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Study on investment mode of watershed downstream pollution abatement based on stochastic differential games

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ABSTRACT

Watershed downstream pollution abatement has become a severe real challenge for country to build up the resource saving and environment-friendly society. Propose a two-player,finite-horizon stochastic differential game and analyze the influence of environment projects under three cases: autarky,individual investment and cooperation. Solving it by Bellman dynamic programming and comparing welfares at different time, the results show that allowing for investment cooperation between lower reaches and upper reaches could conform to the actuality and be beneficial to realize the objective of emission reduction.

KEYWORDS

Watershed downstream pollution; Stochastic differential games; Bellman dynamic programming; Investment mode; Cooperation.

Decision of the CCCPC on Some Major Issues Concerning Comprehensively Deepening the Reform (hereinafter referred to as Decision) was approved at the Third Plenary Session of the 18th Central Committee of the CPC, which required us to deepen the reform of ecological civilization system by centering on building a wild China, accelerate the establishment of ecological civilization system, improve the system and mechanism for territory development, energy conservation and ecological environmental protection and foster a new pattern of modernization construction featuring harmonious development between man and nature. However, as for how to establish ecological civilization, it has been clearly specified in the Decision that it is necessary to establish a systematic and complete ecological civilization system, implement the strictest source protection system, compensation system and accountability system, improve the system of environmental management and ecological remediation in order to protect the ecological environment with such systems. Watershed ecological protection, as an important part of the establishment of ecological civilization, hinges on securing the sustainable utilization of watershed water resources. Nearly 30% of China's territories are distributed in ten watersheds, involving nearly one thousand rivers of various sizes. With the acceleration of urbanization process, urban sewage discharge is increasing day by day. According to statistics, the total discharge of the industrial and urban sewage increased to 105 billion tons in 2010 from 31.5 billion tons in 1980, and now more than 90% of the urban waters have been polluted to different degrees, some rivers have been polluted so seriously that they seriously do harm to the health of residents, result in great damages to economic and social life to the country, therefore, water pollution has become one of the most serious environmental problems China currently faces. Over the years, in order to maintain the ecological safety of such watersheds and ensure sustainable utilization of water resources in such watersheds, a lot of manpower, materials and financial resources have been invested in upstream of most rivers for ecological construction and environmental protection. However, the upstream areas of most rivers are often of relatively poor economy, relatively fragile ecology and it is difficult for them to bear the responsibility for construction and protection of watershed ecological environment alone[1-2]. In recent years, watershed areas, especially the benefited downstream areas, have begun to engage in pollution abatement from their own development and practical needs; however, the main problem is what is the most effective mode for pollution abatement and how to calculate the costs shared by such areas in pollution abatement cooperation.

Now there are mainly two research methods that have been accepted. One is Shapley Value, whose main thought is that the cost or the benefit shared by each participant shall be equal to the average value of marginal contribution of the alliance it participates in. Leon Petrosjan and Georges Zaccour (2003) specified the method about how to calculate the cost shared by each area in pollution abatement cooperation in continuous time[3]. However, the disadvantage of the method lies in the necessity to take all possible alliances in areas into account, while in practice, some alliances may be ineffective or impractical, therefore, the application of this method is limited. The other method is differential games. For any game, if one of its participation may act depending on former action, the game is dynamic; otherwise, it is static. For a dynamic game, if it has two or more stages, it is a discrete dynamic game; if time difference of each stage is narrowed to the lower limit, the game is a continuous dynamic game, also known as differential game. Steffen Jorgensen and Georges Zaccour (2001) made a study about two adjacent areas to calculate the benefits to be shared after pollution abatement with differential game model in order to abate pollution discharge and made a conclusion by comparison that the result of the abatement cooperation is obviously better than that of noncooperation[4]. Steffen Jorgensen (2009) studied three adjacent areas in the watershed, assuming that there are additional pollutions and existing pollution will not be naturally absorbed, the decrease of pollution of each area can only be achieved by discharging it to other areas. Under the condition of given sum of pollution stock, it can be concluded from the analysis of the pollution discharge under cooperation and noncooperation conditions from the differential game perspective that only the effective cooperation made by internal transfer of payment mechanism can solve the problem[5]. Though differential game is frequently used in watershed pollution abatement now, it ignores the influence of many uncertain factors in the abatement process. Therefore, in order to conform to the stochastic dynamic evolution characteristics of watershed downstream pollution abatement, stochastic differential games model is proposed in this paper to make a study from the perspective of the investment in watershed downstream pollution abatement, respectively establish three investment modes: autarky, individual investment and cooperation, solve the problem with Bellman dynamic programming method and finally compare the abatement effect of each mode with numerical example and find the most effective investment strategy.

BASIC ASSUMPTION AND VARIABLE DESIGN

Assumption 1: Pollutants in the rivers mainly include two kinds: organic pollutants and inorganic pollutions; because inorganic pollutants migrate only with water generally and may be subject to simple state transfer, therefore, the watershed pollutants are assumed to be mainly dominated by organic pollutants in this paper.

