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The actuarial valuation of reverse mortgages in China

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ABSTRACT

Owing to high housing self-ownership rate in China, it is a realistic option for the government to implement the housing reverse mortgage to relieve the fiscal pressure and supplement the drop or insufficiency of social security income. But it is also difficult to evaluate reverse mortgage for its payments depend on the housing value at maturity. After describing the stochastic processes of interest rate and the growth rate of housing value, the paper builds the actuarial pricing models of reverse mortgages to evaluate the annuity payments of reverse mortgages. The result reflects that the payments can effectively supplement the social security benefit, and the redemption option is important for the participant even it would reduce the level of payments of reverse mortgage to a large extent.

KEYWORDS

Reverse mortgage; Actuarial valuation; Annuitized payment; No-recourse clause; Social security.

INTRODUCTION

It is an inevitable trend that the social security benefits will drop in view of the ever-increasing dependency rate for the aged population in China. So widening other sources of retirement incomes is important for the government to deal with consequential insufficiency of social security income. On the other hand, the residents in China own relatively high housing self-ownership rate. According to the investigation of Chinese Academy of Social Science, the housing self-ownership rate of the families in China in 2013 reached up to 90%, 30% higher than the world average level. So it is a realistic option for the government to implement the housing reverse mortgage (RM). Under the precondition of not affecting the right of dwelling of the retirees, RM can not only help the retirees transform the housing into income flows to support their retirement lives and solve the liquidity predicament which has been described as "with house but without cash", but also provide the longevity risk guarantee to the retirees with the payments as a life annuity. Therefore, it will be of great significance to solve the social pension problems with RM in China from the actual need of establishing the multi-pillar pension system.

However, RM is being confronted with some practical challenges in China. Different from the traditional home mortgage, the payment of RM depends on the housing value at maturity (usually the house owner dies). Therefore, no matter the lender (such as the insurer) or the borrower (housing owner) is confronted with the risks that housing value at term does not conform to the accumulative loan amount. Specifically speaking, these risks include housing price fluctuation risk, interest rate risk and longevity risk. In addition, the bequest motive in China is also a key factor confining RM [1]. So the trial RM in some regions in China is embedded with the clauses regarding redemption options, namely when the contract expires, if the housing value exceeds the accumulative loan amount, the housing owner or its children can select redeem the house, in this way, it can help the borrower eliminate the worry about the rise in housing price and the housing not functioning as the bequest. So it is important to compare the annuitized payment of RA with redemption option to without redemption option.

Based on this, in next sections, we describe the random process of changes in which the interest rate and the housing price growth rate with CIR Model and the jump-diffusion model correspondingly, and estimates the parameters of the above-mentioned models with Generalized Method of Moments (GMM) and Maximum Likelihood Method (ML) respectively. And then this paper simulates the annuity payments of RM with redemption option and non-redemption option under the actuarial condition with Monte-Carlo method. Further we discuss whether the RA can supplement the social security benefits or not in different conditions.

MODELING RM PRICING

Interest rate model

CIR Model is adopted here to describe the fluctuation of interest rate in continuous and disperse form, indicated as:

$$dr(t) = k_r(\mu_r - r(t))dt + \sigma_r\sqrt{r(t)}dW_r(t) \quad (1)$$

$$r(t+\Delta t) - r(t) = k_r(\mu_r - r(t))\Delta t + \sigma_r\sqrt{r(t)}\Delta W_r(t) \quad (2)$$

Where, $W_r(t)$ is a standard Wiener process, dW can be deemed as a normal variable with the mean of 0 and the variance of dt, k_r, μ_r and σ_r are the constants, among which μ_r is the long-term mean value of the interest rate, σ_r is the fluctuation ratio of the interest rate and k_r is the regulating speed.

Therefore, $E(dr) = k_r(\mu_r - r(t))dt$, $\mathrm{var}(dr) = \sigma^2{}_r \cdot r(t)$.

Furthermore, Formula (2) is written as:

$$r(t)-r(t-1)=\alpha+\beta\cdot r(t-1)+\varepsilon(t) \quad (3)$$

Based on Formula (3), k, μ_r and σ_r need to be estimated; and $E(\varepsilon(t))=0$, $\text{var}(\varepsilon(t))=\sigma^2{}_r\cdot r(t-1)$.

