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Study on Keller's mathematical model in the long-distance running and its application in the tactics and countermeasures

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ABSTRACT

Mid-long Run is Middle and Long Distance Running for short, which is referred to middle distance running and long distance running, is an item of developing physical endurance project. It is required that athletes maintain certain running speed in the entire journey and try to reduce their consumption of physical strength. In terms of technology, athletes are required to run easily, be coordinated, balance body gravity, and have good rhythm. Complete technology of Mid-long Run includes starting running and its speeding-up running, medium-distance running and finish running etc. In this paper, the Keller elementary model is established according to the player's body and physiological conditions. Based on the characteristics of Mid-long Run, full varied pace running tactics used by most excellent athletes can be obtained through the analysis of excellent athletes' competition tactics. However, the tactics is not the optimal strategy. Therefore, in this paper, the improved Mid-long Run tactical mathematic model is established by analyzing the advantages and disadvantages.

KEYWORDS

Track and field;
Mathematic model;
Physiology;
Mid-long run.

INTRODUCTION

Middle and long distance running events can date back to the fourteenth session of the ancient Olympic Games Held in 724 BC. Modern Mid-long Run first started in England. At the beginning of eighteenth Century, professional long-distance runners in the UK had participated long-distance race competition, which is very popular with people. After that, it has been holding widespread. The first modern Olympics In 1896 had a 10000m record. In 1912, International Association of Athletics Federations (IAAF)was established, mid-long distance race individual world records including 9 items accessible to both man and woman were had been admitted, then approved.

At present, the official event of the Olympic Games is the 800m, 1500m, 5000m, 10000m. The IAAF also acknowledged that the official world records of 3000m for man and woman and of 1 mile event. In addition, it is also popular with the events such as 880 yards, 2 miles, 3 miles, 6 miles and other inch distance in some countries. At the beginning of 20th Century, events for man were just held in china. In 1924, there were men's 800m, 1500m, 3000m the Third National Games in china, but the performance was very poor. Woman mid-Long distance running event was not carried out. After

the founding of New China, because the foundation is weak, there existed a big gap between mid-long run level in china and advanced level in the world. After the 1980s, China women running started to achieve world advanced level: 5000m event was ranked eighth in the world in 1987, 10000m ranked sixth. Since the 1990s, Mid-long Run in China has stepped a completely new stage. Especially, women's Mid-long Run level improved rapidly with Liaoning women's team as the representative, a large number of outstanding players were emerged and frequently won in the world events, which re-created the world records of the women's 1500m, 3000m, 5000m and 10000m and set a milestone in the Chinese mid-long run history.

Now, many scholars do research on middle and long distance running. Analysis on Application of Mid-long Run tactics written by niujing, who proposed that players must make many tactics according to its own characteristics in order to achieve good results in the course of the game, and do attempted training and summary aimed at designed tactics in their regular training. In the course of competing, pay close attention to change of the opponents with the combination of dynamic attack against dynamic and acting ability according to circumstances. Huang Tao in the book Study on characteristics of competitive ability of middle and long distance running items did research on skill characteristics of Mid-long Run in terms of the impact to Mid-long Run sports performance factor of physical characteristics of competitive ability and skill characteristics of competitive ability running in the competitive sport project. Zhang Jie, who wrote the article Analysis on Athletes' Speed Training Methods and Means, proposed the main decisive factors for Mid-long run special sports lies in speed, speed endurance and strength, and speed endurance is on the basis of the speed quality, also, velocity reserve directly determines the speed endurance. Only with very high velocity reserve, can athletic performance be made better. It is a trend to establish speed - center training thought in the development of mid-long run training in the future. On the basis of which, further study is done and a mathematical model is established in this paper.

KELLER MODEL

In the course of themed- long-distance run, athletes, for speed distribution, need to consider their physiological conditions. In this paper, according to the Keller model, the speed function of the race is determined according to the 4 physiological parameters in terms of the optimal control.

The athletes overcome the resistance in the course of the game to achieve or maintain a certain speed; need to develop forward momentum to provide energy itself. There are two sources, which are equivalent energy with inspired oxygen produced by the respiratory and circulatory system by way of metabolism. The other is the energy provided for running, which is stored in the athlete's bodies before the race. The former can be reasonably assumed to keep constant during the running, while the latter has a problem on how to distribute the stored energy into each schedule stage, and run out of them exactly before their arrival at the finishing point.