Assumption 2: Make $Q_i(t)$ stand for the industrial production of area i at time t, which may result in pollution discharge $e_i(t)$, assuming a positive relationship between industrial production and pollution discharge, it may be expressed as $Q_i = Q_i(e_i(t))$. Area i will generate profits $R_i(Q_i)$ through industrial production, therefore, profit function may be expressed by the discharge $e_i(t)$ and it is the second concave function of increasing discharge $e_i(t)$[6].

$$R_i(Q_i(e_i(t))) = e_i(t)(b_i - \frac{1}{2}e_i(t)), 0 \le e_i(t) \le b_i \tag{1}$$

Where: b is a given parameter, standing for the discharge value upon maximum profits.

Assumption 3: Industrial production results in pollution to the watershed environment, therefore, it is necessary to pay relevant costs. Make $D_i(s)$ stand for the damage costs resulting from industrial production, which depends on watershed pollution stock s, i.e.

$$D_i(s) = \pi_i s \qquad \pi > 0 \tag{2}$$

Where: π is the degree of damage to the area made by the each unit of pollution stock.

Assumption 4: Each area can control and abate pollution discharge by using environmentally friendly production technologies, establishing pollution abatement infrastructure and taking other environment project investments, and it can invest both in local environment projects and the environment projects in other areas. Investment costs of area i for local environment projects and environment projects in other areas can be respectively expressed as:

$$\begin{aligned} C_{ii}(I_{ii}) &= \frac{1}{2}a_i I_{ii}^2 \qquad a_i > 0 \\ C_{ij}(I_{ij}) &= \frac{1}{2}a_j\left[(I_{jj}+I_{ij})^2 - I_{jj}^{\ 2}\right] \\ &= \frac{1}{2}a_j I_{ij}(2I_{jj}+I_{ij}) \qquad a_j > 0, i \neq j \end{aligned} \tag{3}$$

Where: a stands for the efficiency parameter of investment cost.

Assumption 5: Each area will get Emission Reduction Units ($ERU_i(t)$) by investing in pollution abatement, assuming Emission Reduction Units and investment are directly proportional[7,8], then the Emission Reduction Units of area i in local area and other areas can be respectively expressed as:

$$\begin{aligned} ERU_i(t) &= \gamma_i I_{ii}(t) \qquad \gamma_i > 0 \\ ERU_i(t) &= \gamma_j I_{ij}(t) \qquad \gamma_j > 0 \end{aligned} \tag{4}$$

Where: γ stands for the investment scale parameter.

INVESTMENT MODE OF WATERSHED DOWNSTREAM POLLUTION ABATEMENT

Investments may be divided into three modes: autarky, individual investment and cooperation according to the investment mode of watershed downstream pollution abatement, here two adjacent areas of watershed are taken as study objects (Area 1 stands for upstream areas and area 2 stands for downstream areas), and a model and solution shall be made for each investment mode.

Autarky

In autarky mode, each area is only willing to invest in local environment projects to control the river pollution of the area. Express stochastic differential games between two areas with $\Gamma_1(T\text{-}t_0, s_0)$ and because of dependence of $s(t)$ on some uncertain factors, its development changes depend on the following stochastic differential equation:

$$\begin{aligned} ds(t) &= \left[e_1(t) + e_2(t) - \gamma_1 I_{11}(t) - \gamma_2 I_{22}(t) - \delta s(t)\right]dt \\ &\quad + \sigma s(t)dz(t) \\ s(0) &= s_0 \end{aligned} \tag{5}$$

Where: δ stands for natural absorption rate of pollution of each area, σ stands for noise parameters and $z(t)$ stands for Wiener process. At time t_0, present value of expected profit of area 1 and area 2 are respectively expressed as:

$$\begin{aligned} \max_{e_1, I_{11}} \ W_1 &= E\{\int_{t_0}^{T}\{e_1(b_1 - \frac{1}{2}e_1) - \pi_1 s - \frac{1}{2}a_1 I_{11}^2\}e^{-r(t-t_0)}dt \\ &\quad - g^1\left[s(T) - \bar{s}_1\right]e^{-r(T-t_0)}\} \\ g^1 &\geq 0, \bar{s}_1 \geq 0 \end{aligned} \tag{6}$$

$$\begin{aligned} \max_{e_2, I_{22}} \ W_2 &= E\{\int_{t_0}^{T}\{e_2(b_2 - \frac{1}{2}e_2) - \pi_2 s - \frac{1}{2}a_2 I_{22}^2\}e^{-r(t-t_0)}dt \\ &\quad - g^2\left[s(T) - \bar{s}_2\right]e^{-r(T-t_0)}\} \\ g^2 &\geq 0, \bar{s}_2 \geq 0 \end{aligned} \tag{7}$$

Where: $e_i(b_i-\frac{1}{2}e_i)-\pi_i s-\frac{1}{2}a_i I_{ii}^2$ stands for the profits gotten by area i at time t, under given discount rate $r(t)$ changing over time,the profits gotten by area i at time t shall be discounted according to discount factor $e^{-r(t-t_0)}$. $g^i(s(T)-\bar{s}_i)$ stands for the final profits to be gotten by area i at time T. If the final level of pollution stock is higher than the limit value $s(T)-\bar{s}_i>0$, area i shall pay penalty $g^i(s(T)-\bar{s}_i)>0$; if the final level of the pollution stock is lower than the limit value $s(T)-\bar{s}_i>0$, area i will obtain the reward $g^i(s(T)-\bar{s}_i)<0$. Solve games (5)~(7) with Bellman dynamic programming as follows[9,10]:

Set $\{[e_i^*(t),I_{ii}^*(t)]=[\phi_i^{e*}(t,s),\phi_{ii}^{I*}(t,s)]\}$ as a feedback strategy set of original game Nash equilibrium, when continuous differential expected profit function $V^{(t_0)i}=(t,s)\times R_m\to R$ exists, satisfy the equation:

$$\begin{aligned}
&-V_t^{(t_0)1}(t,s)-\frac{1}{2}\sigma^2 s^2 V_{ss}^{(t_0)1}(t,s)\\
&=\max_{e_1,I_{11}}\{[e_1(t)(b_1-\frac{1}{2}e_1(t))-\pi_1 s-\frac{1}{2}a_1 I_{11}(t)^2]e^{-r(t-t_0)}\\
&+V_s^{(t_0)1}(t,s)[e_1(t)+\phi_2^{e*}(t,s)-\gamma_1 I_{11}(t)-\gamma_2\phi_{22}^{I*}(t,s)-\delta s(t)]\}\\
&-V_t^{(t_0)2}(t,s)-\frac{1}{2}\sigma^2 s^2 V_{ss}^{(t_0)2}(t,s)\\
&=\max_{e_2,I_{22}}\{[e_2(t)(b_2-\frac{1}{2}e_2(t))-\pi_2 s-\frac{1}{2}a_1 I_{22}(t)^2]e^{-r(t-t_0)}\\
&+V_s^{(t_0)2}(t,s)[\phi_1^{e*}(t,s)+e_2(t)-\gamma_1\phi_{11}^{I*}(t,s)-\gamma_2 I_{22}(t)-\delta s(t)]\}\\
&V^{(t_0)1}(T,s)=-g^1\left[s(T)-\bar{s}_1\right]e^{-r(T-t_0)}\\
&V^{(t_0)2}(T,s)=-g^2\left[s(T)-\bar{s}_2\right]e^{-r(T-t_0)}
\end{aligned} \quad (8)$$

To maximize right side of the first and the second equation of partial differential equation set, the maximization requirements are concluded as:

$$\begin{aligned}
&\phi_1^{e*}(t,s)=b_1+V_s^{(t_0)1}(t,s)e^{r(t-t_0)}\\
&\phi_{11}^{I*}(t,s)=-\frac{\gamma_1}{a_1}V_s^{(t_0)1}(t,s)e^{r(t-t_0)}\\
&\phi_2^{e*}(t,s)=b_2+V_s^{(t_0)2}(t,s)e^{r(t-t_0)}\\
&\phi_{22}^{I*}(t,s)=-\frac{\gamma_2}{a_2}V_s^{(t_0)2}(t,s)e^{r(t-t_0)}
\end{aligned} \quad (9)$$

By substituting formula (9) into formula (8) and solve it, it may be drawn that the current value of profit function of two areas is:

$$\begin{aligned}
&V^{(t_0)1}(t,s)=e^{-r(t-t_0)}\left[A_1(t)s+B_1(t)\right]\\
&V^{(t_0)2}(t,s)=e^{-r(t-t_0)}\left[A_2(t)s+B_2(t)\right]
\end{aligned} \quad (10)$$

By substituting formula (10) into formula (9), it may be drawn:

$$\begin{aligned}
&\phi_1^{e*}(t,s)=b_1+A_1(t)\\
&\phi_{11}^{I*}(t,s)=-\frac{\gamma_1}{a_1}A_1(t)\\
&\phi_2^{e*}(t,s)=b_2+A_2(t)\\
&\phi_{22}^{I*}(t,s)=-\frac{\gamma_2}{a_2}A_2(t)
\end{aligned} \quad (11)$$

$A_1(t)$, $B_1(t)$, $A_2(t)$ and $B_2(t)$ in the formula must meet the following dynamic system and margin conditions:

$$\dot{A}_1(t) = \pi_1 + (r+\delta)A_1(t) \quad A_1(T) = -g^1$$
$$\dot{A}_2(t) = \pi_2 + (r+\delta)A_2(t) \quad A_2(T) = -g^2$$
$$\dot{B}_1(t) = rB_1(t) - \frac{1}{2}b_1^2 - (\frac{1}{2} + \frac{\gamma_1^2}{2a_1})A_1(t)^2 - A_1(t)(b_1+b_2) - A_1(t)A_2(t)\left(1+\frac{\gamma_2^2}{a_2}\right)$$
$$\dot{B}_2(t) = rB_2(t) - \frac{1}{2}b_2^2 - (\frac{1}{2} + \frac{\gamma_2^2}{2a_2})A_2(t)^2 - A_2(t)(b_1+b_2) - A_1(t)A_2(t)\left(1+\frac{\gamma_1^2}{a_1}\right)$$
$$B_1(T) = g^1\bar{s}_1 \quad B_2(T) = g^2\bar{s}_2 \tag{12}$$

