Table tennis changes to movement trajectory influence research based on fuzzy comprehensive evaluation and differential equation

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ABSTRACT

The paper firstly makes force analysis of table tennis when lifting the ball, and according to moment of momentum theorem, it gets rotational speed and diameter inverse ratio relations; then it makes force analysis of air movement table tennis, and gets kinematic equation. After diameter getting big, ball speed and rotational speed get small, and ball movement time gets long, let opponent probability of receiving the ball and fighting back increase. And establish analytic hierarchy process and fuzzy comprehensive evaluation test model to test. It concludes that big ball promotes competition appreciation relative to small ball, but players’ competition experiences reduce. © 2014 Trade Science Inc. - INDIA

KEYWORDS

Table tennis diameter; Differential equation; Analytic hierarchy process; Fuzzy comprehensive evaluation.

INTRODUCTION

In 2000, international table tennis federation increased international table tennis professional competition official ball diameter from 38mm to 40mm. The aim is to further increase ball’s air resistance during air running, slow down competition’s ball running speed, so that achieve the purpose of further increasing and enriching table tennis professional athletes hitting techniques and skills, and finally increase table tennis competitions’ overall appreciation. Due to sphere increasing, it causes sphericity, arc, ball speed, rotation, rebound angle and others have a series of changes, which will put forward higher requirements on athlete’s psychological quality, physical quality and technical level so on. It is worth noting that due to professional athletes’ height, playing habit, gripping habit differences, their sensitivities to ball diameter changes are also different.

Table tennis as a kind of sport that extremely exercises participants reactions, coordination and operational thinking abilities, it is well received by Chinese mass. As participants themselves, whether can easier play higher offensive ball is the key to participating experiences, therefore, to athlete’s experience quality, the paper mainly from ball speed, strength, rotational speed three aspects and diameter relations, by consulting corresponding data, it further gets before and after ball diameter changes, ball’s changes in speed, strength, and rotational speed the three aspects sizes, according to final values comparison, it objectively reflects whether athletes can more easily hit high offensive ball and that is closely related to their experience qualities.

As audience, competition intense degree no doubt is the main factor that affects competition appreciation. An intense competition, it is surely that participating two...
parties’ strength difference is not big so that can stimulate players’ highest levels. Players reflected abilities are stronger and then that reflects on competition each round number of hitting rackets increasement as well as each game number of rounds increasement. If every round number of hitting rackets increase, and then athletes surely have higher receiving accuracy, in case ignoring players themselves abilities, only increase players’ receiving reaction time that can extremely increase participating players’ receiving accuracy, and finally improve competition appreciation quality.

**COMPETITION EXPERIENCE QUALITY AND APPRECIATION RESEARCH**

**Table tennis dynamical analysis**

As Figure 1 show, when athlete swings and lifts the ball, racket and horizontal line angle is $\alpha$, make force analysis of ball: in the direction of racket movement, ball suffered friction force is $f$, in racket movement vertical direction, ball suffered force is $F$, racket to ball acting time is $t_0$. Set table tennis quality is $m$, diameter is $D$, speed is $v_0$, set $f$ in its own direction gives table tennis impulse $\int_0^t f \, dt = I_1$, set $F$ in its own direction gives table tennis impulse $\int_0^t F \, dt = I_2$, meanwhile $f$ to table tennis mass center one torque is $M = \frac{D \cdot f}{2}$, set torque $M$ to table tennis impulsive moment $\int_0^\Delta L M \, dt = \Delta L$, table tennis rotational inertia $J = \frac{mD^2}{6}$ is $
abla$

$$I_1 = mv_1 - 0$$

$$I_2 = mv_2 - 0$$

$$\Delta L = J\omega - 0$$

It solves:

$$v_1 = \frac{f \cdot t_0}{m}$$

$$v_2 = \frac{F \cdot t_0}{m}$$

$$\omega = \frac{3f \cdot t_0}{mD}$$

And $n = \frac{\omega}{2\pi}$

Conclusion one: By formula (6), it is clear that when impulsive moment is fixed, table tennis angular speed $\omega$ is inversely proportional to its diameter $D$, by formula (7) it is clear that angular speed $\omega$ is in direct proportion to rotational speed $n$, therefore when $D$ changes from $38\,mm$ to $40\,mm$, $n$ will correspondingly reduce.

