Design of orthogonal waveforms based on continuous chaos frequency modulation

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ABSTRACT

In view of the problem that the orthogonality of MIMO radar waveform decreased with the increase of number of waveforms, using the pseudo-random and initial value sensitivity of continuous chaotic system, this paper presents a design method of orthogonal waveform based on the continuous chaotic frequency modulation. Firstly, the mathematical expression of orthogonal continuous chaos frequency modulation signal (OCCFMS) is given, autocorrelation and quasi-orthogonal property is analyzed, and proposes a progressive chaotic particle swarm optimization algorithm to optimize the chaos initial value. Performance analysis and simulation results show that OCCFMS presents quite good performances in ambiguity function, autocorrelation and pseudo-orthogonal property, the performance of the waveform is affected by the chaos initial value. Using the suggested method to optimize the initial value, can get any number of orthogonal waveforms, the precision of optimization is higher, can further reduce the autocorrelation sidelobe peak and cross-correlation peak of OCCFMS, and the autocorrelation sidelobe peak and cross-correlation peak of each orthogonal signal of the waveform set are very close, and can be applied as a kind of promising MIMO radar signals.

KEYWORDS

MIMO; OCCFMS; Cross-correlation; Autocorrelation.
INTRODUCTION

MIMO radar emits a set of orthogonal waveforms, forming a low gain wide beam, the receiver uses a set of matched filters to separate the orthogonal waveforms, its target detection performance and low probability of intercept are superior to the conventional radar[1-4]. Currently, MIMO radar waveform is generally orthogonal discrete frequency coding waveform and orthogonal phase coding waveform[5-7].

The orthogonal coding waveform design method is usually simulated annealing algorithm and genetic algorithm. The good orthogonality of the waveforms can be obtained with these methods, but the orthogonality of waveform declines with the increase of the number of orthogonal waveforms. So it is practical significance that studying a set of orthogonal waveforms design can contain any number of waveform. The orthogonal waveforms requires two conditions, one is to have low autocorrelation sidelobe peak (ASP), the radar has the characteristics of multi-target detection and high range resolution, and the other is to have good orthogonality, the received signals can be effectively separated. Therefore, the orthogonality and autocorrelation of the waveform is one of the keys to achieve MIMO radar. With the rapid development of electronic warfare, radar waveform requires low probability of intercept and anti-interference ability. Therefore, the design of orthogonal waveform set also need to consider the low probability of intercept and anti-interference ability.

Continuous chaotic frequency modulation signal is a pseudo noise signal, has the initial value sensitivity, pseudo random, low probability of interception and anti-interference performance[8]. This signal has pseudo randomness, so it has quasi orthogonality; because of initial value sensitivity, we can get any number of signals. Therefore, this paper applies continuous chaotic frequency modulation signal to the orthogonal waveform design of MIMO radar, and proposes a progressive chaotic particle swarm optimization algorithm to optimize the chaos initial value. Performance analysis and simulation results show that OCCFMS has low ASP and cross-correlation peak (CP), and can get any number of orthogonal waveforms.

DESCRIPTION OF OCCFMS

OCCFMS can be expressed as

\[
s_l(t) = A \exp \left( j2\pi f_c t + j2\pi k \int_0^t x'(u) du \right),
\]

\[l = 1, 2, \ldots, L, t \in [0, T]\]

Which, A is the signal amplitude, \(f_c\) is the center frequency, \(k\) is the coefficient of frequency modulation, \(T\) is the pulse width. \(x'(t)\) is a continuous chaotic signal, can be obtained with the high order iterative methods, such as the four order Runge Kutta has a faster convergence speed. In the signal duration, the frequency spectrum of the signal is mainly concentrated in \(kx_{\text{max}}^l(t) \leq f \leq kx_{\text{min}}^l(t)\).

Direct digital synthesis (DDS) has the characteristics, such as high frequency resolution, frequency conversion speed and can synthesize complex waveforms. With the development of DDS technology, frequency conversion time has reached ns level. Therefore, the higher intermediate frequency of continuous chaotic frequency modulation signal can be generated by DDS. DDS can assure the phase is continuous when the frequency is agile. The output waveform of DDS can be expressed with the sum of single frequency pulse signals. To simplify the discussion, we only consider the baseband signal. According to the Nyquist sampling theorem, taking \(t\) as the sampling period, the expression of OCCFMS based on DDS:

\[
s_l(t) = \sum_{n=0}^{N} p'_n(t - nt_l)
\]
The autocorrelation function

The autocorrelation function of OCCFMS is

\[
R(\tau) = \int_{-\infty}^{\infty} s_i(t+t) \overline{s_i(t)} dt = \int_{-\infty}^{\infty} A^2 \exp(j2\pi k \sum x'(n)t) dt
\]

