ABSTRACT

Along with the progress of The Times, is both beautiful and passionate tennis is more and more popular among the masses and favor, in order to reduce the probability of people were injured while trying to play tennis, tennis players backhand return in this paper, through research on the dynamics of the arm back to find the athletes the forearm and the speed of the shoulder joint, elbow and wrist joints. Tennis is obtained by analysis of moment of inertia of the momentum depends on wrist shot on the momentum of the backhand and got the tennis player in the back to back when pat arm of potential energy and kinetic energy, the athletes can increase the elbow by shoulder joint effective braking the momentum of the elbow will similarly to brake momentum transfer to the wrist, rise to increase the momentum of the tennis hits. So the forearm angular velocity should be greater than the angular velocity of the upper arm, it will be more conducive to the acceleration of elbow joint. And also can get tennis player in his backhand return to shoot with the player at the moment of stroke sudden braking of wrist joint and buffer time lead to smaller lesions in the wrist and elbow. Volkswagen group according to the parts in this paper, the corresponding protection.

KEYWORDS

Dynamic analysis; The analysis of momentum; Momentum transfer; The moment of inertia and biomechanics.
INTRODUCTION

Tennis, originated in France, is one of indoor stroke game which the missionaries took the palm of the hand as a racket, in the 14th century it became a royal aristocrats entertainment sports. Because there is no tennis rackets and nets, they only set up a rope as a line in the middle of the venue and throw the ball to each other with the hand, so the tennis in French was named “Tenez”, meaning: hold on, throw out. So it was named "Tennis" in English later. Then in the 16th century, this kind of royal sports was discovered and followed by French civilians, what is more, they reformed and improved the quality of the tennis racket, but later the king forbade the citizens to play it. However, the modern tennis originated in the 1870 s, by the British named Winfield who improved the rules of tennis and set the field on the lawn then renamed it the lawn tennis. Since then tennis became a both indoor and outdoor sports. In 1905, the was born, then they becomes the most famous "grand slam" together with the Wimbledon tennis tournament, the French championship, and the United States tennis championship, which means that athletes will award the title of "grand slam" who won the four championships. Tennis scoring method is originated in the royal palace, at that time, they use the clock to score as the convenient, which means the clock would strike a quarter of an hour when the player won a ball, so there is a 15 points and 30 points, but because the pronunciation of the 45 points consists three syllables and difficult to read in English, so they took 40 points instead, this is the origin of scores. The use of the tennis tournament venue is divided into the grass, clay and, different tournament game use different places. Australian open championship is held in every January in Australia Melbourne where the place is hard courts. French tennis tournament is held from May to June, using the site for clay court; the Wimbledon tennis tournament is the oldest tournament holding in June and July in the UK, which uses the site for the grass; the United States tennis championship was held in August and September which uses the site for the hard court. As time goes by, the site category should have represented the tournament site. The length of the tennis stadium is 23.77m, the Width of the tennis stadium is 8.23m. The posts shall be the 15cm side length of square column or the 15cm diameter of the cylinder and whose height can't more than the 2.5cm of the net twine, the height of the net twine should be 1.07m. The diameters of tennis are between 6.541 ~ 6.858cm, the weight of the tennis is 56.7g ~ 58.5g. There are many Tennis skills, such as high pressure ball volley, the net to intercept and so on. This article will focus on studying the backhand return which injured the athletes a lot.

THE ESTABLISHMENT OF THE MODEL AND SOLUTION

When the tennis players use backhand return to shoot, the first step is to the racket back, and let the body around the axis rotate to the left, then let the right foot move toward the left, and bent the body to make the right shoulder focus on the net, at the same time, the left hand will return to the left rear back, and keep the right foot tiptoe 1 inch to the shooting place, they hit the tennis ball below the racket back then the tennis ball is up or move out. Then arm has been taken back to swing to the limit angle, another hand moves back to back, conveniently, the two arms hold out a bosom as a nature extend, after completion of the whole action athletes return to ready state. Figure 1, figure 2 are the front and back of the tennis players’ backhand return.

The dynamics analysis of Tennis taken backhand

Now we take the big arm forearm of the tennis player’s taken backhand as two rigid bodies, and building up three degrees of freedom of the shoulder joint and elbow, as is shown in figure 3.