After athlete swinging and lifting the ball, analyze table tennis speed as Figure 2. Because $f$ and $F$ are perpendicular, $v_1$ and $v_2$ included angle is 90 degree, and $v_2$ and $y$ axis included angle is also $\alpha$. Set $v_1$ horizontal direction component $v_1 \cos \alpha$ is $v_{1x}$, vertical direction component $v_1 \sin \alpha$ is $v_{1y}$; $v_2$ horizontal direction com-
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ponent \( v_2 \sin \alpha \) is \( v_{x_2} \), vertical direction component \( v_2 \cos \alpha \) is \( v_{y_2} \).

Then:

\[
\begin{align*}
    v_x &= v_{x_1} + v_{x_2} \\
    v_y &= v_{y_1} - v_{y_2}
\end{align*}
\]

It solves:

\[
\begin{align*}
    v_x &= \left( f \cos \alpha + F \sin \alpha \right) t_0 \\
    v_y &= \left( f \sin \alpha + F \cos \alpha \right) t_0
\end{align*}
\]

(8)

Table tennis force analysis when moving in the air:

When no rotating table tennis moves in the air, it suffers gravity \( G \), friction force \( F_m \), buoyancy \( F_f \), friction force and speed direction are opposite, buoyancy and gravity direction are opposite. When table tennis rotates at higher speed, it should also consider Magnus force. As Figure 3 show:

Gravity expression:

\[ G = mg \] (\( g = 9.8 \text{m} \cdot \text{s}^{-2} \))

Suffered buoyancy \( F_f \) is equal to table tennis displaced air mass, table tennis volume \( V = \frac{1}{6} \pi D^3 \), set air density is \( \rho \), so

\[ F_f = \rho g V = \frac{\pi}{6} \rho g D^3 \]

(9)

In movement, friction force formula is:

\[ F_m = \frac{1}{8} C_D \pi D^2 v_1^2 \]

(11)

Among them, \( C_D \) is resistance coefficient, \( \rho \) is air density, \( D \) is ball diameter, \( v \) is table tennis speed when it is out of racket.

Magnus force is a force generated due to object two sides air flow speeds are different on condition that object rotational angular speed vector and object flight speed vector don’t overlap, its direction is as Figure 3 show, its direction is vertical to speed direction, its expression is:

\[ F_M = C_L \rho D^3 f v_1 \]

(12)

Among them, \( C_L \) is lift force coefficient, \( \rho \) is air density, \( D \) is table tennis diameter, \( f \) is table tennis rotational frequency, \( v_1 \) is table tennis speed when it is out of racket.

According to Figure 3, establish table tennis rotational movement dynamical equation[21]:

\[
\begin{align*}
    m \alpha_x &= C_L \rho D^3 f v_1 \cdot \sin \alpha - \\
    \frac{1}{8} C_D \pi D^2 v_1^2 \cdot \cos \alpha \\
    m \alpha_y &= \frac{\pi}{6} \rho g D^3 - G - C_L \rho D^3 f v_1 \\
    \cdot \cos \alpha - \frac{1}{8} C_D \pi D^2 v_1^2 \cdot \sin \alpha
\end{align*}
\]

(13)

By Matlab programming, calculate when \( D = 40 \text{mm} \) and \( D = 38 \text{mm} \), assume that initial speed is \( 8 \text{m} \cdot \text{s}^{-1} \), initial displacement is \( 0 \), speed \( v_1 \) relative to time \( t \) image comparison is as following Figure 4, displacement \( s \) relative to time \( t \) is as Figure 5, Figure 6.