(4)

The chaotic signal \( x'(t) \) can be seen as a random process, and the little variation of Initial value can cause that chaotic signals are completely different. Therefore, \( x'(n) \) can be regarded as an independent and identically distributed sequence, and \( \theta = 2\pi k \sum x'(n) \overline{s_i} \) is the sum of independent identically distributed sequences. According to the central limit theorem, the sum of independent and identically distributed random variables sequence approximately obey normal distribution, so a can be approximated as a normal distribution. Assume that the mean of \( \theta \) is \( \mu \), the root mean square is \( \delta^2 \).

\[
\mu = E\{\theta\} = E\left\{2\pi k \sum x'(n)t\right\} = 0
\]

(5)

\[
\sigma^2 = E\{\theta^2\} = E\left\{(2\pi k \sum x'(u) du)^2\right\}
\]

\[
= 4\pi^2 k^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_{x x}(t_1-t_2) dt_1 dt_2
\]

\[
= 8\pi^2 k^2 \int_0^\infty (\tau-t_1) R_{x x}(t) dt
\]

(6)

According to the relationship between normal distribution characteristic function and mean square root

\[
E\{\exp(j\theta)\} = \exp(-\sigma^2/2), \text{ We can get}
\]

\[
R(\tau) = \exp(-4\pi^2 k^2 \int_0^\infty (\tau-t_1) R_{x x}(t_1) dt_1)
\]

(7)

The mean of continuous chaotic signal is approximately zero, according to the relationship between the correlation coefficient with the autocorrelation function, we can know

\[
\rho_x(\tau) = \frac{R_{x x}(\tau)}{\sigma_x^2} = \frac{R_{x x}(\tau)}{\sigma_x^2}
\]

(8)

\[
R_x(0) = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(t)x(t+0) dt
\]

\[
= \mu_x^2 + \sigma_x^2 = \sigma_x^2 = \rho_x(0) \sigma_x^2
\]

(9)

Substituting (8)-(9) into (7), we can get that
\[ R(\tau) = \exp\left(-4\pi^2 k^2 \sigma^2 \int_{-\tau}^{\tau} (t-t_s) \rho_x(t) \, dt \right) \]  

\[ (10) \]

When the continuous chaotic signal is constant, this inference is consistent with that theoretically radar range resolution is inversely proportional to the signal bandwidth \( B \). The sidelobe of autocorrelation function is proportional to frequency modulation coefficient \( k \) and variance of continuous chaotic signal \( \sigma^2 \). With \( k \) and \( \sigma^2 \) increased, the autocorrelation sidelobe is reduced, and the ASP is also affected by \( k \) and \( \sigma^2 \). When the bandwidth \( B \) is constant, \( k \) also is constant, the ASP is related to \( \sigma^2 \). \( \sigma^2 \) is the rms of chaotic signal \( x'(t) \), so \( \sigma^2 \) is related to the initial value of chaotic signal \( x'(t) \). To select and optimize the initial value of chaotic signal \( x'(t) \), we can get a lower ASP.

**The cross-correlation function**

\( s_p(t) \) and \( s_q(t) \) is OCCFMS generated by different initial values, the cross-correlation function between \( s_p(t) \) and \( s_q(t) \) is

\[ R_{pq}(\tau) = \int_{-\infty}^{\infty} s_p^*(t) s_q(t-\tau) \, dt \]

\[ = \int_{-\infty}^{\infty} \sum_{n=0}^{N-1} p_n^*(n t_s - t) \sum_{m=0}^{N-1} p_m^*(t - \tau - m t_s) \, dt \]

\[ = \int_{-\infty}^{\infty} \sum_{n=0}^{N-1} \exp\left(-j 2\pi k \sum_{a=0}^{N-1} x_n^a(n)(t - n t_s)\right) \sum_{m=0}^{N-1} \exp\left(j 2\pi k \sum_{a=0}^{N-1} x_m^a(m)(t - \tau - m t_s)\right) \, dt \]

\[ = \int_{-\infty}^{\infty} \sum_{n=0}^{N-1} \exp(j 2\pi k \sum_{a=0}^{N-1} x_n^a(n)(n t_s - t)) \]

\[ + j 2\pi k \sum_{n=0}^{N-1} \sum_{m=0, m\neq n}^{N-1} \exp(j 2\pi k \sum_{a=0}^{N-1} x_n^a(n)(n t_s - t)) \]

\[ + \sum_{m=0}^{N-1} \sum_{n=0, n\neq m}^{N-1} \exp(j 2\pi k \sum_{a=0}^{N-1} x_n^a(n)(n t_s - t)) \]

\[ + \sum_{m=0}^{N-1} x_m^a(m)(t - \tau - m t_s) \, dt \]

\[ (11) \]

