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Synchronization analysis of multiple systems based on a class of symmetric matrix

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ABSTRACT

Based on a class of symmetric matrix, the synchronization of multiple systems is investigated. The system error, which is different from the error in previous mentioned, of the complex network model is chosen, and a symmetric matrix is gotten. Then the relationship between the eigenvalues of the symmetric matrix and the parameter is exploited for the synchronization of multiple systems. Finally, numerical experiments of hyper-chaotic Chen system show that the proposed method is more efficient and practical.

KEYWORDS

Symmetric matrix; Eigenvalues; Synchronization of multiple systems; Hyper-chaotic Chen system.

INTRODUCTION

The study of chaos synchronization is of great practical significance and has received some results[1-10] in the past few years. But most chaos synchronization is realized between two systems in the above literatures. In this paper, the synchronization problem for multi-chaotic systems[11-13] will be presented by analyzing the relationship between the eigenvalues of a class of symmetric matrix and the parameter. The synchronized simulation results of hyper-chaotic Chen system are given to illustrate the proposed approach.

Let X be a Banach space endowed with the l^2 -norm $\| \ \|$,i.e. $\|x\| = \sqrt{x^T x}$. We consider the following system:

$$\dot{x}(t) = f(x(t)), \qquad (1)$$

where $x(t) \in R^n$, $f(0) = 0$.

Definition 1[11] System (1) is called to be exponentially stable on a neighborhood Ω of the equilibrium point, if there exist constants $\mu > 0, \alpha > 0$, such that

$$\|x(t)\| \le \alpha \exp(-\mu t)\|x_0\|, \quad (t \ge 0),$$

where $x(t)$ is any solution of (1) initiated from $x(t_0) = x_0$.

Definition 2[14] The vector function $f(x,t) \in R^n$ is called to be $f(x,t) \in K\Gamma$, if there exist a constant matrix K and a inner-connected matrix Γ of the complex dynamic network model, such that

$$(x-y)^T (f(x,t) - f(y,t) \le (x-y)^T K\Gamma(x-y),$$

where $x, y \in R^n$.

Lemma [15;16] Cosider the tridiagonal matrix of the form

$$\Lambda = \begin{pmatrix} \sigma & \sigma_1 & 0 & \cdots & 0 \\ \sigma_2 & \sigma & \sigma_1 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \sigma_2 & \sigma & \sigma_1 \\ 0 & \cdots & 0 & \sigma_2 & \sigma \end{pmatrix}_{m\times m}.$$

The eigenvalues $\lambda_i(\Lambda)$ of Λ are given by

$$\lambda_i(\Lambda) = \sigma + 2\sigma_1 \sqrt{\frac{\sigma_2}{\sigma_1}} \cos(\frac{i\pi}{m+1}), \quad i = 1, 2, \cdots, m.$$

THEORY OF MULTI-SYSTEMS SYNCHRONIZATION

Considering the complex dynamic network model

$$\dot{x}_i = F(x_i) + \gamma \sum_{j=1, j\ne i}^{N} a_{ij}\Gamma(x_j - x_i), \quad i = 1, 2, \cdots, N, \qquad (2)$$

where $x_i = (x_{i1}, x_{i2}, \cdots, x_{in}) \in R^n$ is the state vector of the i - th node, $\dot{x} = F(x)$, $x \in R^n$,is the dynamic behavior of each node, $\gamma > 0$ is a coupling factor, $A = (a_{ij})_{N\times N}$ is a weight matrix representing the coupling strength and topological structure , Γ is a inner-connected matrix, *N* is the node number of the complex dynamic network model.

The matrix A satisfies the constraint

$$a_{ii} = -\sum_{j=1, j\ne i}^{N} a_{ij}, i = 1, 2, \cdots, N,$$

$$a_{ij} \ge 0, i \ne j.$$

The model (2) is rewritten as

$$\dot{x}_i = F(x_i) + \eta \sum_{j=1}^{N} a_{ij} \Gamma x_j . \quad (3)$$

and the controlled model of (2) is described as

$$\dot{x}_i = F(x_i) + \eta \sum_{j=1}^{N} a_{ij} \Gamma x_j + u_i , \quad (4)$$

where $u_i = -k(x_i - x_{i-1}),\ i = 1,2,3,\cdots,N, k>0, x_0 = s = s(t),$ which satisfies $\dot{s}(t) = f(s(t)),\ s(t) \in R^n$.

