ABSTRACT

The paper firstly applies Lagrange equations to solve restricted particle kinetic equation, combines with theoretical formulas to analyze when tennis player serves, hand joint mechanical movement, combining with shoulder joint, elbow joint mechanical analysis to study on tennis service technology. According to geometric principle, it establishes tennis service model, when athlete serves, he should try to stretch arms to right ahead and to be vertical to service point, and then probability that ball passes through net will get larger. And apply mechanical conservation law to analyze tennis service problems, it gets that athlete himself can further control rotational angular speed by changing self-rotational inertia. Twist service is when athlete takes off and serves, carries out serving by changing upper body faced direction, when athlete jumps, it should increase athlete himself rotational angular speed, on the contrary, when athlete jumps to the highest point, athlete should try to adjust body stability and let rotational angular speed to reduce as much as possible.

KEYWORDS

Tennis service; Lagrange equation; Mechanical conservation law; Mathematical model.
INTRODUCTION

In tennis, service technology is one of relatively important technologies in tennis, and because service is a unique movement method that is not restricted by opponents, many athletes can get scores after serving, it is concerned and studied by many experts. Consulting relative documents, we can know that many scholars have analyzed tennis service from multiple perspectives, on this basis; the paper systematically looks for tennis service features. And consider tennis technology, field awareness, physical quality, psychological quality and other factors, service connects with numerous factors in the hope of propelling to Chinese tennis development.

MODEL ESTABLISHMENTS

Arms rotational inertia calculation when serving

By Lagrange equations, the paper gets restricted particle dynamical equations, from which Lagrange function $L$ is difference generated between system kinetic energy $K$ and potential energy $P$: $L = K - P$, system dynamical equation is:

$$F_i = \frac{\partial L}{\partial q_i} = \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) \quad i = 1,2,L,n$$

In above formula, $\dot{q}_i$ is corresponding speed, $q_i$ is kinetic energy and potential energy coordinate, $F_i$ is the coordinate acting force, thigh and shank included angles with coordinate axis are respectively $\theta_1, \theta_2$, lengths are respectively $l_1, l_2$, distances that arm front and back’s gravity center position from elbow joint center and knee joint center are respectively $p_1, p_2$, thereupon it is clear that arm gravity center coordinate $(X_1, Y_1)$ is:

$$X_1 = p_1 \sin \theta_1 \quad Y_1 = p_1 \cos \theta_1$$
$$X_2 = l_1 \sin \theta_1 + p_2 \sin(\theta_1 + \theta_2) \quad Y_2 = -l_1 \cos \theta_1 - p_2 \cos(\theta_1 + \theta_2)$$

Similarly, arm gravity center coordinate $(X_2, Y_2)$ can also be solved. System kinetic energy $E_k$ and system potential energy $E_p$ expressions are:

$$E_k = E_{k1} + E_{k2} + E_{k1} = \frac{1}{2} m_1 p_1^2 \dot{\theta}_1^2$$
$$E_{k2} = \frac{1}{2} m_2 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 l_2^2 \left( \dot{\theta}_1 + \dot{\theta}_2 \right)^2 + m_2 l_2 p_2 \left( \dot{\theta}_1 + \dot{\theta}_2 \right) \cos \theta_2$$

$$E_p = E_{p1} + E_{p2} + E_{p1} = \frac{1}{2} m_1 g p_1 \left(1 - \cos \theta_1 \right)$$
$$E_{p2} = m_1 g l_1 \left(1 - \cos \left(\theta_1 + \theta_2 \right) \right) + m_2 g l_2 \left(1 - \cos \theta_2 \right)$$