Housing price model

Jump-diffusion model is adopted to describe the fluctuation of the housing price in China, which can be indicated as:

$$\frac{dH}{H}=\mu dt+\sigma dW+\bar{J}dN \quad (4)$$

In jump-diffusion model, the instantaneous earning rate of the assets is composed of a simple diffusion process and random jump item, among which μ, σ and dW are same to the ones in CIR Model, N is the Poisson process with the intensity λ, namely $P(dN=1)=\lambda dt$, $P(dN=0)=1-\lambda dt$, $\bar{J}$ is the amplitude of random jump, indicated with the percentage of alteration in price.

Formula (4) is subject to logarithm, then:

$$d(\ln H)=\bar{\mu}dt+\sigma\sqrt{t}dz+JdN \quad (5)$$

Where $\bar{\mu}=\mu-\frac{1}{2}\sigma^2$, $dz\sim N(0,1)$, J is the jump amplitude, indicated with the logarithm of the percentage of the alteration in price of the assets in jump plus 1, $J\sim N(\theta,\delta^2)$ and $N\sim P(\lambda)$.

Formula (5) is discretized, then:

$$\ln(H_t)-\ln(H_{t-1})=\bar{\mu}\tau+\sigma\sqrt{t}\eta_t+\sum_{i=1}^{N_\tau}J_i\Delta\mu_\tau+\sigma_\tau\eta_t+\sum_{i=1}^{N_\tau}J_i \quad (6)$$

Where τ is the time interval indicated with the year, $\eta_t\sim N(0,1)$, $J_i\sim N(\theta_\tau,\delta_\tau{}^2)$, $N_\tau\sim P(\tau\lambda)$, $\mu_\tau=\bar{\mu}\tau$, $\sigma_\tau=\sigma\sqrt{\tau}$, $\theta_\tau=\theta\tau$ and $\delta_\tau=\delta\sqrt{\tau}$.

Then, in t time, the change of housing price in continuous compounding with the time interval of τ is:

$$r_H(t)=\ln(H_t)-\ln(H_{t-1})=\mu_\tau+\sigma_\tau\eta_t+\sum_{i=1}^{N_\tau}J_i \quad (7)$$

Actuarial model for RM

In RM contract, the old housing owner, with the house property as the mortgage, obtains the annuity payment flow in lump sum or the fixed annuity payment flow, and it is not necessary for the borrower to repay the annuity in the period of residence till death, house selling or permanent moving out of the house, at this moment, the contract expires, the borrower repays the principal, interest and other expenses of the loan with the income from house selling. This paper mainly evaluates the reverse mortgage (RM) loan repaid with the life annuity, which can protect the longevity risk of the retirees and is the beneficial supplementation of the payment for the social security. In RM, the insurance agents provide the life fixed annuity payment flow to the old housing owner until the borrower dies.

Under the actuarial conditions, according to the traditional method, RM pricing can be indicated that the present value of the expected income of the borrower is equal to the one of the expected expenditure; in case of no redemption option, it can be indicated as:

$$\sum_{t=1}^{\omega-x_0+1} H(t)\cdot {}_tq_{x_0}\cdot e^{-t\cdot r_t} = \sum_{t=1}^{\omega-x_0+1} A(t)\cdot {}_tp_{x_0}\cdot e^{-t\cdot r_t} \quad (8)$$

Where $H(t)$ is the housing value in time t, $t=1,...,\omega-x_0+1$, ω is the max. age of survival, x_0 is the age of the retiree participating in RM, ${}_tq_{x_0}={}_tp_{x_0}-{}_{t+1}p_{x_0}$ is the probability of death of the retiree aged x_0 in time t and $\sum_{t=1}^{\omega-x_0+1} {}_tq_{x_0}=1$, r_t is the loan interest rate of continuous compound interest, $A(t)$ is the payment of fixed annuity in each time. According to (8):

$$A(t)=\left(\sum_{t=1}^{\omega-x_0+1} H(t)\cdot {}_tq_x\cdot e^{-t\cdot r_t}\right)\Big/\left(\sum_{t=1}^{\omega-x_0+1} {}_tp_x\cdot e^{-t\cdot r_t}\right) \quad (9)$$