For the model, three relationships need to be determined as follows: One is the relationship between force and velocity, the second is the relationship between the impact work and the two energy sources mentioned above, the third is the relationship between speed and race performance. Although performance is for the certain schedule, it can be equivalent to the running distance in the limited time in mathematical tools processing. So the best result can come down to extremism problems about function such as function targeting distance, the speed, the momentum, stored energy and so on.

From the analysis above, it can be seen that the model requires 4 physiological parameters, namely, the maximum momentum that players can play, the resistance coefficient in vitro and in vivo, energy produced during oxygen metabolism in unit time, and initial value of stored energy in body. In the model the parameters can be fitted out based on the athlete's average statistics.

Suppose that a player needs the time T, which is to finish schedule D at the speed of function v(t) the race time, then:

$$\mathbf{D(v(t)) = \int_0^t v(t)dt} \quad \mathbf{(1)}$$

When D is known, ask $v(t)$ to make T minimum, which is be equivalent to T, SO, ask $v(t)$ to make D maximum. The latter problem is more convenient to study.

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The athlete's momentum is $f(t)$, from supposed 1, according to Newton's second law, the result can be obtained as follows:

$$\dot{v}(t)+\frac{v}{\tau}=f(t) \quad (2)$$

$$v(0)=0 \quad (3)$$

$$0\le f(t)\le F \quad (4)$$

Stored energy in the body marks as $E(t)$, whose variation rate is the difference value between the energy σ in time unit and consumed energy fv, namely,

$$\dot{E}(t)=\sigma-fv \quad (5)$$

$$E(0)=E_0 \quad (6)$$

$$E(t)\ge 0 \quad (7)$$

Thus, it constructs functional extremism problem targeting formula $D(v(t))$ as object function on the condition of formulas from(2) to (7) above, in which F , τ , σ , E_0 is considered as the known parameters.

The solution to the ordinary model is difficult, because the variation method will be not applicable once the optimal condition (4) and (7) appears on the edge, which can be known from the following analysis as it is. besides, combine formula 1, 2 and 5, and the function expression will be as shown in the following;

$$D(E(t))=\sqrt{2}\int_0^T e^{-\frac{t}{\tau}}\left[\int_0^t(\sigma-\dot{E}(t))e^{\frac{2s}{\tau}}ds\right]^{\frac{1}{2}}dt \quad (8)$$

Formula(8) is beyond complexity, so the solution will be not convenient to obtain even if other methods are used.

THE OPTIMAL MODEL OF MID- LONG RUN TACTICS

Mid- long run tactics

The development process of mid- long run can be roughly divided into four stages: the first one is natural technology stage 30 years ago; whose characteristic is the great stride, slow stride frequency, with the whole foot. The second is improved techno logy stage in the 1930s and 1940s, whose characteristic is "With the front feet outside on the ground first and then to the feet, and then Kick forward". The third one is the pendulum supporting technology in 1950s and 1960s, whose Characteristics is "swing leg, even swinging forward actively and "lying one the stomach", and fully kicking forward. The fourth one is the stage of the whole frequency running since 70's with the Characteristics of actively swing forward in leg and lying on the ground, noticing the cooperation between kicking and swing, and appropriate step size, quick stride frequency, labor-saving.

There are six kinds of tactics. The first one is tactics for creating record which is a tactic without considering the ranking or gain and loss to create tactics on clutter. The second one is championship tactics, which is to seize a place without regard to create excellent results. The third is steady running tactic, which adopts higher-speeding running mostly on the way in addition to speeding running after running start and the final sprint running. The fourth one is the varied pace running tactic: in the whole run, speed changes a lot, which adopts the method of the sudden acceleration or deceleration of the run, so as to keep the opponent behind them or disrupt the adversary rhythm. The fifth one is the leading run tactics, namely, athletes occupy a leading position after the start to try to maintain a high speed of the leading run. The purpose is to use their own good speed endurance to wear down their opponents. The fifth is leading running tactic: the athletes occupy the leading position after the starting run, and manage to keep running at higher speed for the purpose of wearing down the opponents by way of their own stronger endurance. The sixth is following-behind running tactic, that is, keep following behind the leaders or a small group to strive to exceed opponent in the final sprint stage, so as to arrive at the finishing line first.

Through the analysis for relevant data about the world champion, the optimal tactics were found and the new tactics were put forward, by comparing with the original tactical analysis, the speed change time point if new tactics were calculated, here, take 800metre, 5000metre run for an example.