Write above formula into Lagrange function expression, by Lagrange system dynamical equation, it can get hip joint and knee joint moment $M_h$ and $M_k$ as:

$$M_h = D_{11} \dot{\theta}_1 + D_{12} \dot{\theta}_2 + D_{21} \dot{\theta}_1 + D_{22} \dot{\theta}_2 + D_1$$
$$M_k = D_{11} \dot{\theta}_1 + D_{12} \dot{\theta}_2 + D_{21} \dot{\theta}_1 + D_{22} \dot{\theta}_2 + D_2$$
In above formula $D_{jk}$ is as following result:

$$D_{111} = 0 \quad D_{222} = 0 \quad D_{121} = 0$$

$$D_{22} = m_2 p_2^2$$

$$D_{11} = m_1 p_1^2 + m_2 p_2^2 + m_{1} l_1^2 + 2 m_2 l_1 p_2 \cos \theta_2$$

$$D_{12} = m_2 p_2^2 + m_{1} l_1 p_2 \cos \theta_2 \quad D_{21} = m_2 p_2^2 + m_{1} l_1 p_2 \cos \theta_2$$

$$D_1 = (m_1 p_1 + m_{1} l_1) g \sin \theta_1 + m_2 p_2 g \sin (\theta_1 + \theta_2)$$

$$D_{122} = -m_2 l_1 p_2 \sin \theta_2$$

$$D_{211} = m_2 l_1 p_2 \sin \theta_2$$

$$D_{112} = -2m_2 l_1 p_2 \sin \theta_2$$

$$D_{212} = D_{122} + D_{211}$$

$$D_3 = m_2 p_2 g \sin (\theta_1 + \theta_2)$$

Combine with theoretical formula, analyze when tennis player serves, hand joint mechanical movement, combine with shoulder joint, elbow joint mechanical analysis, it studies tennis service technology.

Geometric model establishment

Service trajectory and service drop point solving under geometric model

According to geometric principle, it establishes tennis service model, according to differences from high service and low services, it gets ball trajectory figure and service drop point position as Figure 1 shows.

![Figure 1: High service and low service differences obtained ball trajectory](image)

When athlete serves, he should try to manage to stretch arms to right ahead and remain vertical to service point, athlete take-off height gets higher, and ball over net probability would be larger; with ball located service point height differences, presented over net trajectory and landing point are also different, therefore it gets long ball and short ball; according to service point vertical heights differences, the paper classifies six different heights phases that are respectively (2.6, 2.7, 2.8, 2.9, 3.0, 3.1). And it gets TABLE 1 according to service point to net vertical distance differences.
By TABLE 1, it is clear that athlete service points positions differences, tennis drop point also has differences, athlete service heights differences can let ball over net position to be different.

**Low dropping ball’s service trajectory and service drop point improvement under geometric model**

![Low dropping ball's service trajectory and service drop point under geometric model](image)

Similarly, as Figure 2 show, the paper according to service point vertical heights differences, it classifies six different heights phases that are respectively (2.6, 2.7, 2.8, 2.9, 3.0, 3.1), and according to service point to net vertical distances differences and get TABLE 2.

**TABLE 2: After shortening service point to vertical net 0.1m ball flying across lowest net point landing point data indication**

<table>
<thead>
<tr>
<th>Vertical height / H</th>
<th>1.6</th>
<th>1.7</th>
<th>1.8</th>
<th>1.9</th>
<th>2.0</th>
<th>2.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance between service point and net is 0.75m</td>
<td>45.16</td>
<td>25.01</td>
<td>12.73</td>
<td>9.45</td>
<td>5.6</td>
<td>3.15</td>
</tr>
<tr>
<td>Distance between service point and net is 0.5m</td>
<td>6.32</td>
<td>2.89</td>
<td>2.68</td>
<td>2.48</td>
<td>1.57</td>
<td>0.68</td>
</tr>
<tr>
<td>Distance between service point and net is 0.25m</td>
<td>8.98</td>
<td>6.58</td>
<td>4.08</td>
<td>3.12</td>
<td>2.95</td>
<td>1.68</td>
</tr>
</tbody>
</table>

By TABLE 2 and Figure 2, it is clear that shorten service point to net vertical distance 0.1m, which can clearly get to low service, though only shorten service point to net 0.1m vertical distance, ball service drop point positions have great differences. Service drop point in opponent area occurs to great changes.

