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Study on global bifurcation characteristics of gear system

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ABSTRACT

The global bifurcation characteristics about motion state about dimensionless frequency Ω and the mean meshing force P_m are studied based on the classical nonlinear dynamical model of a gear pair with clearance in this paper. The results reveals that Periodic motion is the gear pair's main motion state in the range of $0.5 \le \Omega \le 2.5$ and it just appears chaos motion in a small range $0.62 \le \Omega \le 0.92$ temporarily; the way to chaos is periodic-doubling bifurcation with dimensionless frequency Ω decreases; Periodic motion is the system's main motion state in the range of $0.1 \le p_m \le 1.4$ and it just appears non-periodic motion in a small P_m range of $0.01 \le p_m \le 0.1$; the way to chaos is periodic-doubling bifurcation with the mean meshing force P_m decreases.

KEYWORDS

Gear system;
Nonlinear vibration model;
Global bifurcation characteristic.

INTRODUCTION

Existing literatures about study on nonlinear dynamics of gear pair are numerous. Kubo[1] observed a jump phenomenon in the frequency response of a gear pair with backlash even though the test set-up was heavily damped. Comparin and Singh addressed subsection nonlinear problem in [2] and pointed out that most techniques available in the literatures cannot be directly applied to solving this problem. Kahraman and Singh[3] made a review of nonlinear gear dynamics available in current literatures. Kahraman and Singh find the motion of a spur gear pair with a set of special parameters maybe chaos[4]. Their study also made contributions to the nonlinear dynamics of a spur gear pair with backlash subject to the static transmission error.

For a nonlinear dynamical system, global bifurcation study is an important target. The bifurcation property of a gear system can provide important information, such as stable parameter region, stable parameter region, the way to chaos, for a designer. However, the study on this aspect is still very limited.

The objective of this paper is to study gear system's global bifurcation characteristics about some parameters based on nonlinear dynamical model of a gear pair with clearance.

NONLINEAR DYNAMICAL MODEL

The classical nonlinear dynamical model of a gear pair with clearance[5] is adopted in this paper, which is shown in figure 1.

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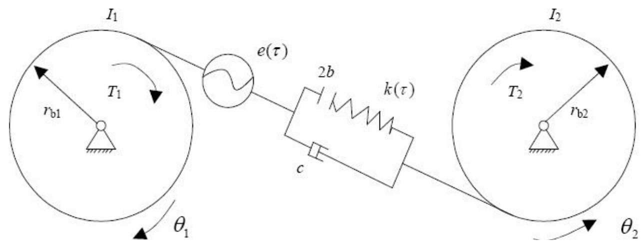

Figure 1 : Nonlinear dynamical model of gear system with backlash

Where, r_{bi}, θ_i, I_{bi} and T_i (i=1,2) denote the base circle radius, rotational motions, the inertia of and the torque of the two gear in pair. $e(\tau)$ and $k(\tau)$ stand for the static transmission error in pressure line and the time-varying meshing stiffness respectively, τ is time; $2b$ is the clearance of the gear pair in pressure line, c is damping coefficient.

The dimensionless differential equations of torsional vibration of the nonlinear planetary gear system which is shown as figure 1 can be established by using the Lagrange principle as following

$$\ddot{x}+2\varsigma\dot{x}+[1+\xi\cos(\Omega t)]f(x)=P_m+P_a\cos(\Omega t) \qquad (1)$$

Where $x=[r_{b1}\theta_1+r_{b2}\theta_2-e(\tau)]/b_c$, $\zeta=c/(2m\omega_n)$, $\xi=k_1/k_0$, $P_m=T_1\cos\alpha/(r_{b1}b_ck_0)$, $P_a=e_a\Omega^2/b_c$ $t=\omega_n\tau$,

$$\Omega=\omega/\omega_n,\quad f(x)=\begin{cases} x-b/b_c & x>b/b_c \\ 0 & -b/b_c\le x\le b/b_c \\ x+b/b_c & x<b/b_c \end{cases}.$$

In the above formulas; x is dimensionless rotational motions; b_c is character length; ζ is dimensionless damping coefficient; m is equivalent mass of the gear pair; ω_n is the natural frequency of the gear pair; ξ is the time-varying amplitude of the stiffness in the meshing pair; k_1 is the first harmonic term of the expanding rectangular waves in Fourier series; k_0 is the mean values of the stiffness; f(x) is a nonlinear function in the relative gear mesh displacement; P_m is the mean meshing force; α is the pressure angle; P_a is the amplitude of the meshing force; e_a is the amplitude of the static transmission error; t is dimensionless time; Ω is dimensionless frequency.

