy substantially equal to the [8], and the movement oxygen uptake functions can be obtained by the energy consumption formula, as shown in formula (8):

\[ V = 207.29E + 9.328 \]  

(8)

Substitute the formula (1) into formula (8); the formula (9) can be obtained:

\[ V = 207.29(aF + b)w + 9.328 \]  

(9)

In Formula (9), \( V \) represents the oxygen uptake, and \( E \) means the energy expenditure value per unit of time.

In order to exclude the effect of exercise time on fat consumption, the paper makes the following two assumptions:

Assumption 1: fat consumption ratio during exercise and relative exercise intensity has a linear relationship on the local;

Assumption 2: with different relative exercise intensity, the size relationship between the fat consumption ratios does not change during the same exercise time.

In the above hypothetical premise, suppose \( p \) indicates that the ratio between the motion oxygen uptake and the maximal oxygen uptake, \( z \) means the consumption ratio of the fat; based on the experimental data when is 40\% \( VO_{2\text{max}} \), the fat consumption ratio was 48\% after exercise for 20 minutes, when \( p \) is 50\% \( VO_{2\text{max}} \), the ratio function of the fat is 40\%, thus the linear equation can be obtained by formula (10) below:

\[ z = -0.8p + 0.8 \]  

(10)

If bring \( p \) into the formula (10) then the relation-

ship shown in formula (11) can be obtained:

\[ z = \frac{-165.832(aF + b)w - 7.4624}{(34.777 + 11.685 \times s - 0.808 \times \text{BMI} + 0.159 \times h) \times W + 0.8} \]  

(11)

Thus the function relationship between fat and stride frequency can be established and find the maximum value, the constraint condition of the objective function is the heart rate’s change range, the mathematical model of the maximized fat consumption is in the formula (12):

\[
\begin{align*}
\max & \quad E \times z \\
\text{s.t.} & \quad H \in [h_0, h_m] \\
& \quad \text{Maximized fat consumption}\n\end{align*}
\]  

(12)

In Formula (12) \( h_0 \) indicates the heart rate in the rest state, \( h_m \) means the maximum safe heart rate that can be achieved in the motion state, typically the computational formula of is shown in the formula (13) below:

\[ h_m = (220 - Age) \times 85\% \]  

(13)

Mechanistic model of fat consumption

Medical science shows that the ratio of sugar consumption ratio can promote the speed of fat consumption ratio, and the speed of fat consumption ratio also receives the inhibition of self-consumption ratio; the corresponding coefficients are denoted by \( a \) and \( -b \), the ratio of fat consumption is denoted by \( z(t) \), the mathematical model of fat consumption over time is shown in formula (14) below:

\[
\begin{align*}
\frac{dz}{dt} = a(1 - z) - bz \\
z(t_0) = z_0 \\
\end{align*}
\]  

(14)

The analytical solution of the ordinary differential equation (14) determined by the initial value is in formula (15) below:

\[ z(t) = \frac{a}{a + b} - \left( \frac{a}{a + b} - z_0 \right) e^{-(a+b)t} \]  

(15)

When the object’s movement reaches a certain time, the body’s breath basically tends to be stable; that is, when the \( t \to \infty \), the consumption ratio of fat and sugar reaches a stable value, as in the formula (16) below:

\[
\lim_{t \to \infty} z(t) = \frac{a}{a + b} 
\]  

(16)
When the original value is \( z_0 = 0.325 \), the formula (16) is as formula (17) below:

\[
z(t) = c_1 - (c_2 - 0.325)e^{-c_2 t}
\]

Therefore, according to the least square method, we have the formula (18):

\[
\min \sum_{k=1}^{n} \left[ c_1 - (c_1 - 0.325)e^{-c_2 t_k} - i_k \right]^2
\]

When the value of \( c_1, c_2 \) is determined, then the value of \( a, b \) can be determined.

**STUDY RESULTS AND ANALYSIS**

The purpose of establishing gray system is to seek the variation law of the system development by the original data’s collection and collation. The information displayed by the raw data is often more complex, and it is difficult to find the variation law. But gray system theory thinks that, the gray sequences of any system can weaken the randomness of data display by some generate, so that it can reflect the regularity of the information and interpret the nature of the system. In this paper, it uses Gray Logistic model to predict the relationship between stride frequency and personalized heart rate, in order to establish the relationship between stride frequency and human energy consumption during movement through personalized heart rate.