**Change service angle**

Service release angle is not in 90° vertical incidence, and ball also doesn’t vertical enter into opponent field, generally speaking, when athlete serves, it will serve at certain angles, and oblique serve ball down to opponent field.

Set service point and field edge vertical distance is 0.5m, when athlete serves, deflects rightward 45° to release and serve, and get service point to drop point distance \( S \) is:

\[
S = \frac{0.5}{\cos 45°} = \frac{0.5}{\sqrt{2}/2} = 0.7072m
\]

Similarly when deflection angle is 30°, it gets dropping point to serving point vertical projection distance \( S \) is:
0.5cos30° = \frac{0.5}{\sqrt{3}} = 0.574m

As Figure 3 shows.

Figure 3: Angle changes service drop point schematic

Figure 4: Athlete service

In general, athlete service will have certain angles, and jumps as Figure 4 shows, the paper according to Figure 3 showing, it gets data and establishes TABLE 3 to analyze.

<table>
<thead>
<tr>
<th>Angle</th>
<th>Vertical angle</th>
<th>45°</th>
<th>Drop point coordinate</th>
<th>30°</th>
<th>Drop point coordinate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.63m</td>
<td>9.85m</td>
<td>13.45m</td>
<td>\begin{pmatrix} 12.05 \ 12.05 \end{pmatrix}</td>
<td>12.09m</td>
<td>\begin{pmatrix} 12.46 \ 5.36 \end{pmatrix}</td>
</tr>
<tr>
<td>1.82m</td>
<td>6.53m</td>
<td>8.67m</td>
<td>\begin{pmatrix} 5.85 \ 5.85 \end{pmatrix}</td>
<td>5.56m</td>
<td>\begin{pmatrix} 6.02 \ 3.476 \end{pmatrix}</td>
</tr>
<tr>
<td>1.68m</td>
<td>4.2m</td>
<td>5.6m</td>
<td>\begin{pmatrix} 4.22 \ 4.22 \end{pmatrix}</td>
<td>5.15m</td>
<td>\begin{pmatrix} 4.14 \ 2.36 \end{pmatrix}</td>
</tr>
<tr>
<td>1.91m</td>
<td>3.15m</td>
<td>4.37m</td>
<td>\begin{pmatrix} 3.25 \ 3.25 \end{pmatrix}</td>
<td>3.57m</td>
<td>\begin{pmatrix} 3.08 \ 1.86 \end{pmatrix}</td>
</tr>
<tr>
<td>2.13m</td>
<td>2.49m</td>
<td>3.52m</td>
<td>\begin{pmatrix} 2.18 \ 2.18 \end{pmatrix}</td>
<td>2.89m</td>
<td>\begin{pmatrix} 2.48 \ 1.42 \end{pmatrix}</td>
</tr>
<tr>
<td>2.25m</td>
<td>2.11m</td>
<td>2.97m</td>
<td>\begin{pmatrix} 1.85 \ 1.85 \end{pmatrix}</td>
<td>2.45m</td>
<td>\begin{pmatrix} 2.05 \ 1.47 \end{pmatrix}</td>
</tr>
</tbody>
</table>

Establish moment of momentum theorem model

When apply mechanical conservation law to solve problems, firstly it should select reasonable research objects, and make correct force analysis of researched objects, secondly on the basis of force analysis, reference conservation law to check problem, finally according to conservation law, establish equation and solve problems.
Set $I$ is one rigid body rotational inertia, it suffers moment $M$ acting. Among them, angular accelerated speed $\beta$ is constant, the rigid body at $t_1$ instant angular speed is $\omega_1$, rigid body at $t_2$ instant angular speed is $\omega_2$, it gets: $M = I\beta = I \frac{\omega_2 - \omega_1}{t_2 - t_1}$

Deform and get: $M(t_2 - t_1) = I(\omega_2 - \omega_1)$

When $M = M(t)$, it has: $M(t)(t_2 - t_1) = I(\omega_2 - \omega_1)$

It gets moment of momentum formula, from which $M(t)(t_2 - t_1)$ is impulsive moment, $I\omega$ is moment of momentum, from formula, it is clear that rigid impulsive moment variable quantity is equal to moment of momentum variable quantity.