BIFURCATION CHARACTERISTICS ABOUT DIMENSIONLESS FREQUENCY

The bifurcation diagram of the system's motion state can be made through the following steps[6]:

(1) Obtain the system's asymptotically stable numerical solutions by using ODE45 to solving state equations corresponding to differential equation (1).

(2) Let the system's Poincare section is $\sum=\{(\tau,X)\in R\times R^N \mid \tau=\mathrm{mod}(2\pi/\Omega), x_i=\max(x_{i0}\to x_{iT})\}$, where $x_i=\max(x_{i0}\to x_{iT})$ denotes the maximum of the i-th state variables in a cycle of the external excitation force, and the system's Poincare map at a determined bifurcation parameter can be obtained.

(3) Change the bifurcation parameter constantly and repeat the step 2 in order to obtain the system's Poincare map at different bifurcation parameter.

(4) Assemble all of the Poincare maps calculated in step 3 from small bifurcation parameter to large one.

The main parameters of the gear system in this paper are: the number teeth of the two gear is $z_1=z_2=25$, module M=3mm, face width B=25mm, the pressure angle $\alpha=20°$, helix angle $\beta=0°$, displacement factor is 0, P_m=0.1, P_a=0.2, $\zeta=0.05$, $\xi=0.4$, the range of bifurcation parameter is $0.5\le\Omega\le2.5$.

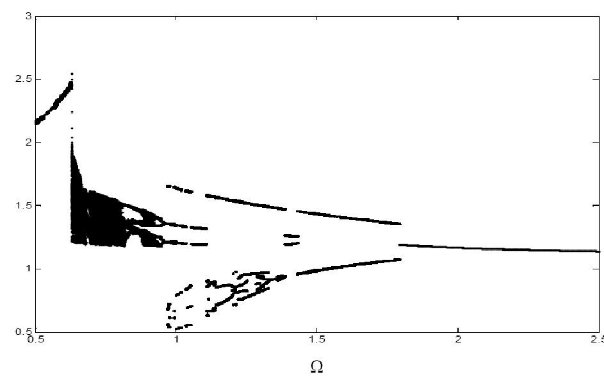

Figure 2 : Bifurcation diagrams of the system's motion state about dimensionless frequency Ω

We can see from figure 2 that periodic motion is the system's main motion state in the range of $0.5\le\Omega\le2.5$ and it just appears non-periodic motion in a small range of $0.62\le\Omega\le0.92$ temporarily.

In order to make sure the specific shape of the non-periodic attractor, the Poincare map, the displacement curve and the phase diagram are calculated at different bifurcation parameter.

The non-periodic attractor is chaos motion in the range of $0.62\le\Omega\le0.92$ according to the topology shape of the above six figures.

It is easy to conclude that the way to chaos is periodic-doubling bifurcation with dimensionless frequency Ω decreases.