Let $e_i = x_i - x_{i-1}$, then the error system is

$$\begin{cases} \dot{e}_1 = F(x_1) - F(s) + \eta(\sum_{j=1}^{N} a_{2j} \Gamma(x_j - s) - ke_1, \\ \dot{e}_m = F(x_m) - F(x_{m-1}) + \eta(\sum_{j=1}^{N} a_{m,j} \Gamma(x_j - s) \\ \qquad - \sum_{j=1}^{N} a_{m-1,j} \Gamma(x_j - s)) - ke_m + ke_{m-1}, \\ \qquad\qquad m = 2,3,\cdots,N. \end{cases} \quad (5)$$

There exist $b_{ij} = \sum_{p=j}^{n} a_{ip}$ such that

$$\sum_{j=1}^{N} a_{ij} \Gamma(x_j - s) = \sum_{j=1}^{N} b_{ij} \Gamma e_j ,\ \ i = 1,2,\cdots,N ,$$

Taking $c_{1j} = b_{1j}, c_{ij} = b_{ij} - b_{i-1,j}, i = 2,3,\cdots,N,$ we obtain

$$\sum_{j=1}^{N} a_{ij} \Gamma(x_j - s) - \sum_{j=1}^{N} a_{i-1,j} \Gamma(x_j - s) = \sum_{j=1}^{N} c_{ij} \Gamma e_j .$$

So the error system (5) is rewritten as

$$\begin{cases} \dot{e}_1 = F(x_1) - F(s) + \eta \sum_{j=1}^{N} c_{1j} \Gamma e_j - ke_1, \\ \dot{e}_m = F(x_m) - F(x_{m-1}) + \eta \sum_{j=1}^{N} c_{m,j} \Gamma e_j - ke_m + ke_{m-1}, \\ \quad m = 2,3,\cdots,N. \end{cases} \quad (6)$$

Theorem Suppose $k > 0, F(x) \in K\Gamma$, $C = (c_{ij})_{N\times N}$, $D = diag(K\Gamma, K\Gamma, \cdots, K\Gamma)_{N\times N}$, $\lambda_1 = \lambda_{\max}(((\eta C\Gamma + D) + (\eta C\Gamma + D)^T)/2)$, $\lambda_2 = \lambda_{\max}(Q)$, where $\lambda_{\max}(Q)$ is the largest eigenvalue of matrix Q,

$$Q = \begin{pmatrix} -k & \frac{k}{2} & & & \\ \frac{k}{2} & -k & \frac{k}{2} & & \\ & \frac{k}{2} & -k & \ddots & \\ & & \ddots & \ddots & \frac{k}{2} \\ & & & \frac{k}{2} & -k \end{pmatrix},$$

then the synchronization of the controlled model (4) is reached if $\lambda_1+\lambda_2<0$.

Proof According to Lemma, we know that the eigenvalues $\lambda_i(Q)=-k(1-\cos\frac{i\pi}{N+1}),\ i=1,2,\cdots,N,$ and are strictly monotone decreasing function on k.

Choose Lyapunov function

$$V(t)=(1/2)e^Te,\ e=(e_1^T,e_2^T,\cdots,e_N^T)^T,$$

then the derivative along (6) on *t* is

$$\begin{aligned}\dot V(t)&=e^T\dot e=(e_1^T,e_2^T,\cdots,e_N^T)(\dot e_1^T,\dot e_2^T,\cdots,\dot e_N^T)^T\\&=e_1^T(F(x_1)-F(s))+e_2^T(F(x_2)-F(x_1))\\&\quad+\cdots+e_N^T(F(x_N)-F(x_{N-1}))\\&\quad+(e_1^T,e_2^T,\cdots,e_N^T)\eta C\Gamma(e_1^T,e_2^T,\cdots,e_N^T)^T\\&\quad+(e_1^T,e_2^T,\cdots,e_N^T)Q(e_1^T,e_2^T,\cdots,e_N^T)^T\\&\le(e_1^T,e_2^T,\cdots,e_N^T)(D+\eta C\Gamma)(e_1^T,e_2^T,\cdots,e_N^T)^T\\&\quad+(e_1^T,e_2^T,\cdots,e_N^T)Q(e_1^T,e_2^T,\cdots,e_N^T)^T\\&\le(\lambda_1+\lambda_2)(e_1^T,e_2^T,\cdots,e_N^T)(e_1^T,e_2^T,\cdots,e_N^T)^T\\&=2(\lambda_1+\lambda_2)V.\end{aligned}$$

So we infer that $V(t)=V(t_0)\exp\{2(\lambda_1+\lambda_2)(t-t_0)\}$.

Consider the condition $\lambda_1+\lambda_2<0$ and Lyapunov stability theory, and we know that the synchronization of the controlled model (4) is reached.