Set $\varepsilon^{(\tau)i}(\tau, s_\tau)$ as the profit function of each area at time τ, $P_i(\tau, s_\tau)$ as the instant profit of each area at time τ in stochastic differential games, it may be calculated as below:

$$\begin{aligned} P_1(\tau, s_\tau) &= -\varepsilon_t^{(\tau)1}(\tau, s_\tau) - \frac{\sigma^2 s_\tau^2}{2}\varepsilon_{s_t s_t}^{(\tau)1}(\tau, s_\tau) \\ &\quad -\varepsilon_{s_t}^{(\tau)1}(\tau, s_\tau)\left[e_1(\tau)+e_2(\tau)-\gamma_1 I_{11}(\tau)-\gamma_2 I_{22}(\tau)-\delta s_\tau\right] \\ &= -(\pi_1 + rA_1(\tau))s_\tau - rB_1(\tau) + \frac{1}{2}b_1^2 - (\frac{1}{2}+\frac{\gamma_1^2}{2a_1})A_1(\tau)^2 \\ P_2(\tau, s_\tau) &= -\varepsilon_t^{(\tau)2}(\tau, s_\tau) - \frac{\sigma^2 s_\tau^2}{2}\varepsilon_{s_t s_t}^{(\tau)2}(\tau, s_\tau) \\ &\quad -\varepsilon_{s_t}^{(\tau)2}(\tau, s_\tau)\left[e_1(\tau)+e_2(\tau)-\gamma_1 I_{11}(\tau)-\gamma_2 I_{22}(\tau)-\delta s_\tau\right] \\ &= -(\pi_2 + rA_2(\tau))s_\tau - rB_2(\tau) + \frac{1}{2}b_2^2 - (\frac{1}{2}+\frac{\gamma_2^2}{2a_2})A_2(\tau)^2 \end{aligned} \tag{13}$$

At the end of time *T*, each area may get its own final profit $-g^i\left[s(T)-\bar{s}_i\right]$.

Individual investment

In individual investment, lower reaches can construct environment projects both in local area and upper reaches. By expressing the stochastic differential games between these two areas with $\Gamma_2(T\text{-}t_0, s_0)$, area 1 only makes investment in local area while area 2 makes environment project investment in area 1, in addition to local investment, then the development changes of $s(t)$ may depend on the following stochastic differential equation:

$$\begin{aligned} ds(t) &= \left[e_1(t)+e_2(t)-\gamma_1 I_{11}(t)-\gamma_2 I_{22}(t)-\gamma_1 I_{21}(t)-\delta s(t)\right]dt \\ &\quad +\sigma s(t)dz(t) \\ s(0) &= s_0 \end{aligned} \tag{14}$$

At time t_0, the present values of expected profits of both area 1 and area 2 can be respectively expressed as:

$$\begin{aligned} \max_{e_1, I_{11}} \ W_1 &= E\{\int_{t_0}^{T}\{e_1(b_1-\frac{1}{2}e_1)-\pi_1 s-\frac{1}{2}a_1 I_{11}^2\}e^{-r(t-t_0)}dt \\ &\quad -g^1\left[s(T)-\bar{s}_1\right]e^{-r(T-t_0)}\} \\ g^1 &\geq 0, \bar{s}_1 \geq 0 \end{aligned} \tag{15}$$

$$\begin{aligned} \max_{e_2, I_{21}, I_{22}} \ W_2 &= E\{\int_{t_0}^{T}\{e_2(b_2-\frac{1}{2}e_2)-\pi_2 s-\frac{1}{2}a_2 I_{22}^2-\frac{1}{2}a_1 I_{21}(2I_{11}+I_{21})\}e^{-r(t-t_0)}dt \\ &\quad -g^2\left[s(T)-\bar{s}_2\right]e^{-r(T-t_0)}\} \\ g^2 &\geq 0, \bar{s}_2 \geq 0 \end{aligned} \tag{16}$$

Set $\{[e_1^*(t), e_2^*(t), I_{11}^*(t), I_{21}^*(t), I_{22}^*(t)] = [\phi_1^{e*}(t,s), \phi_2^{e*}(t,s), \phi_{11}^{I*}(t,s), \phi_{21}^{I*}(t,s), \phi_{22}^{I*}(t,s)]\}$ as a feedback strategy set of original game Nash equilibrium, which shall satisfy the following equation:

$$\begin{aligned}
&-V_t^{(t_0)1}(t,s)-\frac{1}{2}\sigma^2 s^2 V_{ss}^{(t_0)1}(t,s)\\
&=\max_{e_1,I_{11}}\{[e_1(t)(b_1-\frac{1}{2}e_1(t))-\pi_1 s-\frac{1}{2}a_1 I_{11}(t)^2]e^{-r(t-t_0)}\\
&\quad+V_s^{(t_0)1}(t,s)\times[e_1(t)+\phi_2^{e*}(t,s)-\gamma_1 I_{11}(t)-\gamma_2\phi_{22}^{I*}(t,s)-\gamma_1\phi_{21}^{I*}(t,s)-\delta s(t)]\}\\
&\quad-V_t^{(t_0)2}(t,s)-\frac{1}{2}\sigma^2 s^2 V_{ss}^{(t_0)2}(t,s)\\
&=\max_{e_2,I_{21},I_{22}}\{[e_2(b_2-\frac{1}{2}e_2)-\pi_2 s-\frac{1}{2}a_2 I_{22}^2-\frac{1}{2}a_1 I_{21}\left(2\phi_{11}^{I*}(t,s)+I_{21}\right)]\times e^{-r(t-t_0)}\\
&\quad+V_s^{(t_0)2}(t,s)[\phi_1^{e*}(t,s)+e_2(t)-\gamma_1\phi_{11}^{I*}(t,s)-\gamma_2 I_{22}(t)\\
&\quad-\gamma_1 I_{21}(t)-\delta s(t)]\}\\
&V^{(t_0)1}(T,s)=-g^1\left[s(T)-\bar{s}_1\right]e^{-r(T-t_0)}
\end{aligned} \tag{17}$$

By maximizing Formula (17), it may be obtained:

$$\begin{aligned}
&\phi_1^{e*}(t,s)=b_1+V_s^{(t_0)1}(t,s)e^{r(t-t_0)}\\
&\phi_{11}^{I*}(t,s)=-\frac{\gamma_1}{a_1}V_s^{(t_0)1}(t,s)e^{r(t-t_0)}\\
&\phi_2^{e*}(t,s)=b_2+V_s^{(t_0)2}(t,s)e^{r(t-t_0)}\\
&\phi_{22}^{I*}(t,s)=-\frac{\gamma_2}{a_2}V_s^{(t_0)2}(t,s)e^{r(t-t_0)}\\
&\phi_{21}^{I*}(t,s)=-b_1-V_s^{(t_0)1}(t,s)e^{r(t-t_0)}-\frac{\gamma_1}{a_1}V_s^{(t_0)2}(t,s)e^{r(t-t_0)}
\end{aligned} \tag{18}$$

The present values of the profit function of both areas:

$$\begin{aligned}
&V^{(t_0)1}(t,s)=e^{-r(t-t_0)}\left[A_1(t)s+B_1(t)\right]\\
&V^{(t_0)2}(t,s)=e^{-r(t-t_0)}\left[A_2(t)s+B_2(t)\right]
\end{aligned} \tag{19}$$

By substituting formula (19) into formula (18), it can be calculated as:

$$\begin{aligned}
&\phi_1^{e*}(t,s)=b_1+A_1(t)\\
&\phi_{11}^{I*}(t,s)=-\frac{\gamma_1}{a_1}A_1(t)\\
&\phi_2^{e*}(t,s)=b_2+A_2(t)\\
&\phi_{22}^{I*}(t,s)=-\frac{\gamma_2}{a_2}A_2(t)\\
&\phi_{21}^{I*}(t,s)=-b_1-A_1(t)-\frac{\gamma_1}{a_1}A_2(t)
\end{aligned} \tag{20}$$

$A_1(t)$, $B_1(t)$, $A_2(t)$ and $B_2(t)$ in the formula must meet the following dynamic system and margin conditions:

$$\begin{aligned}
&\dot{A}_1(t)=\pi_1+(r+\delta)A_1(t)\qquad A_1(T)=-g^1\\
&\dot{A}_2(t)=\pi_2+(r+\delta)A_2(t)\qquad A_2(T)=-g^2\\
&\dot{B}_1(t)=rB_1(t)-\frac{1}{2}b_1^2-(\frac{1}{2}+\frac{\gamma_1^2}{2a_1}+\gamma_1)A_1(t)^2-A_1(t)\left(\gamma_1 b_1+b_1+b_2\right)\\
&\qquad-A_1(t)A_2(t)\left(1+\frac{\gamma_1^2}{a_1}+\frac{\gamma_2^2}{a_2}\right)\\
&\dot{B}_2(t)=rB_2(t)-\frac{1}{2}a_1 b_1^2-\frac{1}{2}b_2^2-\left(a_1 b_1+b_1\gamma_1\right)A_1(t)\\
&\qquad-\left(2b_1\gamma_1+b_1+b_2\right)A_2(t)-(\frac{1}{2}a_1+\gamma_1)A_1(t)^2\\
&\qquad-(\frac{1}{2}+\frac{3\gamma_1^2}{2a_1}+\frac{\gamma_2^2}{2a_2})A_2(t)^2-A_1(t)A_2(t)\times\left(1+2\gamma_1+\frac{2\gamma_1^2}{a_1}\right)\\
&B_1(T)=g^1\bar{s}_1\qquad B_2(T)=g^2\bar{s}_2
\end{aligned} \tag{21}$$