When \( D = 40 \text{mm} \), speed \( v_1 \) with regard to time \( t \) function:

\[ v_1 = 491/((360 - 2389/8) \cdot \exp(-491/2000000 \cdot t)) \]

(14)

When \( D = 38 \text{mm} \), speed \( v_1 \) with regard to time \( t \) function:

\[ v_1 = 4/((11 - 37/4) \cdot \exp(-21/100000 \cdot t)) \]

(15)

Assume that athlete swings in the external 1m, it drops on the table, length of table tennis table is 2.74m,
so assume that two cases horizontal displacements are equal as 3.5 m, from Figure 5 and Figure 7, it can get ball movement time are 0.45 s and 0.40 s. According to time, it finds out from Figure 4 that table tennis end speeds are $v_1 = 7.80 m \cdot s^{-1}$ and $v_2 = 7.97 m \cdot s^{-1}$.

Due to 40 mm table tennis diameter increasing relative to that of 38 mm, it let drop point speed reduce 0.02 $m \cdot s^{-1}$. 40 mm table tennis mass is $m_1 = 2.7 g$, 38 mm table tennis mass is $m_2 = 2.6 g$, so the two end kinetic energies are respectively:

\[
E_1 = \frac{1}{2} m_1 v_1^2 = 0.081 J
\]

\[
E_2 = \frac{1}{2} m_2 v_2^2 = 0.083 J
\]

Therefore after diameter increasing, its end kinetic energy reduces for instead.

By above, it is clear that after table tennis diameter increasing, arriving at drop point time is 1.12 times previous one, which provides opponent enough reaction time to judge ball trend and drop point, and receiving probability will greatly increase.

In order to check above whether above result conforms to practice or not, the paper investigates on partial audience opinions on competition exciting level after changing into big ball.

Audience’s evaluation on big ball competition intense level relative to small ball one has four measurement indicators: number of rounds, rotation, speed and strength\cite{4}. Different indicators relations are fuzzy, and impacts on competition intense level are different. Therefore, use fuzzy comprehensive evaluation model combining with analytic hierarchy process to solve the problem. First utilize analytic hierarchy process solving each indicator weight vector. And then make grading classification on impacts sizes, it constructs single item indicators to impacts grades membership function $\mu_F(x)$, weight vector combining with membership matrix to solve fuzzy comprehensive evaluation matrix, it can get audience overall evaluation.

**Fuzzy comprehensive evaluation method**

Set factors set as $X = \{x_1, x_2, \ldots, x_m\}$, it represents object possessed m pieces of attributes; evaluation set is $Y = \{y_1, y_2, \ldots, y_m\}$, it represents evaluation that may
adopt to factors. Fuzzy evaluation is a fuzzy set on $Y$. In the paper $Y = \{A, B, C, D\}$, A grade represents: audience evaluation as very well; B grade represents: audience evaluation as good; C grade represents: audience evaluation as normal; D grade represents: audience evaluation as bad. When getting indicator $x$ membership vector, in general, it adopts maximum membership principle to make grades classification on $x$, but the classification way generally is very fuzzy, it ignores $x$ to other grades membership relations. Therefore, to comprehensive carry out grade evaluation on $x$, it scores the grades here that lets $A = 10, B = 7, C = 5, D = 1$. That is changing membership vector into weight relational vector, element $x$ comprehensive grade score is:

$$M = 10A(x) + 7B(x) + 5C(x) + D(x)$$

Now assume that every factor $x_i$ has a fuzzy evaluation $R_i = \{r_{i1}, r_{i2}, \cdots, r_{in}\} \in F(X \times Y)$. Then, to $m$ pieces of factors, it has $m$ pieces of fuzzy evaluation $R_1, R_2, \cdots, R_m$, they always can be expressed by matrix $R$:

$$R = \begin{pmatrix}
R_1 \\
R_2 \\
\vdots \\
R_m
\end{pmatrix} = \begin{pmatrix}
r_{11} & r_{12} & \cdots & r_{1n} \\
r_{21} & r_{22} & \cdots & r_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
r_{m1} & r_{m2} & \cdots & r_{mn}
\end{pmatrix}$$

Call it as single factor evaluation matrix. If known fuzzy relation matrix $R$ and factor weight distribution is $A = (a_1, a_2, \cdots, a_m)$, from which $a_i > 0$ and $\sum_{i=1}^{m} a_i = 1$, then it can solve fuzzy comprehensive evaluation $B$ by $A$ and $R$. The computing can be written as following form: $B = A \circ R$.