However, the pricing method above-mentioned ignores one important clause in RM contract, namely "no-recourse clause", which implies that when RM contract expires, even if the housing value is lower than the accumulative amount of loan, the lender has no right of recourse to the assets of the borrower in other form. Therefore, in RM contract, the lender is confronted with the risks that the housing value is not so sufficient to repay the accumulative amount of loan in expiration of the contract, as is an important reason why the financial institutions feel an objection to carry out RM [4]. In order to encourage the financial institutions especially the insurers to provide RM products, the government often allows them to charge an additional premium to compensate the potential conditional losses. Therefore, considering the no-recourse clause in RM contract, the present value of the insurance premium collected by the lender due to the right of non-recourse shall be equal to the present value of the expected losses in the future.

Given the lender allocates the premium to each installment equally, the extra premium occurred in each installment due to no-recourse clause shall be:

$$P(t)=\left(\sum_{t=1}^{\omega-x_0+1} [B(t)-H(t)]^+\cdot {}_tq_x\cdot e^{-t\cdot r_t}\right)\Big/\left(\sum_{t=1}^{\omega-x_0+1} {}_tp_x\cdot e^{-t\cdot r_t}\right) \quad (10)$$

$B(t)$ is the accumulative amount of loan in installment t, indicated as $B(t)=\sum_{i=1}^{t} A(i)\cdot e^{r_{t-i}}$, when the borrower dies in installment t and $B(t)>H(t)$, $B(t)-H(t)$ is the loss of the lender in installment t.

Therefore, in case of no redemption option, according to Formula (9) and (10), the fixed annuity paid to the borrower in each installment is $A(t)-P(t)$.

In case of redemption option, given the cost of redemption option is also allocated to each installment evenly, to the lender, the cost to be collected for the redemption option in each installment shall be:

$$\pi(t)=\left(\sum_{t=1}^{\omega-x_0+1} [H(t)-B(t)]^+\cdot {}_tq_x\cdot e^{-t\cdot r_t}\right)\Big/\left(\sum_{t=1}^{\omega-x_0+1} {}_tp_x\cdot e^{-t\cdot r_t}\right) \quad (11)$$

When the borrower dies in installment t and $H(t)>B(t)$, the borrower's children can select to redeem the housing, and $H(t)-B(t)$ is the bequest the borrower leaves to its children. As a result, in case of redemption option, the fixed annuity paid to the borrower in each installment shall be $A(t)-P(t)-\pi(t)$.

RESULT AND DISSCUSS

Parameter estimation of interest rate model

The data about the interest rate are from the monthly weighted mean inter-bank offered rate from Jan. 1998 to Dec. 2013 in CSMAR.

The above-mentioned data is regressed with GMM; the instrumental variable is set as $1/r(t-1)$, with the regression results shown in TABLE 1.

TABLE 1 : Estimated results of formula (3)

Parameter	Estimated value	Standard deviation	Approx $\Pr > \|t\|$
α	0.002	0.0007	0
β	-0.109	0.028	0
σ^2	1.5560E-05	0.004	0

It can be seen that all of the parameters in Formula (3) pass the significance test at the significance level of 1%; furthermore, the estimated values of three parameters in Formula (1) can be obtained, namely $k = 0.109$, $\mu_r = 0.022$ and σ_r=0.004 respectively.

Parameter estimation of housing price model

The data about the housing price are from the monthly data of the housing price indexes in China from Jan. 1994 to Dec. 2011 issued by National Bureau of Statistic. Based on Formula $r_H(t) = \ln(H_t / H_{t-1})$, the earning rate of continuous compound interest of the housing price can be obtained in each month.

Compared with the parameter estimation of other random processes, the parameter estimation of jump-diffusion model is more complicated. In the estimation methods, although the maximum likelihood method has greater difficulty in calculation, it is more effective than the GMM because the latter may meet with the problem of non-convergence [5]. Some studies adopt non-parameter estimation method, such as Johannes [6] and Mancini and Renò [7]. This paper tries to adopt Conjugate Gradient in the maximum likelihood method for estimation just because it is one of the most effective methods to settle the large-sized non-linear optimization problems. The parameter estimation results under the condition of convergence are finally obtained through 110 iterations (See TABLE 2).

