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The application of projectile damped motion in the research of basketball movement trajectory based on matlab numerical simulation

Chao Han^{1*}, Yuhao Zhang²

¹Department of Physical Education, College of Education, Nanchang University, Nanchang330031 (CHINA)

¹Email : 37679055@qq.com

²Sport School, Shandong Sport University, Jinan 250101 (CHINA)

²Email : 39310257@qq.com

ABSTRACT

In this paper, under the premise of basic assumptions, it uses Newton's second law to establish two sub-direction differential equation models aiming at the force condition of basketball. In the study, by analyzing the nature of the differential equation and the relationship that kinetic parameters satisfy, it establishes four first order differential equations; then, explores the first-order differential equations method; finally, using Matlab software programming, realizes parameter equation trajectory simulation of basketball centroid analytical solutions and numerical simulation of four first-order differential equations. After the study, build differential equation model on the general projectile damped motion, and achieve the numerical simulation of differential equations and trajectory simulation of parameter equation. Moreover the application effect of the model algorithm and simulation methods is better in the study of basketball centroid moving trajectory under initial assumptions, providing a theoretical basis for the study of projectile damped moving trajectory.

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KEYWORDS

Differential equations;
Numerical analysis;
Trajectory;
Newton's second law.

INTRODUCTION

Basketball motion trajectory reflects the comprehensive characterization of its centroid trajectory. And the factors that impact basketball centroid motion trajectory include people's initial role on the ball, the initial relative position of ball and box, the force of the ball after releasing and so on. Since this paper purely studies the centroid trajectory problems of basketball centroid, the initial conditions can be assumed. The main research content is the force movement process with

certain initial velocity of the center of mass. The solution of force and motion state is based on the theory of Newton's second law. Differential equations are the mathematical representation model to meet Newton's second law. In order to study basketball motion trajectory, first we need establish a reasonable differential equation model, and then obtained the displacement function of basketball changing over time through the differential equation solution.

Many people have made efforts on the research of the numerical solution of ordinary differential equations

and projectile trajectory, and the results of their research promoted the development of these two technologies. Solving ordinary differential equations is the practical problem that modern scientific research and engineering technology often encounters. But the differential equations created out of the practical problems often have complex forms, some analytical formula is difficult to calculate, and some even simply do not have analytic expression. So using the numerical method to solve practical problems has much better and direct effect. This paper, based on previous studies, conducted studied on damped projectile motion; the model is applied to the study of the basketball centroid trajectory; it used the fourth-order Runge-Kutta method in Matlab software to achieve the numerical simulation of ordinary differential equation and trajectory simulation.

PROBLEM ANALYSIS

Kinetic analysis of basketball after releasing and before dropping on the basket

During the movement after releasing and before dropping on the basket basketball receives only gravity and air resistance, wherein gravity is a constant force and plays a leading role in its movement process, thus resulting in basketball trajectory similar to parabola. However, the shape and size of the basketball will determine it will be subject to the effect of air resistance. Air resistance includes friction and other fluid force. In the air resistance friction plays a leading role, the action direction is constantly changing. It is just opposite to the direction of the ball speed and size of the force changes with the speed size. When the ball speed in-

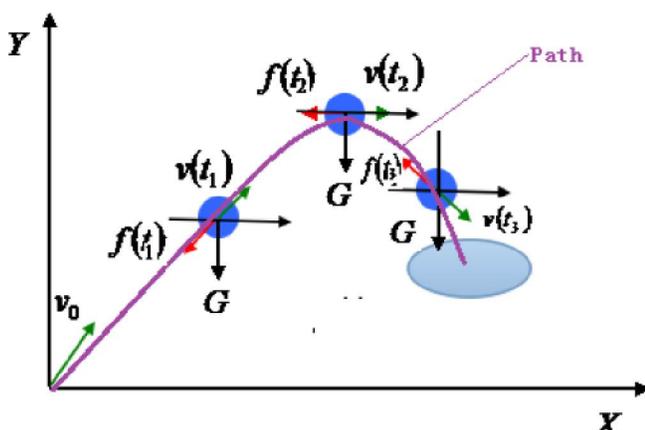


Figure 1 : Stress analysis during basketball moving

creases, its acting forces also increases, correspondingly it will decrease with the decreasing of sphere velocity. Figure 1 shows the force analysis and other kinetic parameters of basketball before dropping on the basket.