Corollary 1 When the coupling matrix $A=(a_{ij})_{N\times N}=0$, suppose $\lambda_1=\lambda_{\max}((D+D^T)/2)$, $\lambda_2=\lambda_{\max}(Q)$, then the synchronization of the controlled model (4) is reached if $\lambda_1+\lambda_2<0$.

Corollary 2 When the model (4) is described as the following pinning controlled model

$$\begin{cases}\dot x_i=F(x_i)+\eta\sum_{j=1}^{N}a_{ij}\Gamma x_j+u_i,\\ \qquad i=1,2,\cdots,l,\\ \dot x_m=F(x_m)+\eta\sum_{j=1}^{N}a_{mj}\Gamma x_j,\\ \qquad m=l+1,l+2,\cdots,N.\end{cases}\qquad(7)$$

Let $\eta C\Lambda=\begin{pmatrix}C_1 & C_2\\ C_3 & C_4\end{pmatrix},\ C_1\in R^{l\times l},\ C_4\in R^{(N-l)\times(N-l)},\ D=diag(K\Gamma,K\Gamma,\cdots,K\Gamma)_{N\times N}$

$$=\begin{pmatrix}D_1 & 0\\ 0 & D_2\end{pmatrix},\ D_1\in R^{l\times l},\ D_2\in R^{(N-l)\times(N-l)},$$

$$Q=\begin{pmatrix}-k & \frac{k}{2} & & & & & & \\ \frac{k}{2} & -k & \frac{k}{2} & & & & & \\ & \frac{k}{2} & -k & \ddots & & & & \\ & & \ddots & \ddots & \frac{k}{2} & & & \\ & & & \frac{k}{2} & -k & \frac{k}{2} & & \\ & & & & \frac{k}{2} & 0 & \ddots & \\ & & & & & \ddots & \ddots & 0\\ & & & & & & 0 & 0\end{pmatrix}=\begin{pmatrix}Q_1 & Q_2\\ Q_2^T & Q_3\end{pmatrix},$$

$Q_1 \in R^{l\times l}$, $Q_3 \in R^{(N-l)\times(N-l)}$, $P_1 = (((C_1 + D_1) + (C_1 + D_1)^T) / 2) + Q_1$,

$P_2 = (C_2 + C_3^T + Q_2)/2$, $P_3 = (D_2 + D_2^T)/2 + (C_4 + C_4^T)/2$, On the basis of Schur complement theory[17;18], the synchronization of the controlled model (7) is reached if the conditions $P_1 < 0$,

$P_3 - P_2 P_1^{-1} P_2^T < 0$ are satisfied, which the notation $P_1 < 0$ means that the matrix P_1 is real symmetric and negative definite.

SYNCHRONIZATION OF MULTI-HYPER-CHAOTIC CHEN SYSTEMS

We consider hyper-chaotic Chen system[19-22]

$$\begin{cases} \dot{x} = 35(y - x) + w, \\ \dot{y} = 7x - xz + 12y, \\ \dot{z} = xy - 3z, \\ \dot{w} = yz + 0.5w, \end{cases}$$

as a example to verify that the conclusion of Theorem, Corollary 1 and Corollary 2 are efficient. In the simulations, we choose $k = 16, N = 5, l = 3$, the coupling matrix

$$A = \begin{pmatrix} -3 & 1 & 2 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ 1 & 14 & -15 & 0 & 0 \\ 1 & 14 & 3 & -18 & 0 \\ 1 & 10 & 3 & 2 & -16 \end{pmatrix},$$

$\Gamma = I, \eta = 1$, and the initial conditions $x_0 = (3, 4.3, 17, 13.8, 7, 2, 0.1, 8.2, 2.3, 0.4, 4.5, 16, 13.8, 9.4, 12.6, 0.4)^T$, $s_0 = (1, 15.5, 11, 20)^T$, respectively. See Fig.1, for the states x_1, x_2, x_3, x_4, x_5 of system (4) to asymptotically synchronize with the state $s(t)$, which satisfies the conditions of Theorem. It can be shown that the synchronization results to the states x_1, x_2, x_3, x_4, x_5 and $s(t)$, which satisfies the conditions of Corollary 1, Corollary 2, respectively, in Fig.2 and in Fig.3.