TABLE 2 : Parameter estimation results of formula (7)

Parameter	Estimated value	t value	Approx $\Pr > \|t\|$
λ	1.206	6.837	0
μ_H	0.043	6.349	0
μ_J	0.003	1.236	0.22
σ_H^2	0.021	1.763	0
σ_J^2	0.019	13.686	0

It is shown by the results that other parameters pass the significance test at the significance level of 1% except the mean μ_J of the jump amplitude.

The result of Monte-Carlo simulation for RA payments

Given the value of the housing of the borrower at the beginning of period is RMB 2,000,000 Yuan, and the Experience Life Table of China Life (2000-2003) is adapted to calculate mortality rates. Since the parameter estimation of CIR model of interest rate adopts the risk-free interest rate of

interbank lending, with which an interest rate premium needs to be added to obtain the loan interest rate, usually higher than the risk-free interest rate. Here it is firstly supposed the interest rate premium is 300BP. According to the housing price fluctuation model, interest rate fluctuation model and RM pricing model under the actuarial condition for the above-mentioned estimation, we calculate $A(t), P(t)$ and $\pi(t)$ with Monte-Carlo method. After the simulation for 1000 times, the fixed payment to the borrowers at different ages participating in RM in each year is shown in TABLE 3.

TABLE 3 : Pricing for RM of the borrowers at different ages (RMB yuan, %)

Sex	Age	$A(t)$	$P(t)$	$\pi(t)$	**(1)** $A(t)-P(t)$	**(2)** $A(t)-P(t)-\pi(t)$	**(2)/(1)**
Male	60	118803.6	2616.8	63219.2	116186.8	52967.6	45.6
	65	139941.2	5240.0	69256.6	134701.2	65444.6	48.6
	70	170546.8	10296.2	78046.8	160250.6	82203.8	51.3
	75	213415.4	19877.2	90370.0	193538.2	103168.2	53.3
Female	55	91931.0	649.6	55415.8	91281.4	35865.6	39.3
	60	106850.0	1279.6	60934.0	105570.4	44636.4	42.3
	65	124682.0	2625.0	66288.8	122057.0	55768.2	45.7
	70	151129.2	5582.8	74131.6	145546.4	71414.8	49.1
	75	188705.8	11920.6	84882.8	176785.2	91902.4	52.0

It can be seen from TABLE 3:

No matter the men or women, the annuitized payments of RM with redemption option is far lower than the one of RM without redemption option. For example, as to the male housing owner aged 65, the annual payment for RM with redemption option is RMB 65,445 Yuan, only equivalent to 48.6% of the payment for RM without redemption option.

The payment to the female borrower participating in RM is commonly lower than the one to the male borrower; as to the female borrower aged 55, the payment of RM with redemption option is less than RMB 36,000 Yuan every year and the ratio of the payment of RM with redemption option to the one without redemption option is only 39.3%. Since the payment will not be adjusted with the inflation rate or the growth rate of the social average wage, it is predicted that the insufficiency of retirement income may take place in the later stage after retirement. Just like the male borrower, the ratio of the payment of RM with redemption option to the one without redemption option increases with the age in same manner; however, as to the female borrower at the same age, this ratio is commonly less than the one of the male borrower. For instance, as to the female borrower at the age of 60, 65, 70 and 75, this ratio is respectively 3.3%, 2.9%, 2.2% and 1.3% lower than the one of the male borrower. However, with the increase in age, the gap between the ratio of the male borrower and the female borrower is being reduced because the lower mortality rate of the female borrower than the one of the male borrower. Therefore, the younger the borrower participating in RM is, the lower the fixed payment.