Chaotic frequency modulation signal has the randomness and ergodicity, so we can be obtained

\[ \lim_{N \to \infty} \int_{-\infty}^{\infty} \sum_{n=0}^{N-1} \exp\left(j 2\pi k \sum_{a=0}^{N-1} x_n^a(n)(n t_s - t)\right) \]

\[ + j 2\pi k \sum_{n=0}^{N-1} x_n^a(n)(n t_s - t) \] \( dt = 0 \)

\[ \lim_{N \to \infty} \sum_{m=0}^{N-1} \sum_{n=0, m\neq n}^{N-1} \exp(j 2\pi k \sum_{a=0}^{N-1} x_n^a(n)(n t_s - t)) \]

\[ + \sum_{m=0}^{N-1} x_m^a(m)(t - \tau - m t_s) \, dt = 0 \]

\[ (12) \]

\[ R_{pq}(\tau) = 0 \]

As can be seen from (12), when the continuous chaotic signal is long enough, OCCFMS has a good orthogonality. In fact, the length of signal is limited in MIMO radar, and so the OCCFMS is not
completely orthogonal. Cross-correlation function is affected by the initial value of the signal, and optimize the initial value that can make the waveform of the radar has greatly improved, and the orthogonality of the waveform does not decreases with the number of the waveform increase, so we can get any number of good performance of the orthogonal waveform in theory.

DESIGN OF OCCFMS BASED ON PROGRESSIVE CHAOTIC PARTICLE SWARM OPTIMIZATION ALGORITHM

MIMO radar waveform requires the orthogonal signal having a low ASP and a low correlation peaks. According to the analysis of the autocorrelation function and the cross-correlation function in the previous section, The initial value of chaotic signal is related to the ASP and CP. By the calculation, we found that not all the chaos initial value can be low ASP and CP. In order to get a lower ASP and a lower CP, we need to optimize the initial chaotic signal.

When the pulse width \( B \) and frequency modulation coefficient \( k \) is constant, using PSO to optimize the initial value of chaotic signal, we can get a lower ASP and a lower CP. Assuming the transmitted signals contain \( L \) OCCFMS, the objective function is the sum of the ASP and the CP

\[
\text{fitness} = \sum_{l=1}^{L} \text{ASP}_l + \sum_{l=1, j=1, j \neq l}^{L} \text{CP}_{lj}, l = 1, 2, \ldots, L
\]

(13)

Which, \( \text{ASP}_l \) is the auto-correlation peak of \( S_l \), \( \text{CP} \) is CP between the signals \( S_l \) and \( S_j \). The objective function is the sum of \( \text{ASP} \) and \( \text{CP} \). If PSO is used to optimize the chaos initial value, the value of the objective function may be very small, but ASP or CP of some signals may be large, the overall performance of orthogonal waveform will be affected, And with the increase in number of signals, the orthogonality decreased. This paper presents a method of progressive optimization based on variable interval to avoid that the ASP or CP of individual signal is too large and orthogonality decreased, the detailed steps is

1. The search space of the chaos initial value is \([L \cdot x_{l}, (l+1) \cdot x] \), \( l = 0 \), the Objective function is

\[
\text{fitness} = \text{ASP}_l + \sum_{j=1}^{l} \text{CP}_{lj}
\]

(14)

2. \( l = l + 1 \), using PSO to optimize the chaos initial value of \( S_l \).

3. Repeat step 2, until \( l = L \).

Because chaotic signal has initial value sensitivity, the chaos initial values are different, every OCCFMS is completely different. When optimized, the initial chaos of each signal uses different search interval, it is easier to find the optimal solution, and to avoid duplication solution and orthogonality decreased.