In Figure 1, the red arrow indicates the direction of the air resistance, the green arrow indicates the direction of basketball speed, the black arrow indicates the horizontal direction and vertical direction of the coordinates, oval indicates basket, blue ball indicates basketball, purple parabola represents the movement trajectory of the basketball centroid. In the illustration four movement moments of the basketball were analyzed, wherein v_0 means that the initial velocity of instantly releasing the basketball, $v(t_1)$ represents basketball's speed at certain moment in the upward phase, $v(t_2)$ means the velocity that basketball reached the highest point in the vertical direction, $v(t_3)$ means basketball's speed at certain moment in the downward phase. Stress analysis shows that: in the upward phase basketball receives gravity vertically downward, air resistance in the horizontal direction and air resistance in the vertical direction, wherein resistance in the vertical direction is the sum of gravity and air resistance; while in the downward phase the resistance in the vertical direction is the difference of gravity and air resistance.

Model assumption

- 1) During basketball movement only consider the gravity and air friction;
- 2) Air friction is proportional to the speed of basketball ball center, and its proportionality coefficient is k ;
- 3) The acceleration of gravity is g , taking 9.8m/s^2 ;
- 4) Do not consider the case when the basketball drop on the basket, and only study the movement after releasing and before dropping;
- 5) The position of basketball releasing is the origin in the coordinate system.

MODEL BUILDING AND SIMULATION

Model building of second-order differential equation

In order to facilitate research the movement is divided into the horizontal direction and the vertical di-

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rection. Suppose the horizontal speed is denoted by $v_{//}(t)$, the vertical speed is denoted by $v_{\perp}(t)$, the horizontal displacement is denoted by $x(t)$, the vertical displacement is denoted by $y(t)$, the air friction in the horizontal direction is denoted by $f_{//}(t)$, the air friction in the vertical direction is denoted by $f_{\perp}(t)$, the force of gravity is denoted by G , basketball's mass is denoted by m , and set the positive direction of the horizontal direction is the positive direction of X axis, the positive direction of the vertical direction is the positive direction of Y axis. There are second-order differential equations in the horizontal direction as formula (1) below:

$$m \frac{d^2 x(t)}{dt^2} = -k \frac{dx(t)}{dt} \quad (1)$$

Formula (1) hides the relationship of formula (2):

$$\begin{cases} v_{//}(t) = \frac{dx(t)}{dt} \\ -kv_{//}(t) = \frac{dv_{//}(t)}{dt} \end{cases} \quad (2)$$

Second-order differential equation in vertical direction is as formula (3) below:

$$m \frac{d^2 y(t)}{dt^2} = -G - k \frac{dy(t)}{dt} \quad (3)$$

Formula (3) hides the relationship of formula (4):

$$\begin{cases} v_{\perp}(t) = \frac{dy(t)}{dt} \\ -g - \frac{kv_{\perp}(t)}{m} = \frac{dv_{\perp}(t)}{dt} \end{cases} \quad (4)$$

When $t=0$, then $v_0 = v(0)$ and the angle of initial velocity and the X axis is θ , then given formula (5):

$$\begin{cases} v_{//}(0) = v_0 \cos \theta \\ v_{\perp}(0) = v_0 \sin \theta \end{cases} \quad (5)$$