The instant profit of each area at time τ is:

$$P_1(\tau, s_\tau) = -(\pi_1 + rA_1(\tau))s_\tau - rB_1(\tau) + \frac{1}{2}b_1^2 - (\frac{1}{2} + \frac{\gamma_1^2}{2a_1})A_1(\tau)^2$$

$$\begin{aligned} P_2(\tau, s_\tau) &= -(\pi_2 + rA_2(\tau))s_\tau - rB_2(\tau) + \frac{1}{2}a_1 b_1^2 + \frac{1}{2}b_2^2 \\ &+ (a_1 b_1 + b_1\gamma_1) \times A_1(\tau) + b_1\gamma_1 A_2(\tau) \\ &+ \left(\frac{1}{2}a_1 + \gamma_1\right)A_1(\tau)^2 - \left(\frac{1}{2} - \frac{\gamma_1^2}{2a_1} + \frac{\gamma_2^2}{2a_2}\right)A_2(\tau)^2 \\ &+ \left(\gamma_1 + \frac{\gamma_1^2}{a_1}\right)A_1(\tau)A_2(\tau) \end{aligned} \quad \textbf{(22)}$$

The profit of each area at the terminal: $-g^i\left[s(T) - \bar{s}_i\right]$.

Cooperation

In the cooperation, both areas want to optimize watershed ecological environment through cooperative investment in environmental projects, therefore, they shall make the final cooperation with cooperative game. By expressing stochastic differential game with $\Gamma_c(T\text{-}t_0,\ s_0)$, area 1 and area 2 reach an cooperation agreement about investment in environment projects in area 1 in upper reaches and development changes of the pollution stock $s(t)$ may be subject to stochastic dynamic system.

$$\begin{aligned} ds(t) &= \left[e_1(t) + e_2(t) - \gamma_1(I_{11}(t) + I_{21}(t)) - \gamma_2 I_{22}(t) - \delta s(t)\right]dt \\ &\quad + \sigma s(t)dz(t) \\ s(0) &= s_0 \end{aligned} \quad \textbf{(23)}$$

At time t_0, the present values of expected profits of both areas are:

$$\begin{aligned} \max_{\substack{e_1,e_2 \\ I_{11},I_{21},I_{22}}} W = E\{&\int_{t_0}^{T}\{e_1(b_1 - \frac{1}{2}e_1) + e_2(b_2 - \frac{1}{2}e_2) - \pi s - \frac{1}{2}a_1(I_{11} + I_{21})^2 - \frac{1}{2}a_2 {I_{22}}^2\}e^{-r(t-t_0)}dt \\ &-\sum_{i=1}^{2} g^i\left[s(T) - \bar{s}_i\right]e^{-r(T-t_0)}\} \\ g^i \geq 0, \bar{s}_i &\geq 0 \end{aligned} \quad \textbf{(24)}$$

Set $\{[e_1^*(t), e_2^*(t), I_{11}^*(t), I_{21}^*(t), I_{22}^*(t)] =$ $[\phi_1^{e*}(t,s), \phi_2^{e*}(t,s), \phi_{11}^{I*}(t,s), \phi_{21}^{I*}(t,s), \phi_{22}^{I*}(t,s)]\}$ as a feedback strategy set of cooperative game Nash equilibrium, when continuous differential expected profit function $V^{(t_0)i} = (t,s) \times R_m \to R$ exists, satisfy the following partial differential equation:

$$\begin{aligned} &-W_t^{(t_0)}(t,s) - \frac{1}{2}\sigma^2 s^2 W_{ss}^{(t_0)}(t,s) \\ &= \max_{\substack{e_1,e_2 \\ I_{11},I_{21},I_{22}}} \{[\phi_1^{e*}(t,s)(b_1 - \frac{1}{2}\phi_1^{e*}(t,s)) + \phi_2^{e*}(t,s)(b_2 - \frac{1}{2}\phi_2^{e*}(t,s)) \\ &\quad - \pi s - \frac{1}{2}a_1(\phi_{11}^{I*}(t,s) + \phi_{21}^{I*}(t,s))^2 - \frac{1}{2}a_2\phi_{22}^{I*}(t,s)^2]e^{-r(t-t_0)} \\ &\quad + W_s^{(t_0)}(t,s)[\phi_1^{e*}(t,s) + \phi_2^{e*}(t,s) - \gamma_1(\phi_{11}^{I*}(t,s) + \phi_{21}^{I*}(t,s)) \\ &\quad - \gamma_2\phi_{22}^{I*}(t,s) - \delta s(t)]\} \\ &W^{(t_0)}(T,s) = -\sum_{i=1}^{2} g^i\left[s(T) - \bar{s}_i\right]e^{-r(T-t_0)} \end{aligned} \quad \textbf{(25)}$$

By maximizing formula (25) above, it may be obtained:

$$\phi_1^{e*}(t,s)=b_1+W_s^{(t_0)}(t,s)e^{r(t-t_0)}$$
$$\phi_2^{e*}(t,s)=b_2+W_s^{(t_0)}(t,s)e^{r(t-t_0)}$$
$$\phi_{11}^{I*}(t,s)+\phi_{21}^{I*}(t,s)=-\frac{\gamma_1}{a_1}W_s^{(t_0)}(t,s)e^{r(t-t_0)}$$
$$\phi_{22}^{I*}(t,s)=-\frac{\gamma_2}{a_2}W_s^{(t_0)}(t,s)e^{r(t-t_0)} \quad \textbf{(26)}$$