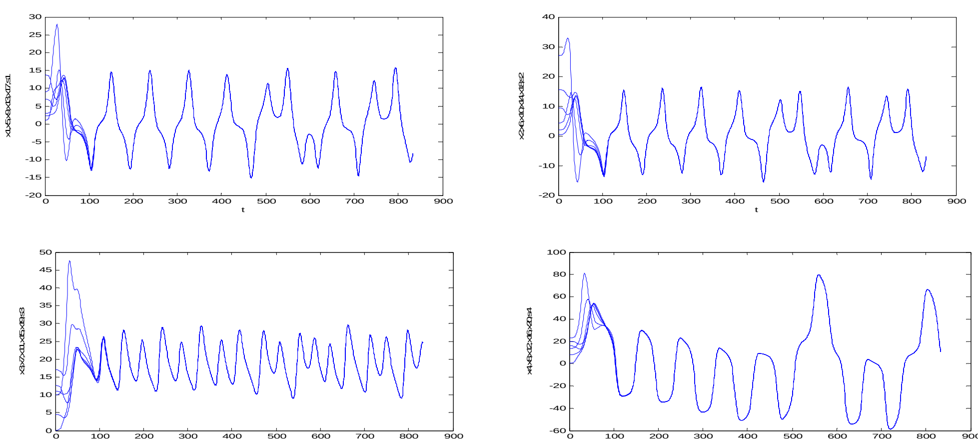

Fig.1. Synchronization of x_1, x_2, x_3, x_4, x_5 and $s(t)$, which satisfies the conditions of Theorem.

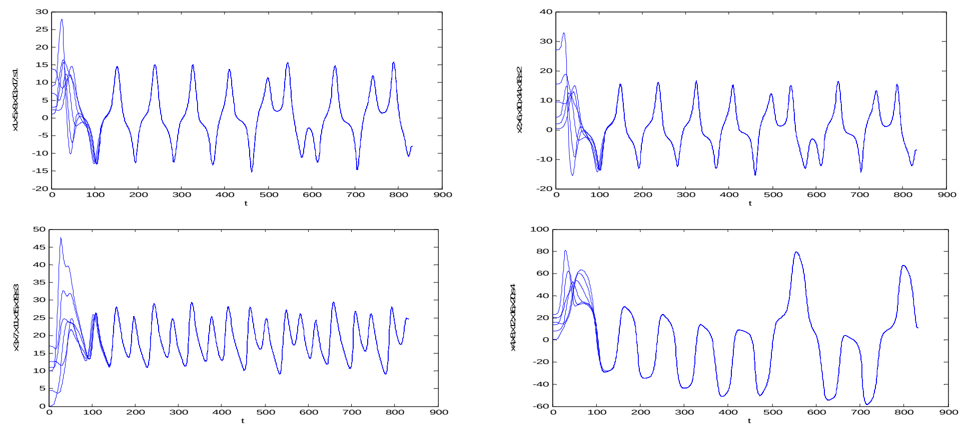

Fig.2. Synchronization of x_1, x_2, x_3, x_4, x_5 and $s(t)$, which satisfies the conditions of Corollary 1.

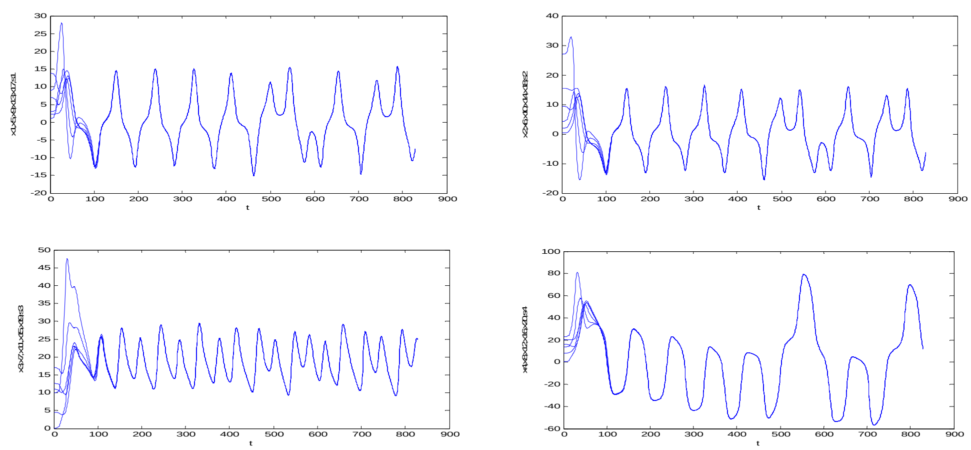

Fig.3. Synchronization of x_1, x_2, x_3, x_4, x_5 and $s(t)$, which satisfies the conditions of Corollary 2.

CONCLUSIONS

In this paper, the synchronization problem of multiple systems have been presented by having the aid of the relationship between the eigenvalues of a class of symmetric matrix and the parameter. Strong properties of global and asymptotic synchronization and numerical simulations have been achieved to hyper-chaotic Chen system. So it is verified that the method is effective.

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