### The data relationship between stride frequency and heart rate

**TABLE 1 : The comparison table of stride frequency and heart rate**

<table>
<thead>
<tr>
<th>Gender</th>
<th>Age</th>
<th>Height</th>
<th>Body weight</th>
<th>Cadence</th>
<th>Heart Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>18</td>
<td>183cm</td>
<td>92kg</td>
<td>0</td>
<td>78</td>
</tr>
<tr>
<td>Male</td>
<td>18</td>
<td>183cm</td>
<td>92kg</td>
<td>104</td>
<td>105</td>
</tr>
<tr>
<td>Male</td>
<td>18</td>
<td>183cm</td>
<td>92kg</td>
<td>111</td>
<td>110</td>
</tr>
<tr>
<td>Male</td>
<td>18</td>
<td>183cm</td>
<td>92kg</td>
<td>126</td>
<td>129</td>
</tr>
<tr>
<td>Male</td>
<td>18</td>
<td>183cm</td>
<td>92kg</td>
<td>163</td>
<td>163</td>
</tr>
</tbody>
</table>

The trend chart of heart rate with the stride frequency can be obtained from TABLE 1, as shown in Figure 1:

By a non-equidistant Logistic gray modeling theory, the relationship between heart rate and stride frequency can be drawn as in formula (19) below:

\[
H = 45.604e^{0.008F}
\]

**Figure 1 : The trend chart of heart chart – stride frequency**

In Formula (19), \( F \) means the stride frequency, \( H \) means the heart rate corresponding to the stride frequency; by the formula (19) the function relationship of stride frequency and heart rate can be drawn as formula (20) below:

\[
F = 125\ln \left(0.02193H\right)
\]

In Formula (20), when \( F \) is 0, the corresponding heart rate value is 45, which has a larger difference with the data in TABLE 1; thus the applied range of the function relationship shown in formula (20) is the relationship between heart rate and stride frequency in the movement process.

By the determined function in formula (20) the optimum model can be determined as in formula (21) below:

\[
\begin{align*}
\max & (aF + b) \\
& = \left\{ \frac{-165.832(aF + b) - 7.4624\max}{34.777 + 11.685r - 0.808\max + 0.1596_{\rm{min}}} + 0.8 \right\} \\
& \quad \text{if} \quad F \in [40, a, \ln b, b]
\end{align*}
\]

By the formula (21) the optimal stride frequency is \( 1400 \sim 150\text{l/min} \), the relative motion intensity is \( 46.9\% \text{VO}_{2\max} \), similarly the optimal stride frequency for women aged 18 is \( 122\text{l/min} \) and the corresponding motion intensity is \( 53\% \text{VO}_{2\max} \).

**Research on the fat consumption ratio data**

**TABLE 2 : The fat consumption ratio data**

<table>
<thead>
<tr>
<th>Time(min)</th>
<th>Fat consumption ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.325</td>
</tr>
<tr>
<td>20</td>
<td>0.425</td>
</tr>
<tr>
<td>30</td>
<td>0.493</td>
</tr>
<tr>
<td>60</td>
<td>0.737</td>
</tr>
<tr>
<td>120</td>
<td>0.808</td>
</tr>
<tr>
<td>180</td>
<td>0.844</td>
</tr>
<tr>
<td>240</td>
<td>0.880</td>
</tr>
</tbody>
</table>
From the data in TABLE 2 the trend figure of fat consumption ratio over time can be obtained, as shown in Figure 2:

![Figure 2: The trend figure of fat consumption ratio - time](image)

According to the data in TABLE 2 and the Logistic model with unequal interval, the function relationship between the fat consumption ratio and the time can be drawn, as shown in the formula (22) below:

$$Z(t) = \frac{0.8637}{1 + 1.6575e^{-0.0313t}}$$ (22)

The data in TABLE 3 shows the average error between the analog data and the experimental data is 3.9%.

### TABLE 3: The error comparison between the experimental data and the model data

<table>
<thead>
<tr>
<th>Time(min)</th>
<th>Fat consumption ratio</th>
<th>Analog data</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.325</td>
<td>0.325</td>
<td>0.0%</td>
</tr>
<tr>
<td>20</td>
<td>0.425</td>
<td>0.458</td>
<td>7.7%</td>
</tr>
<tr>
<td>30</td>
<td>0.493</td>
<td>0.524</td>
<td>6.3%</td>
</tr>
<tr>
<td>60</td>
<td>0.737</td>
<td>0.689</td>
<td>6.5%</td>
</tr>
<tr>
<td>120</td>
<td>0.808</td>
<td>0.831</td>
<td>2.9%</td>
</tr>
<tr>
<td>180</td>
<td>0.844</td>
<td>0.859</td>
<td>1.7%</td>
</tr>
<tr>
<td>240</td>
<td>0.880</td>
<td>0.863</td>
<td>1.9%</td>
</tr>
</tbody>
</table>

### Solution method on fat consumption mechanistic model

Using Gauss - Newton method to conduct least-squares fitting on the data in TABLE 3, the results can be obtained by formula (23) and the judgment matrix of the second layer indicators is shown in formula (23):

$$\begin{align*}
  c_1 &= 0.8958 \\
  c_2 &= 0.0153 \\
  z(t) &= 0.8958 - (0.8958 - 0.325)e^{-0.0153t} \\
        &= 0.9395 - 0.5708e^{-0.0153t}
\end{align*}$$ (23)

The data in TABLE 4 shows, the average relative error of the simulated data obtained by the fat consumption mechanism model is 4.6%.