Figure 1 : The front figure of the tennis players’ backhand return
In figure 3, $P$ is the shoulder joint of the athlete, and $P_1$ is the elbow joint. The upper arm and forearm is $J_1, J_2$. The anatomical angles are $\phi_1, \phi_2$. Among them, $P$ and $P_1$ have three degrees of freedom and the trivector are $\dot{\phi}_1, \dot{\phi}_2$.

Now we regard the earth as the reference frame. $\lambda_1$ and $\lambda_2$ are the angular velocity, relative to the reference frame, of the upper arm and forearm of the arm the athlete used to apply return shoot. The velocity of the elbow joint is $\dot{\phi}_2$, and its value is:

$$\dot{\phi}_2 = \lambda_2 - \lambda_1$$

When the tennis player deliver a backhand return to shoot, the speed the tennis is shot out depends on the instantaneous velocity of the shooting arm. And, the angular velocity of wrist $O$ is subject to the angular velocities of the upper arm, the forearm, and the elbow. The reason is that when elbow joint $P_1$ is circling, it will force the forearm to make translation and rotation in the relative coordinate system $P_1 - xyz$. Besides, the translation and rotation are relatively independent, and therefore, it won’t affect the motion vectors of the hip joint and the elbow joint and the angular velocity of the crus. But the speed of wrist $O$ is connected to its relative velocity and the speed of elbow joint $P_1$. So:

$$\vec{C}(P_1)_O = \dot{\phi}_1 \times H_1 = \dot{\phi}_1 \times H_1, \quad \vec{C}(O)_L = \dot{\phi}_2 \times H_2$$

In the equation, $\vec{C}(P_1)_O$ is the velocity vector of the elbow joint in the reference frame, and $\vec{C}(O)_L$ is the speed of $O$ relative to elbow joint $P_1$. $\dot{\phi}_1, \dot{\phi}_2$ are the angular velocities of shoulder joint $P$ and elbow joint $P_1$. $H_1$ is the position vector between the shoulder joint and the knee joint, and $H_2$ is the position vector between the elbow joint and wrist joint $O$. 
To get the speed of wrist joint $O$ relative to the earth $\mathbf{\dot{O}_G} = \mathbf{\dot{C}(O)G}$, we must know the influence of the local motion of the upper arm and the forearm to the elbow joint. According to the vector theorem, we can get:

$$\mathbf{\dot{O}_c} = \phi_1 \times \mathbf{H}_1 + \phi_2 \times \mathbf{H}_2 + \phi_3 \times \mathbf{H}_3$$

$$\mathbf{\dot{O}_c} = \phi_1 (\mathbf{H}_1 + \mathbf{H}_2) + \phi_3 \times \mathbf{H}_3$$

To simplify is:

$$\mathbf{\dot{O}_c} = \mathbf{\dot{O}_c} \times \phi_1 + \phi_3 \times \mathbf{\dot{H}_2}$$

The shoulder joint makes the speed produced by the wrist joint to be cross product $\mathbf{\dot{O}_G} \times \phi_1$, and cross product $\lambda_2 \times \mathbf{V}_2$ is the speed produced by the wrist joint made by the elbow joint. $\mathbf{\dot{O}_c}$ is the position vector of the wrist joint in the reference frame.

To describe the velocity relation of the shoulder joint, elbow joint and the wrist joint further and accurately, we write down the relationship between the angles of the shoulder joint and the elbow joint and the position of the wrist joint as:

$$\begin{align*}
x_p &= H_1 \cos \phi_1 + H_2 \cos(\phi_1 + \phi_2) \\
y_p &= H_1 \sin \phi_1 + H_2 \sin(\phi_1 + \phi_2) \\
z_p &= H_1 \cos \phi_1 + H_2 \sin(\phi_1 + \phi_2)
\end{align*}$$

To perform differential to the angles of the shoulder joint and the elbow joint, the relationship between the position vector of the wrist joint and them can be obtained by derivation:

$$\begin{align*}
dX &= \frac{\partial X(\phi_1, \phi_2)}{\partial \phi_1} d\phi_1 + \frac{\partial X(\phi_1, \phi_2)}{\partial \phi_2} d\phi_2 \\
dY &= \frac{\partial Y(\phi_1, \phi_2)}{\partial \phi_1} d\phi_1 + \frac{\partial Y(\phi_1, \phi_2)}{\partial \phi_2} d\phi_2 \\
dZ &= \frac{\partial Z(\phi_1, \phi_2)}{\partial \phi_1} d\phi_1 + \frac{\partial Z(\phi_1, \phi_2)}{\partial \phi_2} d\phi_2
\end{align*}$$