In moment of momentum theorem, time and moment product is equal to impulsive moment; it represents object rotational accumulative effect under external force moment effects. Angular speed and rotational inertia product is state when rigid body rotates. With external force moment increasing and acting time enlarging, rigid body rotational state changes are increasing accordingly.

When human body moves, human body generated rotational inertia is changing, due to rotational variable changes, different time rotational inertia is different, set $t_1$ instant rotational inertia is $I_1$, $t_2$ moment rotational inertia is $I_2$, therefore, above formula can be changed as: $M(t)(t_2 - t_1) = I_2(\omega_2 - \omega_1)$

For human body sports rules, it should meet: $I\omega = 0, \sum M \cdot t = 0$

Now it enters into soaring phase, if human body meets: $I_2(\omega_2 + \omega_1) = 0$

In addition, it should also meet that human body rotates around $I_1\omega_1$, then tennis service sports form is opposite movement, during service process, human body moment of momentum vectors sum is 0, according to correlation law, we get that human body will suffer a reactive force from ball that let people to generate moment of momentum, so that reduce service process strength size, so it is bad for service stability, but if in service process, due to body each part suffered active force effects, it causes rotational inertia increasing, so that it will generate an advancing moment of momentum effect, according to energy conservation law, we know that now human body similarly will generate a reactive force effect, so that let human body to move relative to ball, so that increase arm swinging distance, concentrate whole body strength to serve.

In whole service process, each limb will generate moment that on the opposite direction but size is the same, and every pair can offset, when athlete lands, sole part rapidly lands to support the whole body, and meanwhile it will occur to abdomen contraction, knees bending and others to buffer diminished strength to make preparation for next motion.

In the air, angular speed changes, when moment of momentum remains unchanged, rotational inertia will reduce with angular speed increasing, when athlete jumps and soars, athlete himself can further control rotational angular speed by changing self-rotational inertia.

Twist service is when athlete takes off and serves, he serves by changing upper body faced direction, when athlete jumps, it should increase athlete self-rotational angular speed, therefore, athlete jumping leg arrives at flat and straight, lets gravity center and body rotational axis to come to terms, so that it can reduce rotational inertia, so that arrives at effects of increasing rotational angular speed, and at the same time of taking-off, twist upper body can also continue to increase self-rotational angular speed and let ball contacting to be more rapid.

When athlete takes off and arrives at highest point, athlete should try to adjust body stability, let rotational angular speed to reduce as much as possible, now, athlete should raise two legs backward, let gravity center to be far away from rotational axis, and arrive at state of steady ball contacting.

**CONCLUSION**

The paper utilizes Lagrange equations to get restricted particle kinetic equation, combines with theoretical formulas to analyze when tennis player serves, hand joint mechanical movement, combining
with shoulder joint, and elbow joint mechanical analysis to study on tennis service technology. And according to geometric principle, it establishes tennis service model, when athlete serves, he should try to stretch arms to right ahead and to be vertical to service point, when athlete takes-off height gets higher and then probability that ball passes through net will get larger. And apply mechanical conservation law to analyze tennis service problems, it gets that athlete himself can further control rotational angular speed by changing self-rotational inertia. Twist service is when athlete takes off and serves, carries out serving by changing upper body faced direction, when athlete jumps, it should increase athlete himself rotational angular speed, on the contrary, when athlete jumps to the highest point, athlete should try to adjust body stability and let rotational angular speed to reduce as much as possible.

REFERENCES