Compositing formula (1) (2) (3) (4) (5) we can draw the parameter equation of basketball centroid moving trajectory, as shown in formula (6):

$$\begin{cases} x(t) = \frac{mv_0 \cos \theta}{k} \left(1 - e^{-\left(\frac{k}{m}t\right)} \right) \\ y(t) = -\frac{mg}{k} t - \left(\frac{m^2 g}{k^2} + \frac{mv_0 \sin \theta}{k} \right) \left(e^{-\left(\frac{k}{m}t\right)} - 1 \right) \end{cases} \quad (6)$$

Numerical solution of four differential equations

To simplify the expression forms differential equations introduce constant two parameters b_1, b_2 , wherein the significance is in formula (7):

$$\begin{cases} b_1 = \frac{m}{k} \\ b_2 = b_1 g = \frac{mg}{k} \end{cases} \quad (7)$$

According to formula (1) and (3) differential equations can be obtained as shown in formula (8):

$$\begin{cases} \frac{d(v_{//}(t)/b_2)}{d(t/b_1)} = -\frac{kb_1 v_{//}(t)}{mb_2} \\ \frac{d(v_{\perp}(t)/b_2)}{d(t/b_1)} = -\frac{gb_1}{b_2} - \frac{kb_1 v_{\perp}(t)}{mb_2} \end{cases} \quad (8)$$

Suppose $\frac{t}{b_1} = t^*$, $\frac{v_{//}(x)}{b_2} = v_{//}^*(t)$, $\frac{v_{\perp}(x)}{b_2} = v_{\perp}^*(t)$ in

formula (8), it can be converted to the equations as shown in formula (9):

$$\begin{cases} \frac{dv_{//}^*(t^*)}{dt^*} = -v_{//}^*(t^*) \\ \frac{dv_{\perp}^*(t^*)}{dt^*} = -1 - v_{\perp}^*(t^*) \end{cases} \quad (9)$$

Numerical solution of four differential equations can not only reflect trajectory coordinates of the basketball centroid, but also reflect basketball speed at certain time. To simplify the equations introduce a constant parameter b_3 on the basis of formula (7), its expression form is as formula (10) below:

$$b_3 = \frac{m^2 g}{k^2} \quad (10)$$

Then according to formula (1) and formula (3) second-order differential equations can be obtained shown in formula (11):

$$\begin{cases} \frac{d^2(x(t)/b_3)}{d(t/b_1)} = -\frac{kb_1}{m} \frac{d(x(t)/b_3)}{d(t/b_1)} \\ \frac{d^2(y(t)/b_3)}{d(t/b_1)} = -\frac{gb_1^2}{b_3} - \frac{kb_1}{m} \frac{d(y(t)/b_3)}{d(t/b_1)} \end{cases} \quad (11)$$

Suppose $\frac{x(t)}{b_3} = x^*(t^*)$, $\frac{y(t)}{b_3} = y^*(t^*)$ in formula

(11) and then according to the formula (7) and formula (10), second-order differential equations can be derived as shown in formula (12):

$$\begin{cases} \frac{d^2 x^*(t^*)}{dt^{*2}} = -\frac{dx^*(t^*)}{dt^*} \\ \frac{d^2 y^*(t^*)}{dt^{*2}} = -1 - \frac{dy^*(t^*)}{dt^*} \end{cases} \quad (12)$$

According to the formula (12) four first-order differential equations can be drawn as formula (13) below:

$$\begin{cases} \frac{dx^*(t^*)}{dt^*} = v_{//}^*(t^*) \\ \frac{dy^*(t^*)}{dt^*} = v_{\perp}^*(t^*) \\ \frac{dv_{//}^*(t^*)}{dt^*} = -v_{//}^*(t^*) \\ \frac{dv_{\perp}^*(t^*)}{dt^*} = -1 - v_{\perp}^*(t^*) \end{cases} \quad (13)$$

The initial conditions are: $x^*(0) = 0, y^*(0) = 0$, and $v_{//}^*(0) = \frac{v_0 \cos \theta}{b_2}, v_{\perp}^*(0) = \frac{v_0 \sin \theta}{b_2}$, which can reflect the Basketball centre trajectory and speed condition.