Here “$\circ$” represents compositional operation.

**Make use of analytic hierarchy process to solve each indicator weight**

Analytic hierarchy process: analytic hierarchy process is a method that according to total target, each layer sub target, evaluation criterions till concreted switching programs orders, decompose decisive problems into different hierarchical structures, and then apply judgment matrix feature vectors solution solving every layer each element to previous layer one element priority weight, finally use weighting sum method hierarchically merging each alternative schemes to total target final weight. The maximum final weight one is optimal scheme.

According to nine scale method, it constructs indicator judgment matrix, nine-scale method is as TABLE 1.

Indicator judgment matrix is: $A = \begin{pmatrix}
1 & 3 & 6 & 9 \\
1/3 & 1 & 3 & 5 \\
1/6 & 1/3 & 1 & 2 \\
1/9 & 1/5 & 1/2 & 1
\end{pmatrix}$

Matrix $A$ maximum feature value is $\lambda_{\max}$, corresponding feature vector is $u = (u_1, u_2, \cdots, u_n)^T$, normalize $u$, solve with the help of MATLAB software, it gets matrix $A$ maximum feature value is $\lambda_{\max} = 4.0340$, corresponding weight vector that normalized feature vector is:

$$U = \begin{pmatrix}
0.5990 \\
0.2505 \\
0.0962 \\
0.0543
\end{pmatrix}$$

**Consistency test**

Test principle: consistency test indicator is $CI = \frac{\lambda_{\max} - n}{n - 1}$ (n is matrix order, here $n=3$), consis-

**TABLE 1: Nine-scale method**

<table>
<thead>
<tr>
<th>Definition</th>
<th>$u_i$ and $u_j$ are equal</th>
<th>$u_i$ is slightly important than $u_j$</th>
<th>$u_i$ is important than $u_j$</th>
<th>$u_i$ is intensely important than $u_j$</th>
<th>$u_i$ is extremely important than $u_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_{ij}$</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>9</td>
</tr>
</tbody>
</table>

2, 4, 6, 8 between adjacent two judgment scale
tency ratio \( CR = \frac{CI}{RI} \) , when \( CR < 0.1 \) , and then it calls the matrix has consistency. Consistency indicator \( RI \) value is as TABLE 2 show.

Consistency test result is as TABLE 3.

**Establish comprehensive evaluation matrix**

**Membership function**

To membership function, the paper makes comparison on relative data; it gets single indicators memberships TABLE 4;

**TABLE 2 : Consistency indicator \( RI \) value**

<table>
<thead>
<tr>
<th>Matrix order number</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( RI ) value</td>
<td>0</td>
<td>0.58</td>
<td>0.90</td>
<td>1.12</td>
<td>1.24</td>
<td>1.32</td>
<td>1.41</td>
<td>1.45</td>
</tr>
</tbody>
</table>

**TABLE 3 : Consistency test result**

<table>
<thead>
<tr>
<th>( \lambda_{\max} )</th>
<th>CI</th>
<th>RI</th>
<th>CR</th>
<th>Whether meet consistency</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0340</td>
<td>0.0113</td>
<td>0.90</td>
<td>0.0126</td>
<td>Yes</td>
</tr>
</tbody>
</table>