### TABLE 4: The comparison between data simulation results and experimental data of the mechanistic model

<table>
<thead>
<tr>
<th>Time(min)</th>
<th>Fat consumption ratio</th>
<th>Analog data</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.325</td>
<td>0.325</td>
<td>0.0%</td>
</tr>
<tr>
<td>20</td>
<td>0.425</td>
<td>0.476</td>
<td>11.9%</td>
</tr>
<tr>
<td>30</td>
<td>0.493</td>
<td>0.535</td>
<td>8.5%</td>
</tr>
<tr>
<td>60</td>
<td>0.737</td>
<td>0.668</td>
<td>9.4%</td>
</tr>
<tr>
<td>120</td>
<td>0.808</td>
<td>0.805</td>
<td>0.4%</td>
</tr>
<tr>
<td>180</td>
<td>0.844</td>
<td>0.860</td>
<td>1.8%</td>
</tr>
<tr>
<td>240</td>
<td>0.880</td>
<td>0.881</td>
<td>0.1%</td>
</tr>
</tbody>
</table>

### STUDY RESULTS

Use the unequal interval Logistic model and fat consumption mechanism model as the ratio function of fat consumption, the ratio function is multiplied by the total energy consumption function, and then the consumption fat can be obtained by integrating this product in time; Suppose a man carries through walking and running movement with a speed of 1401/min, the fat consumption is the maximum, exercise lasts for 120 minutes, and using the non equidistant Logistic model as fat consumption ratio function, then the results in formula (24) can be obtained below.

$$f(t) = 92.5 \times \frac{e^{0.8637(0.9395 - 0.5708e^{-0.0153t})}}{1+1.6575e^{-0.0313t}} \, dt = 645\text{kcal}$$ (24)

If we take the fat consumption mechanism model as the fat consumption ratio function, then the results can be derived as shown in formula (25).

$$f(t) = 92.5 \times \int_{0}^{t} (a \times x + b) \times e^{0.8637(0.9395 - 0.5708e^{-0.0153t})} \, dt = 664.3\text{kcal}$$ (25)

Formula (25) shows that a man exercises continually for 120 minutes, the total energy consumption is 979.2kcal, fat consumption is 664.1kcal, the supply ratio of the fat energy accounts for 67.8%; if the movement time 120min is changed to 30min, the total energy consumption is 244.8kcal, and fat consumption is 117.9kcal, the supply ratio of the fat energy accounts for 48.1%.

From the above analysis, the fat energy supply ratio of the study object reaches 50% when exercising for 30
minutes, while the fat energy supply ratio reaches 80.8% when exercising for 120 minutes, at this moment the fat energy supply accounts for 67.8% of the total energy consumption during exercise; therefore the study object exercise with low-intensity for a long time can reach the purpose of maximum fat consumption.

According to the biological heat price of sugar and fat, the consumption mass of sugar and fat can drawn, the heat price of sugar 17.7kJ/g, the heat price of fat is 38.94kJ/g, and 1kcal = 4.186kJ; based on the above results that the study object to exercise 120 minutes every day can reduce fat of 70g, after a week time it can reduce 490g, and the results are shown in some important literature, which is more consistent with the safety week weight loss 0.5-1.0kg; from another side the model and research approach of the text is reasonable.

CONCLUSIONS

This paper uses Logistic model to study the relationship between stride frequency and heart rate, meanwhile also uses this model to study the case of fat consumption ratio; through the results’ verification, the use of the model is reasonable with high persuasion; in the paper it establishes a function of fat consumption and stride frequency, and builds the optimization model, which provides a mathematical method for optimal fat consumption, and gets a reasonable result; in order to verify the rationality of the Logistic model, the paper establishes a mechanistic model of the body’s fat consumption, the result shows that the error of the two is less than 10%; then it conducts case study on the fat function proportion problem of a man after 120 minutes and 30 minutes of exercise, a long time exercise with low-intensity can achieve the best results of weight loss.

REFERENCES