Tennis movement trajectory and drop point anticipation analytical research based on biomechanical model

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ABSTRACT

In tennis, every stroke cannot do without anticipating oncoming ball, moving pace, meeting the ball, fast returning these four basic segments, while in the four segments, anticipate oncoming ball is the first and also the most important segment, especially for oncoming ball drop point prediction and judgment, it would be particularly important. It carries out prediction after getting tennis one point three dimensional coordinate and speed vector, applies physics projectile and turbulence flow method, establishes corresponding model. In ideal case that ignores air resistance and flow resistance, it adopts projectile model solving tennis drop point issue that considers tennis rotation with Bernoulli’s principle, using consulted turbulence flow knowledge, establishes corresponding mechanical model, and utilizes mathematical differential equation knowledge to make prediction on tennis drop point.

KEYWORDS

Turbulence flow; Differential equation; Tennis drop point.
INTRODUCTION

Correct prediction and judgment ability is a key ability in net games, meanwhile is also thought to be high level athletes’ crucial quality. As the typical representative of net games, tennis game fully reflects speed, rotation; strength, drop point as well as each tactics combination and other factors concentrate contests. In game, athletes’ actions are constantly suffered opponent returning drop point, rotation influences and so change its own technical and tactics combination, conditions of two parties’ attack and defense are constantly changing, opportunity would be fleeting, the situation is uncertain; athlete should constantly adjust their technical motion according to change status on court, and adopt corresponding way to return. It is clear that athletes’ stroke basis is built on correct prediction and judgment of oncoming ball.

In tennis, predict and judge oncoming ball is the first and also the most important segment, especially for oncoming ball drop point prediction and judgment, it would be particularly important. Whether anticipation is timely, correct, it will direct affect other segments. Only on the premise that make timely correct judgment of oncoming drop point, it will fast move and correct return the ball; once miss or delay prediction and judgment, stroke motion will be affected, it will surely generate return miss and lose the score. Research on each factor that affects tennis athletes’ predicting and judging opponents’ smash drop point, let them master opportunity and method of judging smash drop point, it is of great guiding significance to improve tennis athletes’ smash drop point prediction and judgment accuracy and timeliness as well as improve returning quality; in addition, by analytical researching on different level and training years’ athletes generated differences on prediction and judgment, it can provide some beneficial reference for tennis players’ selection.

At present, from the perspective of international, domestic researches, many scholars have made researches on prediction and judgment problems in respectively different items, as (Abernethy,1988) badminton player drop point judgment expert-beginners comparative analysis, on this account, he researched age and professional skill influences on badminton sports perception development; Expert that applied filming technique on badminton-made correlation research on information selection and prediction judgment ability under beginner mode, so as to reveal sports specialty essence; in the following, Wang Bing, Fu Quan etc. also made correlation analytical research in successive on handball and fencing in sports decision-making ability and other aspects. But it is different to find similar research on tennis event, therefore no matter in theory or from practical perspective, it is necessary to carry out deep exploration on the subject.

PREDICTION MODEL ESTABLISHMENT AND SOLUTION

Basic environment establishment
Tennis diameter: 6.5cm mass : 56.7g
Air density $\rho = 1.205kg \cdot m^{-1}$ air viscosity coefficient $\eta = 18.1 \times 10^{-6} \text{ pa}$

Establish coordinate axis: with base line middle coordinate as origin, x axis is parallel to base line direction, y axis is parallel to sideline direction, z axis is vertical and turns upwards, establish three dimensional coordinate system, unit length is m.