Analysis on mid- long run tactics

This paper selects cases of elite athletes participating in major mid- long run competitions in recent years, the results are shown in TABLE 1, TABLE 2

TABLE 1 : Mid-long run performance of some excellent man athletes

period of time	Conche Lagher		Aing		Raggatt		Tanui		Routot	
	split time	Time difference	Split time	Time difference	Split time	Time difference	Split time	Time difference	Split time	Time difference
200m	24"32	24"32	24"1	24"1	25"2	25"2	24"	24"	24'7	24"7
400m	50"59	24"27	51"1	27"	51"5	26"3	49"99	25"99	51"2	26"5
600m	1'16"85	26"21	1'18"	26"9	1'18"3	26"8	1'17"09	27"1	1'17"9	26"7
800m	1'43"06	24"26	1'43"45	25"5	1'43"99	25"7	1'43"30	26"21	1'44"71	26"8

TABLE 2 : Mid-long run performance of some excellent woman athletes

Period of time	Wudasi		DEFAR		Nur Rudy Nova		Rengan		Maria de Lurdes MUTOLA	
	split time	time difference	split time	time difference	split time	time difference	split time	time difference	split time	Time difference
200m	26"9	26"9	27"1	27"1	26"8	26"8		26"1	27"3	27"3
400m	56"3	29"4	56"4	29"3	56"4	29"6	55"73	29"63	56"3	29"0
600m	1'29"	32"7	1'27"7	31"3	1'28"7	32"3	1'26"74	31"01	1'26"0	29"6
800m	1'55"26	26"3	1'56"10	28"4	1'57"50	28"8	1'55"54	28"8	1'55"43	29"4

Through the analysis of the current mid-long running tactics and of Olympic athletes, it turned out that the leading run and following run were not chosen by most athletes, for the reason that they are in relation with the opponents' running speed and does not belong to the athletes' own rhythm.

From the dates in TABLE 1 and TABLE 2, it can be inferred that excellent athletes' run tactics are basically the same. The first phase, in which basic tactical running is applied and the fastest speed is kept, is the initial 200m. In order to occupy a good position, athletes dash in the first period; in the second, the third, athletes run at low speed in order to maintain physical strength; in the fourth stage, athletes are extremely exhausted, but they begin to sprint for the championship or good ranking. Based on the analysis above, the velocity time diagram is drawn as Figure 1.

Analysis on the tactics is as shown: during stage 1 and stage2, athletes don't do full sprint so that they keep certain strength. In stage4, they can do full sprint. For 800m, the battle is very fierce, sometimes the time difference is less than a second, and athletes maintain physical strength. Thus, athletes do a full sprint in this stage, which is in accordance with the psychological characteristics. However, the best running strategy should be to run out of the energy stored in the body for a race, because, it is oblivious that there is much energy in their bodied now that they can do full sprint in stage4. if using this tactic to rush to the finishing line, athletes didn't use up their energy, that is, there is still available energy in the bodies for transforming completely into speed. Some energy is wasted, thus, athletes can not show their strength fully. Therefore, this model is not optimal.

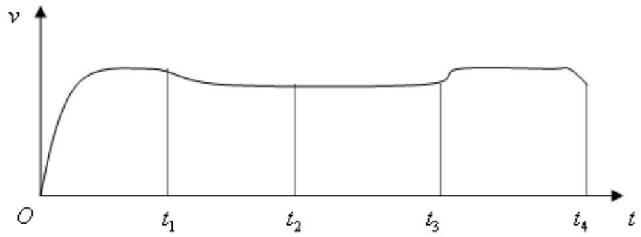

Figure 1 : The velocity time diagram in four periods of time

The optimal model

The model is divided into three stages, the first stage is to sprint, so as to achieve maximum speed in a short time; the second stage is to maintain uniform speed, which is to run out of all the energy in the body; the third stage depends on inertial sprint, velocity and time image as follows in Figure2.

stage1: $0 \le t \le t_1$, $v_1(t)$ can be expressed by formula(9).