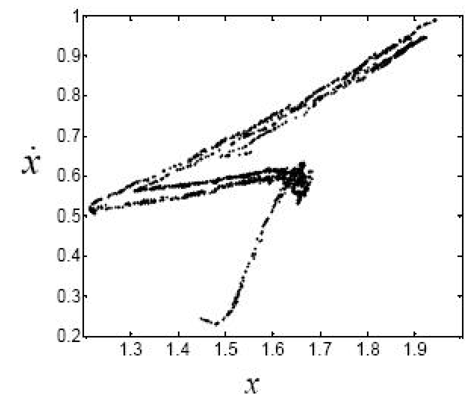

Figure 3 : Poincare map when $\Omega = 0.63$

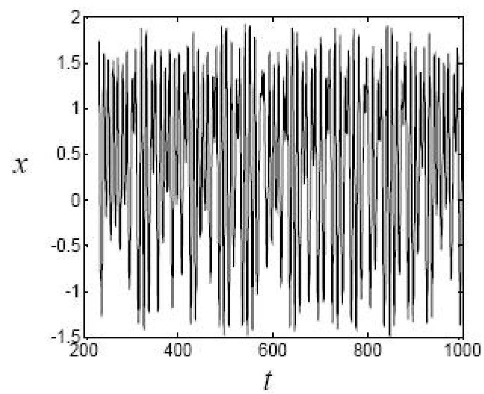

Figure 4 : displacement curve when $\Omega = 0.63$

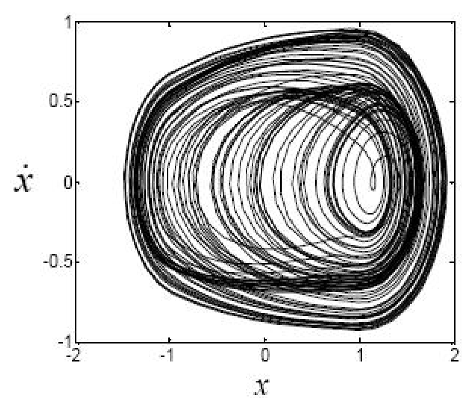

Figure 5 : Phase diagram when $\Omega = 0.63$

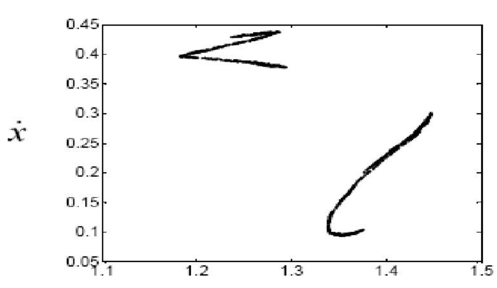

Figure 6 : Poincare map when $\Omega = 0.63$

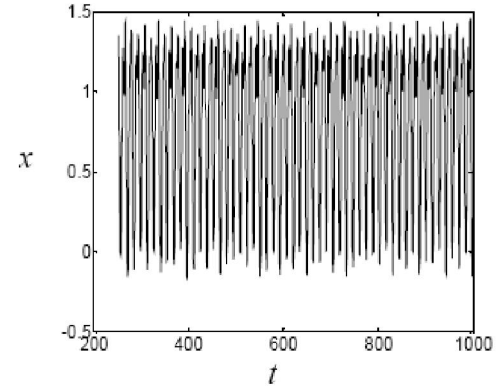

Figure 7 : displacement curve when $\Omega = 0.63$

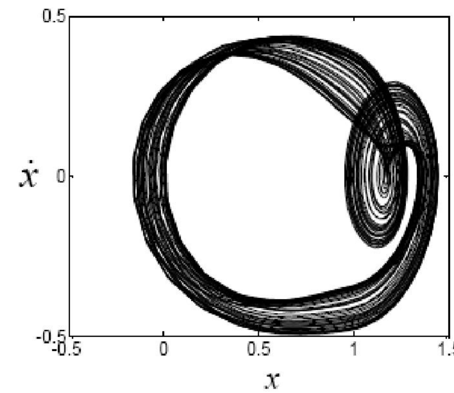

Figure 8 : phase diagram when $\Omega = 0.63$

BIFURCATION CHARACTERISTICS ABOUT MEAN MESHING FORCE P_M

According to the step to calculate bifurcation diagram of Ω, one can calculate the bifurcation diagrams about mean meshing force P_m shown as figure 9.