Start the timer from casting moment, and casting moment $t = 0$, ball lies in point $(x_0, y_0, z_0)$. Let $v_0$ showing object initial speed, $t$ to be movement time, $r$ to be displacement vector, $\alpha$ to be $v_0$ and x axis included angle, $\beta$ to be $v_0$ and y axis included angle, $\gamma$ to be $v_0$ and z axis included angle, then $v_0$ in 0x axis, 0y axis, 0z axis components are respectively:

$$v_{0x} = v_0 \cos \alpha$$
\[ v_{0y} = v_0 \cos \beta \]
\[ v_{0z} = v_0 \cos \gamma \]

When ignoring air resistance, object accelerated speed in the whole movement process is:
\[ a = g = -gk \]

Make use of these conditions, it can solve object any time speed in the air as :
\[ v = (v_0 \cos \alpha)i + (v_0 \cos \beta)j + (v_0 \cos \gamma - gt)k \]  \hspace{1cm} (1)

Because \[ v = \frac{dr}{dt} \], it accordingly gets object sports expression as :
\[ r = \int_0^t vdt = (v_0 t \cos \alpha)i + (v_0 t \cos \beta)j + (v_0 t \cos \gamma - \frac{1}{2}gt^2)k \]  \hspace{1cm} (2)

Therefore, \( t \) moment ball position is:
\[ x = x_0 + v_0 t \cos \alpha \]
\[ y = y_0 + v_0 t \cos \beta \]
\[ z = z_0 + v_0 t \cos \gamma - \frac{1}{2}gt^2 \]

Calculate drop point, when ball landing, \( z = 0 \), and then:
\[ t = \frac{v_0 \cos \gamma + \sqrt{v_0^2 \cos^2 \gamma + 2gz_0}}{g} \]  \hspace{1cm} (3)

Input \( x, y \) it gets:
\[ x = x_0 + v_0 \cos \alpha \frac{v_0 \cos \gamma + \sqrt{v_0^2 \cos^2 \gamma + 2gz_0}}{g} \]
\[ y = y_0 + v_0 \cos \beta \frac{v_0 \cos \gamma + \sqrt{v_0^2 \cos^2 \gamma + 2gz_0}}{g} \]

With base line midpoint as coordinate origin, \( x \) axis is parallel to base line direction, \( y \) axis is parallel to sideline direction, \( z \) axis is vertical and turns upward, establish three dimensional coordinate system, unit length is meter. If tennis point \((0, 0, 1)\) gets \((4\sqrt{2}, 16, 1)\) direction speed at 16m/s, it solves tennis drop point as \((2.89, 8.43, 0)\)

**Turbulent flow fluid mechanical model establishment**

In movement process, tennis suffers gravity, air resistance and movement direction vertical lift force-Magnus, as well as air acting on rotational ball internal fraction torque effects.
At first, calculate tennis movement air Reynolds number: \( \text{Re} = \frac{\rho v l}{\eta} = 6.92 \times 10^4 \)  

It is known \( R_e > 4000 \), fluid is in turbulent flowing. At this time, tennis suffered air resistance: 

\[
f = c_p A v^2 / 2
\]

Let: \( k = \frac{c_p A}{2} = 7.993 \times 10^{-4} \)

Then: \( f = kv^2 \)

Tennis suffered force that vertical to rotational angle direction plane, the concrete direction can be defined by right hand rule: right hand thumb, index finger and middle finger are paired vertical, thumb points to rotational angular speed direction, index finger points to movement speed direction, and then middle direction is Magnus direction.

Magnus force: 

\[
F_M = \frac{8}{3} \pi \rho a \omega l^3 v
\]

Let: \( G = \frac{8}{3} \pi \rho a^3 = 3.464 \times 10^{-4} \)

Then: 

\[
F_M = G \omega v
\]

Rotational angular speed direction defining: Actually ball rotation can approximate to top spin or back spin; left spin or right spin. According to right hand rule, top and back spins mainly due to rotational angular speed along x axis direction generates Magnus force along y axis direction, which lets ball left and right deviate.

Therefore, according to ball deviation distance, it can judge rotational angular speed direction, as Figure 1 and Figure 2.

Because ball will also pass through point (0.2, 1.51, 1.04) supposed ball movement trajectory, nearest point to such point is (0.53, 1.51, 1.04), so that the ball belongs to right spin.