Equation (11) is a multi-peak nonlinear function, the chaos initial value is optimized with standard PSO, the Objective function usually stable convergence in a local optimal solution, It is the emergence of "premature". Chaos is a universal nonlinear phenomenon, has the ergodicity and randomicity. The chaos is introduced into PSO to solve the "premature" phenomenon, it has attracted more and more attention, and achieved certain results\[^{[9,10]}\]. The optimum variables are transformed into chaotic variables by the chaotic mapping, to search the optimal solution using the ergodicity of chaotic variables and to jump out of local optimal solution for the global optimization. This paper presents an improved chaotic particle swarm optimization algorithm, the particles were chaotic motion based on Logistic map, and carry out the following improvement to the PSO update formula.
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\[ v_{i_d}(t+1) = \alpha v_{i_d}(t) + c_1 \text{rand} \cdot (p_{i_d}(t) - x_{i_d}(t)) + c_2 \text{rand} \cdot (p_{g_d}(t) - x_{i_d}(t)) \]  
\[ a_{i_d}(t+1) = \frac{1}{1 + e^{-10(t-c)}} \]  
\[ b_{i_d}(t+1) = 1 - a_{i_d}(t+1) \]  
\[ x_{i_d}(t+1) = b_{i_d}(t) \left[ (x_{i_d}(t) - x_{\min}) \right] + \left[ 4 - \frac{4}{x_{\max} - x_{\min}}(x_{i_d}(t) - x_{\min}) \right] + x_{\min} \]  
\[ + a_{i_d}(t) \left[ x_{i_d}(t) + v_{i_d}(t) \right] \]  
\[ 1 \leq i \leq N, 1 \leq d \leq D \]

Which, \( D \) is the population dimension, \( N \) is the population size, \( t \) is the current number of iterations, \( p_{i_d} \) is the best position of particle have experienced, \( p_{g_d} \) is the best position of the whole particle swarm have experienced, the search space is \( [x_{\min}, x_{\max}] \), \( a_{i_d} \) is a control coefficient, \( c \) is used to control the chaos degree of particles. In the beginning of the optimization iterations, chaos plays a major role. After iterated \( c \) times, the standard particle swarm optimization plays a major role. through the chaos optimization, the particles are close to the optimal solution, in accordance with the standard particle swarm optimization algorithm to update the particle’s speed and position, to be close to the optimal solution and speed up the convergence. When the standard particle swarm optimization plays a major role, if the particle is in a stable state, making chaos to play a major role with adjusting \( c \), and jumping out of the local optimal solution, the particles near to optimal solution by alternating between the chaotic state and stable state. By calculation, the ASP and CP of OCCFMS is almost no change in \( |x_{i_d}(t+1) - x_{i_d}(t)| < 10^{-6} \), it is in a stable state. If \( |x_{i_d}(t+1) - x_{i_d}(t)| < 10^{-6} \), adjusting \( c = c + t \), the particle is in a chaotic state.

The initial value of continuous chaotic system contains three variables, the small variation of each variable will cause that OCCFMS become completely different, it will cause the ASP and CP greatly changed. When the standard PSO updates the location, the three variables of the chaos initial values are updated as a whole to calculate \( p_{i_d} \) and \( p_{g_d} \). Considering the sensitivity of the chaos initial values, PSO is improved that each variable of the chaos initial values is individually search. After each variable is updated, \( p_{i_d} \) and \( p_{g_d} \) are calculated. In this improved PSO, three chaos initial values respectively are search and updated in each iteration. Although the search time increased, but the search precision is improved, and closer to the optimal solution.

**SIMULATION ANALYSIS AND DISCUSSION**

Suppose that the pulse width of OCCFMS is \( 10 \mu s \), the bandwidth is \( 500 MHz \). Figure 1 gives the average ambiguity function of OCCFMS based on Rossler system. As you can see from Figure 1, the ambiguity function of OCCFMS has an approximate thumbtack shape and a sharp main lobe, and has good range and velocity resolution, and well ability of anti-interference, so OCCFMS is a good performance of the pulse compression signal.
The continuous chaotic frequency modulation signal is applied to the design of MIMO radar signal, mainly due to the continuous chaotic frequency modulation signal is a kind of chaotic signal. The signal is random, and it is a kind of pseudo noise signal, so as to obtain a large number of orthogonality signals for MIMO radar. Figure 2 is average autocorrelation function and average cross correlation function of OCCFMS based on the Lorenz system, the bandwidth is $500 \text{MHz}$ and the pulse width is $10 \mu s$. MIMO radar need to accumulate multiple echoes to detect the target, the results show that we can get a better detection when the number of accumulations is 100 times. Therefore, average times are 100 for average autocorrelation function and average cross correlation function. As you can see from Figure 2, OCCFMS has better orthogonality and low ASP.