MATLAB SIMULATION AND ANALYSIS

Simulation trajectory of parametric equations

Take the initial data $v_0 = 12\text{m/s}, m = 0.55\text{kg}, \theta = 53^\circ, k = 0.15, g = 9.8\text{m/s}^2$, Matlab code is shown in Figure 2:

```

1 clear
2 v0=input(' v0= kv0/mg: ');
3 t=0:0.1:4;
4 x=v0*0.55*0.6*(1-exp(-t*(0.15/0.55)));
5 y=-35.9333*t-(131.7556+2.9333*v0)*(exp(-0.27273*t)-1);
6 fs=16;
7 figure
8 plot(x,y)
9 grid on
    
```

Figure 2 : Simulation code of basketball centroid trajectory in parametric equation

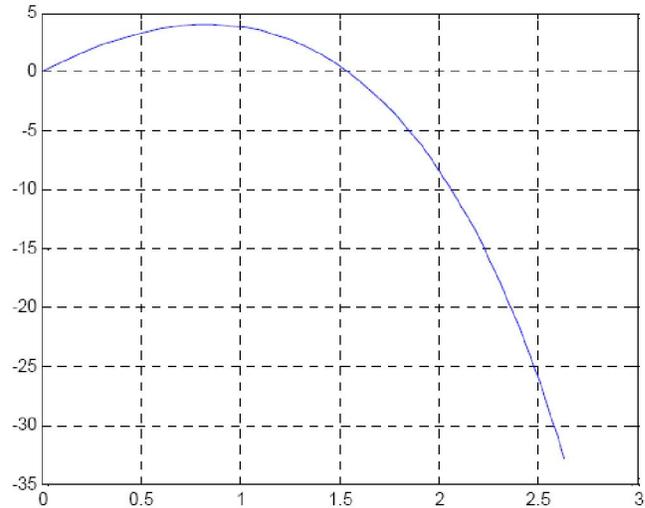


Figure 3 : Simulation Trajectory

Simulation trajectory is shown in Figure 3:

Numerical simulation of four differential equations

First establish function named function, save the file to fun 2 in the Matlab Work folder, and define the function code as shown in Figure 4:

```

1 function f=fun(t,r)
2     f=[ r(3);
3         r(4);
4         -r(3);
5         -1-r(4)];
    
```

Figure 4 : Define function code

Solve the numerical coordinates of basketball centroid trajectory, and carry through numerical simulation, the code is shown in Figure 5:

```

1 [t0,R]=ode45('fun',t,[0,0,9.6,7.2]);
2 plot(R(:,1),R(:,2),'ko')
    
```

Figure 5 : Numerical simulation code

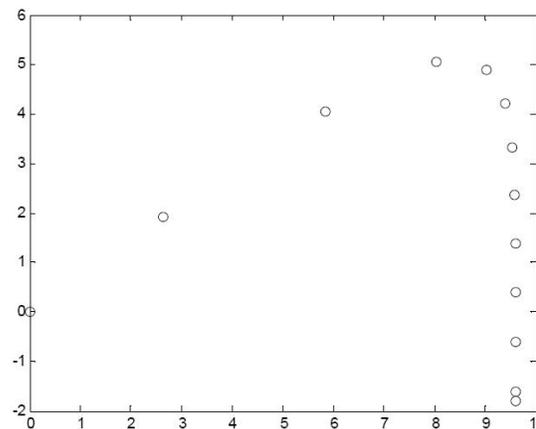


Figure 6 : Simulation Trajectory

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Simulation trajectory is shown in Figure 6

CONCLUSIONS

This article well used Matlab software simulation and achieved a damped motion of basketball providing a theoretical basis and simulation for the study of basketball moving trajectory; it established differential equation model on the general projectile damped motion, achieving numerical simulation of differential equations and trajectory simulation of parametric equation; the application effect of the model algorithm and simulation methods is better in the study of basketball centroid moving trajectory under initial assumptions providing a theoretical basis for the study of projectile damped moving trajectory.

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