Stage2: $t_2 \le t \le T$, put $E(t) = 0$ into formula(5),

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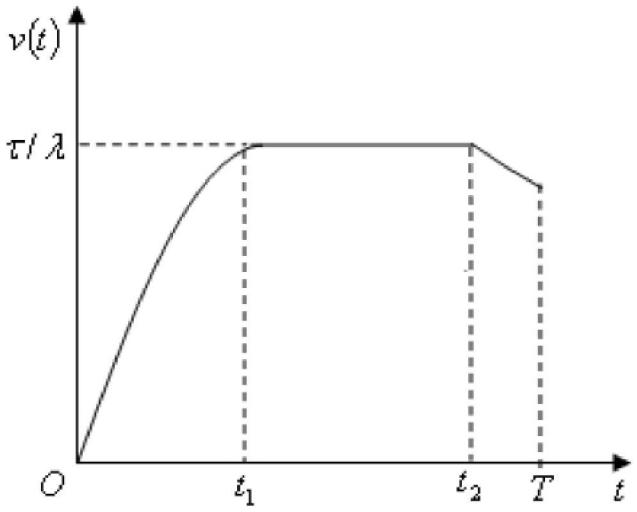

Figure 2 : The velocity time diagram in three periods

(2), obtain

$$:\frac{1}{2}\frac{d}{dt}v_3^2+\frac{v_3^2}{\tau}=\sigma \tag{9}$$

On the condition of $v_3(t_2)=v_2$, the solution above is

$$v_3(t)=[(v_2^2-\sigma t)e^{-\frac{2(t-t_2)}{\tau}}+\sigma\tau]^{\frac{1}{2}} \tag{10}$$

Stage3: $t_1 \le t \le t_2$, v_2 is determined when the value of $D(v(t))$ is maximum in object function namely:

$$D(v(t))=\int_0^{t_1} v_1(t)dt+v_2(t_2-t_1)+\int_{t_2}^{T} v_3(t)dt \tag{11}$$

To get the condition of $E(t_2)=0$, from equation (2), (5) and initial condition (6), solve $E(t)$, graphs:

$$E(t)=E_0+\sigma t_2-\frac{v^2(t)}{2}-\frac{1}{\tau}\int_0^t v^2(t)dt \tag{12}$$

Let $t=t_2$ and Substitute formula (9) into it, obtain:

$$E(t_2)=E_0+\sigma t_2-\frac{v_2^2}{2}-\int_0^{t_1}F^2\tau(1-e^{-\frac{t}{\tau}})dt-\frac{v_2^2(t_2-t_1)}{\tau} \tag{13}$$

On the condition of $E(t_2)=0$, ask v_2, v_2, make $D(v(t))$ in formula(11) maximum use the undetermined constants λ to construct function:

$$I(v(t),t_2)=D(v(t),t_2)=D(v(t))+\frac{\lambda}{2}E(t_2) \tag{14}$$

Suggest leaving out the items unrelated to v_2 and t_2 on the right end, rewrite like this:

$$I(v_2,t_2)=\int_{t_2}^{T} v_3(t)dt+\frac{\lambda\sigma}{2}t_2-\frac{\lambda}{4}v_2^2+(v_2-\frac{\lambda v_2^2}{2\tau})(t_2-t_1) \tag{15}$$

For optimal v_2, t_2, the necessary condition is:

$$v_2=\frac{\tau}{\lambda} \tag{16}$$

$$2\int_{t_2}^{T}[(V_2^2-\sigma\tau)e^{-\frac{2(t-t_2)}{\tau}}+\sigma\tau]^{-\frac{1}{2}}e^{-\frac{2(t-t_2)}{\tau}}dt=\lambda \tag{17}$$

Now, $v(t)$ in the three stages is given respectively by formula(9), (10)and (16), the problem left is how to determine t_1, t_2 and λ.

For Determining t_1, t_2, λ :employ the continuity of $v(t)$ in $t=t_1$, from formula (9) and (16), it can be obtained

$$\lambda F(1-e^{-\frac{t_1}{\tau}})=1 \tag{18}$$

Substitute formula (16) into formula (13), then, let it be zero, so:

$$E_0+\sigma t_2-\frac{\tau^2}{2\lambda^2}-\int_0^{t_1}F^2\tau(1-e^{-t/\tau})^2dt-\frac{\tau}{\lambda^2}(t_2-t_1)=0 \tag{19}$$

Substitute formula (16) into formula (17) and accumulate points, thus:

$$2[(\tau^2-\lambda^2\sigma\tau)e^{-\frac{2(T-t_2)}{\tau}}+\lambda^2\sigma\tau]^{\frac{1}{2}}-2\tau=\lambda^2\sigma-\tau \tag{20}$$

Parameter F,τ in the modeling above has been estimated in model, use some data from mid-long run championship (in table 2, man) to get the estimation value of σ and E_0 : $\sigma=41.5$, $E_0=2403.5$.