Sensitivity analysis

Firstly, the change in housing price growth rate is considered. Theoretically, the higher the growth of housing pricing, the higher the payment of RM with redemption option and the one without redemption option, which is just verified by the changes in payments in TABLE 4. Here the male and female normal borrowers aged 60 are regarded as the comparison reference. Without change in fluctuation and jump, it shows the influences on the payments of RM when the mean of the housing price growth rate fluctuates about 1~2%. When the mean of the housing pricing growth rate increases by

2%, the payments to the male and female borrowers for RM without redemption option increases to RMB 180,101 Yuan and RMB 169,361Yuan, with the growth rate of 55% and 60% respectively compared to the reference, the ratios of the payment of RM with redemption option and the one without redemption option respectively drops by 4.3% and 5%; while the mean of housing price growth rate drops by 2%, the payments of RM with redemption option to the male and female borrowers drops by 37-38% while the ratios of payment of RM with redemption option and the one without redemption option does not change greatly, among 0.5~1%.

TABLE 4 : RM pricing in change in the mean of housing price growth rate (RMB Yuan, %)

Sex	change of μ_H	$A(t)$	$P(t)$	$\pi(t)$	(1) $A(t)-P(t)$	(2) $A(t)-P(t)-\pi(t)$	(2)/(1)
Male	0.02	180311.2	210.6	105667.4	180100.6	74433.2	41.3
	0.01	144679.0	1125.0	80246.6	143554.0	63307.4	44.1
	-0.01	96521.8	4545.0	49688.0	91976.8	42288.8	46.0
	-0.02	79685.2	6453.2	40289.8	73232.0	32942.2	45.0
Female	0.02	169403.0	41.6	106252.6	169361.4	63108.8	37.3
	0.01	133916.0	405.2	79917.2	133510.8	53593.6	40.1
	-0.01	85533.8	2486.2	47011.0	83047.6	36036.6	43.3
	-0.02	68145.0	3811.2	36396.2	64333.8	27937.6	43.4

Secondly, the interest rate risk is another important issue in the implementation of RM. It can be seen from TABLE 5 that the increase in loan interest rate will significantly reduce the payment for RM with redemption option and the one without redemption option; in addition, with the expansion of the interest rate premium, the amplitude of drop in payment for RM with redemption option exceeds the payment for RM without redemption option. When the difference between the loan interest rate and the interbank offered rate exceeds 6%, the payment of RM with redemption option to the male borrower is only RMB 32,094 Yuan while the one to the female borrower is less than RMB 27,636 Yuan, respectively reducing by 39% and 38%. Due to no inflation adjustment, it can be forecast that it is difficult for the payment of RM with redemption option to provide the sufficient retirement income supplementation to the retirees with the increase in age of the participants in case of high loan interest rate.

TABLE 5 : RM pricing in expanding interest rate premium (RMB Yuan, %)

Sex	Margin	$A(t)$	$P(t)$	$\pi(t)$	(1) $A(t)-P(t)$	(2) $A(t)-P(t)-\pi(t)$	(2)/(1)
Male	100BP	105865.0	5199.0	54298.8	100666.0	46367.2	46.1
	150BP	100488.8	6618.0	50866.6	93870.8	43004.2	45.8
	200BP	95649.0	8115.8	48106.8	87533.2	39426.4	45.0
	250BP	91166.4	9615.0	45573.8	81551.4	35977.6	44.1
	300BP	86247.6	11117.2	43036.8	75130.4	32093.6	42.7
Female	100BP	93423.2	2966.0	50825.4	90457.2	39931.8	44.1
	150BP	88147.0	4025.0	47111.8	84122.0	37010.2	44.0
	200BP	82657.0	5158.0	43526.2	77499.0	33972.8	43.8
	250BP	77506.6	6321.6	40361.4	71185.0	30823.6	43.3
	300BP	72416.6	7396.8	37383.8	65019.8	27636.0	42.5

CONCLUSION

The paper builds the actuarial pricing models of reverse mortgages with and without redemption option. It can be seen that even though the redemption option will reduce the level of payment of RM to a large extent; however, considering the benefit the existing social security, the sum of them should be able to realize 70% of the income replacement rate, which can guarantee the retirees maintain the living standard before retirement. Therefore, actuarially, the payments of reverse mortgage can effectively offset the decline in social security benefits caused by the aging of the population for retiree to keep the retirement income adequacy. Further, taking into account the important position of house in the household wealth, the payments of reverse mortgage can become one of important source of retirement income.

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