**TABLE 4 : Single indicators memberships**

<table>
<thead>
<tr>
<th>Factor</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of rounds</td>
<td>0.53</td>
<td>0.25</td>
<td>0.17</td>
<td>0.05</td>
</tr>
<tr>
<td>Rotation</td>
<td>0.48</td>
<td>0.43</td>
<td>0.09</td>
<td>0.00</td>
</tr>
<tr>
<td>Speed</td>
<td>0.33</td>
<td>0.42</td>
<td>0.16</td>
<td>0.09</td>
</tr>
<tr>
<td>Strength</td>
<td>0.29</td>
<td>0.54</td>
<td>0.08</td>
<td>0.09</td>
</tr>
</tbody>
</table>

**Fuzzy comprehensive evaluation matrix**

According to each indicator weight vector \( U \) , combine with membership matrix \( V \) , and get fuzzy comprehensive evaluation matrix \( R \) . According to fuzzy matrix model that is \( R = U \circ V \) , from which to simplify algorithm, convert \( \circ \) into common matrix product computing that

\[
R = U^T \ast V
\]

It solves:

\[
U^T \ast R = (0.4852 \quad 0.3272 \quad 0.1441 \quad 0.0435)
\]

Qualitative analysis uses maximum membership principle and knows that 0.4852 is the maximum value, that is to say audience evaluation on competition is A grade, which shows audience evaluation on competition is very well, score is

\[
M = 10 \times 0.4852 + 7 \times 0.3272 + 5 \times 0.1441 + 0.0435 = 7.9064
\]

By testing, above computing result conforms to fact.

**TABLE TENNIS BEST DIAMETER RESEARCH**

By above, it is known that athlete experience quality and audience appreciation quality interact on each other. Table tennis movement speed has connections with diameter, when its displacement is fixed, the smaller diameter is, the shorter experienced time is, and smaller probability of athlete receiving would be. When athlete failed receiving ball, his reaction time \( t \) , reaction time when receiving is \( T \) . Set \( x = \frac{T}{t} \) , when \( x \) gets closer to most of players’ two reaction times ratios, athlete receiving probability is bigger, player can more give his level into play, experience effect would also be better, meanwhile the number of rounds would be more, competition will also be more exciting, and audience appreciation quality will also improve. Therefore select best ratio \( x \) , and then it can get best table tennis diameter.

According to formula(13), it can solve:

\[
\begin{align*}
\alpha_x &= \frac{C_1 \rho D^3 v_{f1} \cdot \sin \alpha}{m} \\
\pi g D^2 v_{f1} \cdot \cos \alpha \\
8m \\
\alpha_y &= \frac{\pi g D^2 v_{f1} \cdot \cos \alpha}{6m} - \frac{C_1 \rho D^3 v_{f1} \cdot \cos \alpha}{m} \\
\pi g D^2 v_{f1} \cdot \sin \alpha \\
8m \\
\end{align*}
\]

(16)

Set table tennis moves in the air at initial speed \( v_x \) , dropping to table moment horizontal end speed is \( v_x' \) , in horizontal direction, according to theorem of kinetic en-
energy, it gets:

\[
\max_x = \frac{1}{2}mv_x^2 - \frac{1}{2}mv_x^2
\]  

(17)

By formula (14), it is clear that

\[
a_x = \left(\frac{C_L \rho D f v_i \sin \alpha - C_D \pi p v_i^2 \cos \alpha}{m} \right) D^2
\]

(18)

and \(v_i\) is a variable, in order to easier to calculate, assume \(k = \max \left[ \frac{C_L \rho D f v_i \sin \alpha}{m}, \frac{C_D \pi p v_i^2 \cos \alpha}{8m} \right] \),

Therefore formula(17) can be converted as:

\[-mkD^2 = \frac{1}{2}mv_x^2 - \frac{1}{2}mv_x^2\]

(19)

That:

\[
v^2_x - v^2_x = kD^2
\]

(20)

Now accelerated speed is a fixed value, so \(v_x = \frac{1}{2}(v_x + v_i)\), that \(v' = 2v_x - v_x\), therefore

\[
v_x - v^2_x = v_x - (2\bar{v}_x - v_x) = 2(v_x - \bar{v}_x)
\]

\[
v^2_x - v^2_x = (v_x - v_x)'(v_x + v_x)' = 2(v_x - \bar{v}_x) \times 2\bar{v}_x = kD^2
\]