By substituting formula (26) into formula (25) and solving formula (25), it may be obtained profit function of both areas at time Zone $[t_0,T]$:

$$W^{(t_0)}(t,s)=e^{-r(t-t_0)}\left[A(t)s+B(t)\right] \quad \textbf{(27)}$$

By substituting formula (27) into formula (26), it may be obtained:

$$\phi_1^{e*}(t,s)=b_1+A$$
$$\phi_2^{e*}(t,s)=b_2+A$$
$$\phi_{11}^{I*}(t,s)+\phi_{21}^{I*}(t,s)=-\frac{\gamma_1}{a_1}A$$
$$\phi_{22}^{I*}(t,s)=-\frac{\gamma_2}{a_2}A \quad \textbf{(28)}$$

$A(t)$ and $B(t)$ in the formula must meet the following dynamic system and margin conditions:

$$\dot{A}(t)=\pi+(r+\delta)A(t)$$
$$A(T)=-g^1-g^2$$
$$\dot{B}(t)=rB(t)-(1+\frac{\gamma_1^2}{2a_1}+\frac{\gamma_2^2}{2a_2})A(t)^2-(b_1+b_2)A(t)-\frac{1}{2}b_1^2-\frac{1}{2}b_2^2$$
$$B(T)=g^1\bar{s}_1+g^2\bar{s}_2 \quad \textbf{(29)}$$

In cooperation game, for the expected profit of mutual management of both areas, the additional expected profit may be allocated according to the profit proportion of the area under noncooperation. The present value of the profit of each area $\varepsilon^{(\tau)i}(\tau,s_\tau)$ may be expressed as:

$$\varepsilon^{(\tau)i}(\tau,s_\tau)=V^{(\tau)i}(\tau,s_\tau)+\frac{V^{(\tau)i}(\tau,s_\tau)}{\sum_{j=1}^{n}V^{(\tau)j}(\tau,s_\tau)}\times\left[W^{(\tau)}(\tau,s_\tau)-\sum_{j=1}^{n}V^{(\tau)j}(\tau,s_\tau)\right]$$
$$=\frac{A_i(\tau)s_\tau+B_i(\tau)}{\sum_{j=1}^{n}\left[A_j(\tau)s_\tau+B_j(\tau)\right]}\times\left[A(\tau)s_\tau+B(\tau)\right] \quad \textbf{(30)}$$

Formula (30) shows that the expected profit of each area is equal to the sum of the expected noncooperation profit and the one shared in the additional profits under cooperation according to its own proportion in the expected noncooperation profit. The profit of area *i* at each time point under cooperation mode except final payment may be expressed as:

$$
\begin{aligned}
P_i(\tau,s_\tau) = &-\frac{(A_i(\tau)s_\tau + B_i(\tau))(\dot{A}(\tau)s_\tau + \dot{B}(\tau))}{\sum_{j=1}^{n}[A_j(\tau)s_\tau + B_j(\tau)]} \\
&-\frac{(\dot{A}_i(\tau)s_\tau + \dot{B}_i(\tau))(A(\tau)s_\tau + B(\tau))}{\sum_{j=1}^{n}[A_j(\tau)s_\tau + B_j(\tau)]} \\
&+\frac{(A_i(\tau)s_\tau + B_i(\tau))[\sum_{j=1}^{n}(\dot{A}_j(\tau)s_\tau + \dot{B}_j(\tau))](A(\tau)s_\tau + B(\tau))}{[\sum_{j=1}^{n}(A_j(\tau)s_\tau + B_j(\tau))]^2} \\
&-\sigma^2 s_\tau^2\{\frac{A(\tau)A_i(\tau)}{\sum_{j=1}^{n}[A_j(\tau)s_\tau + B_j(\tau)]} \\
&-\frac{\sum_{j=1}^{n}A_j(\tau)(2A(\tau)A_i(\tau)s_\tau + A(\tau)B_i(\tau) + A_i(\tau)B(\tau))}{[\sum_{j=1}^{n}(A_j(\tau)s_\tau + B_j(\tau))]^2} \\
&+\frac{[\sum_{j=1}^{n}A_j(\tau)]^2(A_i(\tau)s_\tau + B_i(\tau))(A(\tau)s_\tau + B(\tau))}{[\sum_{j=1}^{n}(A_j(\tau)s_\tau + B_j(\tau))]^3}\} \\
&-[\frac{(2A(\tau)A_i(\tau)s_\tau + A(\tau)B_i(\tau) + A_i(\tau)B(\tau))}{\sum_{j=1}^{n}[A_j(\tau)s_\tau + B_j(\tau)]} \\
&-\frac{\sum_{j=1}^{n}A_j(\tau)(A_i(\tau)s_\tau + B_i(\tau))(A(\tau)s_\tau + B(\tau))}{[\sum_{j=1}^{n}(A_j(\tau)s_\tau + B_j(\tau))]^2}] \\
&+(b_1 + b_2 + 2A(\tau) + \frac{\gamma_1^2}{a_1}A(\tau) + \frac{\gamma_2^2}{a_2}A(\tau) - \delta s_\tau)
\end{aligned}
\qquad \textbf{(31)}
$$

The profit of each area at the terminal: $-g^i\left[s(T)-\bar{s}_i\right]$.