Figure 1: The average ambiguity function of OCCFMS

Figure 2: The autocorrelation function

Figure 3: The cross-correlation function
The probability distribution of each continuous chaotic system state variables are different, so we need to consider that the influence of the different continuous chaotic system on the performance of OCCFMS. TABLE 1 shows the average ASP and average CP of OCCFMS, Chaos frequency modulation signals are three common continuous chaotic signals, average times are 100, bandwidth is 500MHz, chaos initial value is random. As can be seen from TABLE 1, the ASP of orthogonal waveform based on Chua is very high, far higher than the other two kinds of signal waveform, not suitable for waveform design requirements, the former two signals have good orthogonality and low ASP, the autocorrelation properties of orthogonal waveform based on Rossler is better than the orthogonal waveform based on Lorenz, has low ASP, the cross-correlation peak is higher than the waveform based on Lorenz. The orthogonal waveform based on Rossler and the orthogonal waveform based on Lorenz have good autocorrelation and orthogonality, they all meet the design requirements of orthogonal waveform, can be applied to MIMO Radar.

**TABLE 1: Comparison between mean correlation for three continuous chaotic systems**

<table>
<thead>
<tr>
<th>Signal</th>
<th>Lorenz</th>
<th>Rossler</th>
<th>Chua</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASP (dB)</td>
<td>-22.48</td>
<td>-24.12</td>
<td>-9.34</td>
</tr>
<tr>
<td>CP (dB)</td>
<td>-26.59</td>
<td>-24.65</td>
<td>-22.07</td>
</tr>
</tbody>
</table>

Figure 4 and Figure 5 respectively show the histogram of ASP and CP of OCCFMS based on Lorenz. As can be seen from Figure, different initial values have a great impact on the ASP and the CP. There are a lot of signal which ASP and CP is low, is applied to the potential of MIMO radar, but the autocorrelation and orthogonality of some signal is poor. Therefore, the optimization of the chaos initial values is very necessary in the design of OFFCMS.

**Figure 4: Histogram of ASP**

**Figure 5: Histogram of CP**
Currently orthogonal waveform design generally use heuristic optimization algorithm, we can get the better performance of orthogonal waveform. However, with the increase in the number of orthogonal waveforms, it is difficult to optimize, the orthogonality of the wave will gradually decrease. OCCFMS is a chaotic signal with a pseudo-random, can have any number of orthogonal waveform in theory, the orthogonality of the waveform does not decrease with the number of waveforms increases. The autocorrelation function and cross-correlation function is affected by the initial value of the chaotic system, to optimize the initial value can get good performance of the orthogonal waveform. The objective function is the sum of multiple ASP and CP, if using standard heuristic optimization algorithm to optimize the objective function, there will be unequal distribution of ASP and CP, and some ASP and CP's value is too large, and the objective function is a nonlinear multimodal function, easy to appear "premature" phenomenon, resulting in optimal accuracy is not high.

| TABLE 2: The correlation peak of waveforms generated by the suggested method |
|----------------|----------------|----------------|----------------|----------------|----------------|
|                S1  | S2             | S3             | S4             | S5             | S6             |

| TABLE 3: The correlation peak of waveforms generated by PSO |
|----------------|----------------|----------------|----------------|----------------|----------------|
|                S1  | S2             | S3             | S4             | S5             | S6             |
| S5             -25.78 | -26.90         | -25.81         | -26.11         | -19.87         | -23.77         |
| S6             -25.48 | -27.01         | -25.57         | -24.46         | -23.77         | -20.12         |

TABLE 2 and TABLE 3 show the ASP and CP of OCCFMS generated by the proposed progressive chaos PSO algorithm and PSO algorithm. Compared with PSO, using the design method proposed in this paper, the ASP and CP of each waveform is very close, and the accuracy of optimization is higher, has the lower ASP and CP. Figure 6 shows the ASP and CP of the different number of waveforms, it can be seen from Figure 6, with the increase of the number of waveforms, ASP and CP of OCCFMS did not significantly change, and ASP and CP is lower, this verifies the correctness of the above conclusion and the method in this paper is effective.
CONCLUSION

OCCFMS has the initial value sensitivity, pseudo random, low probability of interception and anti interference performance. Because of chaos signal being introduced into OCCFMS, OCCFMS has the pseudo randomness, the autocorrelation sidelobe of waveform is reduced greatly; chaos signal has initial value sensitivity, the small variation of initial value will cause the signal is completely different, so OCCFMS has a quasi orthogonal. Comparison of the simulation results of three typical continuous chaotic system, the performance of OCCFMS based on Lorenz and Rossler are relatively good, can be used as a MIMO radar waveform.

The autocorrelation function and the cross-correlation function of OCCFMS are affected by the chaos initial value, to further improve the performance of orthogonal waveforms, and proposed a progressive chaotic particle swarm optimization algorithm to optimize the chaos initial value. The simulation results show that this method can effectively reduce the ASP and CP of waveforms, the ASP and CP of each waveform are very close, and with the increase of the number of orthogonal waveforms, orthogonality did not significantly reduce, can obtain any number of orthogonal waveform.

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