We transfer the equations above into matrix:

$$\begin{pmatrix}
dX \\
dY \\
dZ
\end{pmatrix} = \begin{pmatrix}
\frac{\partial X(\phi_1, \phi_2)}{\partial \phi_1} & \frac{\partial X(\phi_1, \phi_2)}{\partial \phi_2} \\
\frac{\partial Y(\phi_1, \phi_2)}{\partial \phi_1} & \frac{\partial Y(\phi_1, \phi_2)}{\partial \phi_2} \\
\frac{\partial Z(\phi_1, \phi_2)}{\partial \phi_1} & \frac{\partial Z(\phi_1, \phi_2)}{\partial \phi_2}
\end{pmatrix}
\begin{pmatrix}
d\phi_1 \\
d\phi_2
\end{pmatrix}$$

Based on the property of the matrix and Vector product method, the equations above can be written as $d\mathbf{O_G} = \mathbf{U} d\phi$. $\mathbf{U}$ is:

$$\mathbf{U} = \begin{pmatrix}
\frac{\partial X}{\partial \phi_1} & \frac{\partial X}{\partial \phi_2} \\
\frac{\partial Y}{\partial \phi_1} & \frac{\partial Y}{\partial \phi_2} \\
\frac{\partial Z}{\partial \phi_1} & \frac{\partial Z}{\partial \phi_2}
\end{pmatrix}$$
$\vec{U}$ is the differential relation between the displacement of node-angle and infinitesimal displacement of the ankle joint. We substitute the matrix into the equations above and get:

$$\frac{d \hat{O}_b}{dt} = U \frac{d \phi}{dt}, \text{ or } \hat{O}_b = U [\phi_1, \phi_2]^T$$

To substitute it into the equation calculating the relative speed of the wrist joint, we can get:

$$\dot{O}_b = \begin{pmatrix} \frac{\partial X}{\partial \phi_1} & \frac{\partial X}{\partial \phi_2} \\ \frac{\partial Y}{\partial \phi_1} & \frac{\partial Y}{\partial \phi_2} \\ \frac{\partial Z}{\partial \phi_1} & \frac{\partial Z}{\partial \phi_2} \end{pmatrix} [\dot{\phi}_1, \dot{\phi}_2]^T \dot{\phi}_1 + \dot{\phi}_2 \times \ddot{H}_z$$

As the arm that the player used to perform returning shoot will perform fixed-axis rotation, now we regard the upper arm and forearm of the returning arm as two rigid bodies, and they are $I_1$ and $I_2$. Figure 4 is a diagram of the rotation when the player perform returning shoot.

![Figure 4: Schematic diagram of athletes’ backhand return shot rotation](image)

So the entire moment of inertia of net men should be:

$$L = \Sigma M_i$$

$M_i$ are qualities of all parts of the body. $D$ are lengths from all parts of the body to body axis. Because this text treats net men’ bodies as rigid bodies whose qualities are continuous distributed. So:

$$L = \int \int \rho dm = \int \int \rho dV$$

$\rho$ is the body density. When tennis players are doing backhand return shot, the rotation tensor of rotating arms $\overset{\rightarrow}{W}_c$ is:

$$\overset{\rightarrow}{W}_c = \int \int \rho (D^2 \overset{\rightarrow}{\omega} - \overset{\rightarrow}{D} \overset{\rightarrow}{\omega} D) dV$$

The vector expression of arbitrary point $P$ on tennis players is $\overset{\rightarrow}{D} = D_1 \overset{\rightarrow}{e}_1 + D_2 \overset{\rightarrow}{e}_2 + D_3 \overset{\rightarrow}{e}_3$; two vectors product is $\overset{\rightarrow}{D} \overset{\rightarrow}{D}$; unit tensor of tennis player is:
\[ E = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \]

\((C; \bar{e}_1, \bar{e}_2, \bar{e}_3)\) is the unit orthogonal curvilinear frame.

**The moment equation of rotation tensor when tennis players are doing backhand return shot**

When tennis players are doing backhand return shot, the resultant moment vector of rotating arms is \(\Sigma \bar{i}_c \cdot \omega\) is the tennis player’s angular speed vector in inertial coordinate. Angular acceleration vector is \(\alpha\). So when tennis player is doing backhand return shot, the moment equation of rotation tensor is: \(\Sigma \bar{i}_c = W \bar{e} \cdot \alpha + \omega \times W \bar{e} \cdot \omega\).