ANALYSIS OF EXAMPLES

Assuming relevant parameters[11] involved in the examples are as below: $\gamma_1 = 0.5, \gamma_2 = 1, a_1 = 0.5, a_2 = 1, b_1 = 20, b_2 = 40, g_1 = 1, g_2 = 3, \bar{s}_1 = 25,$ $\bar{s}_2 = 60, \pi_1 = 4, \pi_2 = 5, \pi = 9, \delta = 0.01, \sigma = 0.05, r = 0.05, T = 3,$ and assuming the pollution stock of each time in autarky mode is 40, 43 and 45 subsequently, while in individual investment mode and cooperation mode, because of nonlocal investment in addition to local investment in environment projects and in order to better compare the differences between individual investment cooperation and noncooperation, it is assumed that the pollution stock of these two modes at each time are same and less than that of autarky mode, 38, 41 and 43 respectively. The expected profits of each area at each time point under these three modes are as below:

TABLE 1 : Expected profit of each area under autarky mode

			Area 1				Area 2		
t	s_τ	$A_1(t)$	$B_1(t)$	$P_1(\tau,s_\tau)$	$P_1(T,s_\tau)$	$A_2(t)$	$B_2(t)$	$P_2(\tau,s_\tau)$	$P_2(T,s_\tau)$
1	40	-9	-4934	244	—	-12	-3663	663	—
2	43	-6	-4597	244	—	-8	-2847	681	—
3	45	-2	50	—	-40	-3	180	—	45

TABLE 2 : Expected profit of each area under individual investment mode

			Area 1				Area 2		
t	s_τ	$A_1(t)$	$B_1(t)$	$P_1(\tau,s_\tau)$	$P_1(T,s_\tau)$	$A_2(t)$	$B_2(t)$	$P_2(\tau,s_\tau)$	$P_2(T,s_\tau)$
1	38	-9	-2577	133	—	-12	-3787	611	—
2	41	-6	-2574	150	—	-8	-3863	700	—
3	43	-2	50	—	-36	-3	180	—	51

TABLE 3 : Expected profit of each region under cooperation mode

				Area 1		Area 2	
t	s_τ	$A(t)$	$B(t)$	$P_1(\tau,s_\tau)$	$P_1(T,s_\tau)$	$P_2(\tau,s_\tau)$	$P_2(T,s_\tau)$
1	38	-22	-2711	133	—	233	—
2	41	-14	-3871	108	—	321	—
3	43	-5	230	—	-36	—	51

It may be drawn from the above calculation:

(1) Under three modes, A(t)'coefficients is negative and gradually increasing, indicating the negative relationship between the pollution stock and the expected profit; the expected profit is gradually increasing along with a slowdown of discharge amount.

(2) Under autarky mode, area 1 and area 2 can make profits in first two years and the profit is increasing. Due to developed economy and strong productivity, area 2 makes more profits than area 1. However, in the third year, area 1 needs to undertake relevant penalties due to failure to meet the given pollution stock requirement, its final profit is negative, while area 2 still makes profits because it meets the given requirement.

(3) Under individual investment, the profit of area 1 in first two years and the profit of area 2 in first year are both less than that of autarky mode, which is caused by increase of investment cost and decrease of pollution stock of area 2 due to its investment in environmental projects in area 1. In third year, because the watershed abatement effect is relatively better than that of autarky mode, area 1 undertakes less penalties and Area 2 gets more profits.

(4) Under cooperation mode, by assuming the pollution stock as same as that of individual investment mode, the final profits of both areas under these two modes are also the same. However, by comparing their instant profits, it may be drawn that the profits of both areas in first two years under cooperation mode are obviously less that of these two modes above, which is because the pollution management in area 1 of poor economy and hard pollution abatement task has become cooperative investment from original scattered investment and the initial investment increases. Along with the better performance of the investment in pollution abatement, the profit will surely increase in coming years.

CONCLUSION

Two adjacent watershed areas are taken as study object in this paper and three game modes including autarky, individual investment and cooperation are established respectively according to the different investment modes of water pollution abatement and have good practical guiding significance: 1. The model established based on stochastic differential games theory gives full consideration to the feature of pollution abatement, which is a long-term stochastic dynamic negotiation process developing with environment development, and it is more practical. 2. The areas at upper reaches are usually of poor economy and also the key areas of pollution abatement; by encouraging developed lower reaches to invest environment projects in upper reaches, it not only yields twice the result with half the effort from the source, but also records the abatement indexes into the assessment of the area, helpful to arouse the enthusiasm of investors. Based on the comparison of the expected profits at each time point under these three modes, it may be drawn that the way that the lower reaches engage in the investment in the environment projects at upper reaches can best realize the win-win situation and sustainable watershed development for a long time.

During model establishment in this paper, profits and cost variables constituting the objective function are simplified and each area is regarded as a player of the game, without consideration of the differences in objective function resulting from the different roles of the government and the enterprises in pollution abatement in each area, which may be studied further.

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