From continuity of $t=t_1$, the formula

$$\lambda F(1-e^{-\frac{t_1}{\tau}})=1 \tag{21}$$

can be obtained, first Substitute formula (21) into formula (23)

$$\int_0^{t_1}F^2\tau(1-e^{-t/\tau})^2dt,\ \int_0^{t_1}\frac{\tau}{\lambda^2}dt=\frac{\tau}{\lambda^2}t_1 \tag{22}$$

can be obtained, then Substitute formula (26) into for-

mula (19), thus:

$$\lambda^2 = \frac{\tau\ (\tau + t_2)}{E_0 + 6t_2} \quad (23)$$

Simplify formula (20), so: $2[(\tau^2 - \lambda^2\sigma\tau)e^{-\frac{2(T-t_2)}{\tau}} + \lambda^2\sigma\tau]^{\frac{1}{2}} = \lambda^2\sigma + \tau$, continue Simplifying, the result can be like this:

$$t_2 = T + \frac{\tau}{2}\ln\frac{(\lambda^2\sigma - \tau)^2}{4(\tau^2 - \lambda^2\sigma\tau)} \quad (24)$$

Substitute formula (23) into formula (24), the value of t_2 can be obtained, Substitute into formula (23) again, and λ can be out, Substitute solved t_2 and λ into formula (19), graphs t_1 can be obtained.

CONCLUSION

In this paper, the optimal tactics model of mid-long run is constructed. Athletes found the optimal tactics which is suitable to them using this model with their own situation with the combination of the athletes' own body condition. This model can guide the athletes to carry on game plan so as to obtain good results. In the special physical training, they should check, control and feedback the training course comprehensively and completely based on the system training so as to make it scientific, accurate, timely and to truly improve the quality of training. Special physical training plays a very important role in the track and field sports, which is asked to practice frequently, as the saying goes, 'economize and avoid running short should often practice', thus, only by this way can the special performance of athletes be improved. Mid-long running training task is about how to run as fast as possible in a specific distance, that is to say, about how to achieve absolute maximum speed in some limited amount. Therefore, the methods and means of all training should focus on the purpose for designing. Without exception, the excellent athletes in the world are to put "improving the athletes' speed" in the first place. The rational Speed training method can improve the performance of mid-long run athletes, which especially is fit for those who usually ignore the speed training of athletes, whose performance will be improved greatly.

REFERENCE

[1] Wu Ye-hai, Yu Bao-Ling, Lou Lan-Ping; Analysis on Competition Result and Ranking Prediction of University Athletics Competition in China[J]. China Sport Science and Technology, **41(2)**, 21-24 **(2005)**.

[2] Zhang Yun-Long; Analysis of the Problems Among Athletic Teams of College Students of China with the Statistics of Their Performance[J]. Shandong Sports Science & Technology, **25(1)**, 31-38 **(2003)**.

[3] Zhai Hua-Nan; Grey Correlation Analysis of Decathletes, Competitive Achievements of the 6th and the 7th National University Games[J]. Journal of Shenyang Sport University, **26(4)**, 100-102 **(2007)**.

[4] Wei Chun-Ling, Sun Jin-Hai; Research on Developing Trend and Grey Prediction of Athletic Performance in Olympic Games[J]. China Sport Science and Technology, **41(2)**, 18-20 **(2005)**.

[5] Luo Yi, Xu Ming, Fu Gui_Hong; Research on the Technological Characteristics of Men's Individual Events of Gymnastics Competition in the 14th Asian Games[J]. Journal of Guangzhou Physical Education Institute, **24(1)**, 79-81 **(2004)**.

[6] Chen Zhi-Qiang; Physical Education.Evaluation and Analysis on Kinematic Characteristics of 400 m Hurdlers[J]. China Sport Science and Technology, **40(2)**, 15-19 **(2004)**.

[7] Chen Zhi Qiang, Huang Ming Jiao, Yang De Fang; Extrapolation and Prediction of Trend of Dynamic Change in Decathlon Results[J]. Journal of Beijing Sport University, **24(2)**, 212-213, 242 **(2001)**.

[8] Chen Liang, Sai Qing-Bin; An Analysis on the Developing Trend of the Performance of Track and Field in National Sports Games and Its Study with Grey Prediction[J]. Journal of Capital College of Physical Education, **18(5)**, 85-88 **(2006)**.