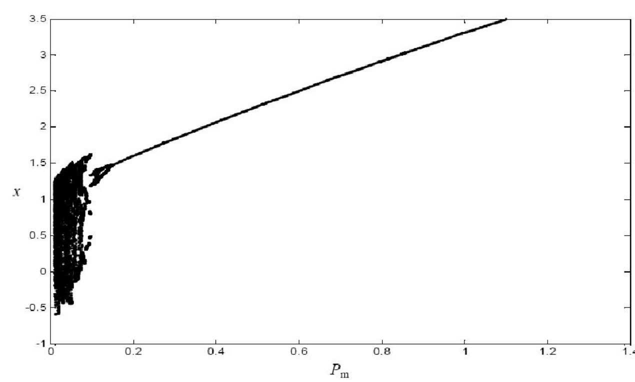

Figure 9 : Bifurcation diagrams of the system's motion state about mean meshing force P_m

We can see from figure 9 that the bifurcation characteristics about Ω is simpler than the one about P_m. Periodic motion is the system's main motion state in the

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range of $0.1 \le p_m \le 1.4$ and it just appears non-periodic motion in a small P_m range of $0.01 \le p_m \le 0.1$. In order to make sure the specific shape of the non-periodic attractor, the Poincare map, the displacement curve and the phase diagram are calculated when $\Omega = 0.63$.

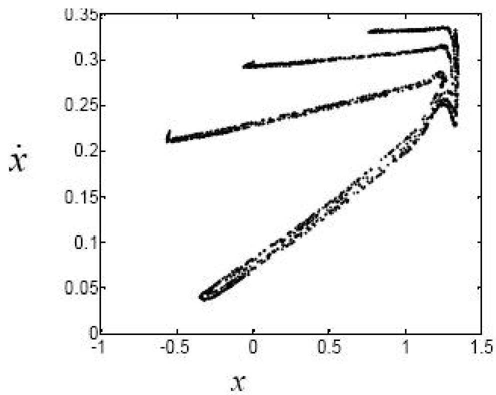

Figure 10 : Poincare map when $\Omega = 0.63$

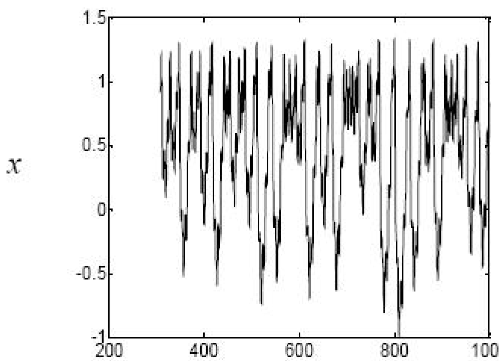

Figure 11 : displacement curve when $\Omega = 0.63$

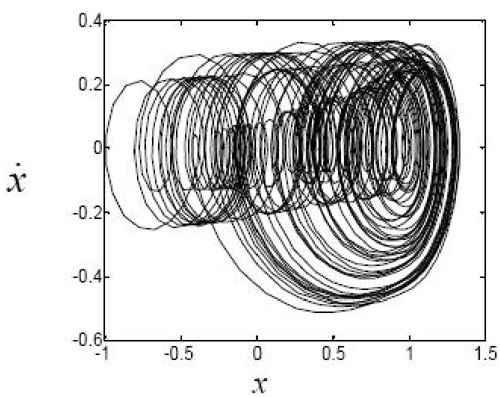

Figure 12 : phase diagram when $\Omega = 0.63$

The non-periodic attractor is chaos motion in the range of $0.01 \le p_m \le 0.1$ according to the topology shape of the above three figures.

It is easy to conclude that the way to chaos is periodic-doubling bifurcation with the mean meshing force P_m decreases.

CONCLUSIONS

The global bifurcation characteristics of motion state about dimensionless frequencyand the mean meshing force P_m decreases are studied in this paper. The results reveal two bifurcation laws.

(1) Periodic motion is the gear system's main motion state in the range of $0.5 \le \Omega \le 2.5$ and it just appears chaos motion in a small range $0.62 \le \Omega \le 0.92$ temporarily; the way to chaos is periodic-doubling bifurcation with dimensionless frequency Ω decreases.

(2) Periodic motion is the system's main motion state in the range of $0.1 \le p_m \le 1.4$ and it just appears non-periodic motion in a small P_m range of $0.01 \le p_m \le 0.1$; the way to chaos is periodic-doubling bifurcation with the mean meshing force P_m decreases.

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