**Figure 1 : Magnus force direction judgment**

**Figure 2 : Tennis force analysis**

**Right spin dynamical model**

Given beginning ball rotational direction (angular speed direction) is along 0z axis, inner frictional moment is quite small so that can ignore its influences, and then according to angular momentum conservation law, it can be thought that ball always rotates along 0z axis, from Bernoulli’s
equation, it can judge that tennis suffered Magnus force always lies in horizontal plane and vertical to sphere speed direction, and points to sphere movement trajectory curvature center, as Figure 3 and Figure 4 show.

By force analysis and movement analysis, horizontal plane movement is equivalent to circular movement, according to Newton Second Law; it can get dynamical differential equation:

\[
\begin{align*}
  m \frac{d^2 z}{dt^2} &= -mg \\
  m \frac{dv}{dt} &= -kv^2 \\
  m \frac{v^2}{\rho} &= G \omega v
\end{align*}
\]  

Among them, \( \rho \) is trajectory curvature.
Make integral on (4), it gets: 
\[ \frac{1}{v} = \frac{k}{m} t + C_1 \]

It has initial conditions: \( v(0) = v_0 \), it gets 
\[ v = \frac{mv_0}{m + kv_0 t} \]

Because \( ds = v dt \), and \( \rho = \frac{ds}{d\theta} = \frac{ds}{dt} \frac{dt}{d\theta} = v \frac{dt}{d\theta} \), input formula (5):

\[ \frac{d\theta}{dt} = \frac{G\omega}{m} \tag{5} \]

So it has 
\[ \theta = \int_0^t G\omega \, dt + \theta_0 = \frac{G\omega}{m} t + \theta_0 \]

It is speed on XOY plane two components included angle in the beginning. Due to speed included angle is quite small, approximates it again to 0. Therefore, it has

\[ v_x = v \sin \theta = \frac{mv_0}{m + kv_0 t} \sin \frac{G\omega}{m} t \tag{6} \]
\[ v_y = v \cos \theta = \frac{mv_0}{m + kv_0 t} \cos \frac{G\omega}{m} t \tag{7} \]

Due to actually tennis air movement time is quite small, and \( kv_0 = 0.0126 < m = 0.0567 \)

So:

\[ \frac{mv_0}{m + kv_0 t} \approx v_0 \]

So make integral of (6), (7), according to initial conditions: \( x(0) = 0, y(0) = 0 \), it can get:

\[ x = \frac{v_x m}{G\omega} (1 - \cos \frac{G\omega}{m} t) \tag{8} \]
\[ y = -\frac{v_y m}{G\omega} \sin \frac{G\omega}{m} t \tag{9} \]

We also know that ball passes through point \( (0, 2, 1.51, 1.04) \) \( \omega = 246.1 \)
Therefore, it can get:

\[ x = -16(t - \cos t) \]
\[ y = 16 \sin t \]

Finally, calculate vertical direction:

\[ z = z_0 + v_z t - \frac{gt^2}{2} \tag{10} \]
That is: \[ z = 1 + 0.9695t - 4.9t^2 \]

Drop point area \( z = 0 \), and then solve the equation and get \( t=0.561 \)

Input into suppose, \( x,y \) equation, it gets:
\[ x=-1.6, y=8.56 \]

We use mathematical describing trajectory, it gets simulation trajectory Figure 5.

![Figure 5: Tennis movement trajectory simulation](image)

**CONCLUSIONS**

The main purpose of the research is to analyze tennis players’ smashing drop point prediction and judgment accuracy rate influence factors, explore high level athletes’ and beginners’ tennis drop point prediction and judgment generated difference factors. It provides some beneficial reference from athletes’ selection on smash drop point judgment ability; meanwhile it also provides some basis for tennis players’ training, hints athletes which aspects information should be paid attention to so as to make correct judging on smash point in training and game, which provides a new simulation training way for tennis players’ training and also of certain guiding significance to effective return smash technique teaching. Through tennis drop point calculation, all-around simulate tennis movement curve, further assist referee to define tennis drop point position in game, guarantee game rationality and fairness, so that more helpful for improving tennis public recognition degree, propels to tennis booming development. The model can be applied into large game, judging dispute drop points through electronics equipment, and can guarantee game process justice and fairness.

**REFERENCES**