**Moment of inertia of athletes’ elbow joint**

When tennis player is doing backhand return shot, we can treat his or hers elbow joint as rotation around the centroid axis. Supposing in this process, the axis over the elbow joint is almost parallel to the athlete’s body axis, like figure 5.

![Figure 5: The schematic diagram of rotation of tennis player’s elbow joint](image)

From figure 5, we can see that moment of inertia of athlete’s forearm \(L_2\) is:

\[ L_2 = L_C + MD^2 \]

Then we project moment equation on \(x\), \(y\) and \(z\) axis in three coordinate system. We can get moment equation of athlete’s backhand arm in each axis. The rotation of backhand arm produced the resultant moment \(K_1\) when tennis player is doing backhand return shot is: \(K_1 = \alpha \cdot L_1\). In the formula, \(\alpha\) is the angular acceleration of backhand arm. \(L_1\) is the moment of inertia. While \(L_1 = \frac{M_i r_i^2}{2}\), \(r_i\) is radius of backhand arm’ big arm. \(M_i\) is big arm’ quality. And big arm’s angular acceleration \(\alpha_1\) is:

\[ \alpha_1 = \frac{d\omega_1}{dt} = \frac{d^2 \alpha_1}{dt^2} \].

So initial angular acceleration of backhand arm’s forearm is equal to big arm’s angular speed \(\alpha_1\). So forearm’s angular speed \(\alpha_2\) is:

\[ \alpha_2 = \frac{d\omega_2}{dt} + \frac{d\omega_1}{dt} = \frac{d^2 \alpha_2}{dt^2} + \frac{d^2 \alpha_1}{dt^2} \].

Through formal analysis, we can know that momentum of tennis depends on momentum of wrist hitting balls. And when tennis player is doing backhand return shot, in order to make tennis get max impulse, player must fasten backhand speed. Only in this way can \(O\) get speed maximum in z axis. And only when \(\phi_1, \phi_2\) meet with condition \(40^\circ < \phi + \phi^2 < 90^\circ\) and \(0 < \phi_1 < \phi_2\), wrist joint \(O\) ’s speed can get maximum on vertical plane. And when \(\phi_1, \phi_2\) meet with their constraint condition, with the growth of \(\phi_1, \phi_2\), the process that tennis player doing backhand return shot requests athlete’s big arm and forearm’
change rate of anatomical angle in unit time to reach maximum. And in this process, forearm’s change rate of anatomical angle must be bigger than thigh’ angle changing rate. According to can transfer theorem of rigid body, at the moment that the backhand arm hitting the ball, the angle between big arm and forearm is almost $180^\circ$, the power will transfer from big arm axis to forearm. But because big arm and forearm are connected, so the power will lose during the transferring process. So the angular speed of forearm bigger than big arm will be more benefit to elbow joint’s acceleration. And from the dynamic analysis results of athletes’ backhand return shot, when cheer leading players are doing backhand return shot, they must fasten forearms’ speed to make forearms’ angle bigger than big arms’ in order to speed up elbow joints’ acceleration. But when men shot balls, it will make every joint sudden stopping, so joint $O$ and joint $p_1$ will injury because of joint $p_1$ ’ sudden braking on the moment when athlete is shooting balls.

CONCLUSION

Through kinetic analysis of backhand arms when tennis players are doing backhand return shot, this text found the speed relationship of athletes’ big arms, forearms, shoulder joint, elbow joint and wrist joint. Through analysis of its moment of inertia, we knew that momentum of tennis depends on momentum of wrist hitting balls, and we got backhand arms produced potential energy and kinetic energy during the process that tennis players doing backhand return shot, i.e. athletes can increase elbow joints’ kinetic energy by shoulder joints’ effective braking. In the same way, they can stop elbow joints to transfer kinetic energy to wrist joints in order to increase kinetic energy that the ball is being shot. So angular speed of forearms should be bigger than big arms’, and in this way can be more benefit to fasten elbow joints’ acceleration. And we can also knew that when athletes are doing backhand return shot, the elbow joints and wrist joints will injury because the sudden braking of shooting balls and too short buffertime.

REFERENCES