(21)

\[
D_0 \text{ is } 38\text{mm ball, } D_1 \text{ is an unknown quantity, } D_1 \text{ average speed is } \bar{v}_{x_1}, \text{ due to hitting speed (that is also initial speed) basically do not change with ball diameter changing, } D_1 \text{ initial speed is also } v_x. \text{ Assume same excellent athlete plays } D_0, D_1 \text{ two different diameters table tennis, and roughly thought that table tennis displacements are equal.}

\[
\frac{(v_x - \bar{v}_{x_1})}{\bar{v}_{x_1}} = \frac{D_0^2}{D_1^2}
\]

(22)

Set \(\frac{\bar{v}_{x_0}}{\bar{v}_{x_1}} = x\), then \(\bar{v}_{x_0} = x \times \bar{v}_{x_1}\)

(23)

Assume same athlete plays \(D_0, D_1\) two different diameters table tennis, and roughly thought that table tennis displacements are equal

\[
t_0 \times \bar{v}_{x_0} = t_1 \times \bar{v}_{x_1}, \text{ therefore } \frac{t_1}{t_0} = x \times t_0 \text{ is diameter as table tennis movement time, is diameter as table tennis movement time.}
\]

Table tennis air movement time \(t\) is athlete reaction judging time \(t\). Watch 40 athletes competition video, and get their receiving ball’s reaction time and missing ball reaction time ratios data is as TABLE 5 (reserves two decimal fractions).

By TABLE 5, it is clear that most of athletes data is equal or approximate to 1.18, so take \(x = 1.18\)

\[
\text{According to table tennis association investigation group provided information, when smashing } 38\text{mm table tennis, hitting speed is nearly } 26, 35 \text{m/s, ball average flight speed is nearly } 17.8 \text{m/s. Due to hitting speed (that is also initial speed) basically will not change with ball diameter changing, diameter as } D_1 \text{ table tennis hit-}
\]

| TABLE 5 : Receiving ball reaction time and missing ball reaction time ratios |
|-------------------------------|-----------------|-----------------|-----------------|-----------------|
| 1.18                         | 1.18            | 1.20            | 1.16            | 1.21            |
| 1.22                         | 1.20            | 1.22            | 1.22            | 1.19            |
| 1.17                         | 1.18            | 1.18            | 1.19            | 1.19            |
| 1.16                         | 1.21            | 1.17            | 1.20            | 1.18            |
| 1.18                         | 1.18            | 1.18            | 1.18            | 1.18            |
| 1.17                         | 1.22            | 1.20            | 1.17            | 1.17            |
| 1.18                         | 1.20            | 1.28            | 1.16            | 1.21            |
| 1.20                         | 1.28            | 1.23            | 1.18            | 1.27            |
Input data into (21) and solve: \( D_i = 43.6177 \text{mm} \).

**CONCLUSIONS**

The paper gets 38mm and 40mm two table tennis influences on athlete experience quality and audience appreciation quality by analyzing different radius table tennis force and flight trajectory, and uses analytic hierarchy process carrying out weight measurement and fuzzy comprehensive evaluation method to make test, it eliminates subjective factors caused weight quantity not clear, it improves test result accuracy. It further solves best table tennis diameter on the condition that ensure athlete experience quality and audience appreciation quality, the result has higher reliability.

**REFERENCES**


4. Dong Yang, Zhu FengJun, Gu JianPing; Analysis on Technique and Tactics of the Excellent Foreign Doubles in the 48th World Table Tennis Championship[J]. Journal of Chengdu Physical Education Institute, **31**(4), 73-75 (2005).


8. Zhang Qiu-Fen, Su Jing; Analysis on Medal Distribution and Medallist’s Playing Type of Table Tennis in the Olympic Games[J]. China Sport Science and Technology, **41**(5), 90-92 (2005).

