ISSN : 0974 - 7435 Volume 10 Issue 12

2014

BioTechnology

An Indian Journal

FULL PAPER

BTAIJ, 10(12), 2014 [6332-6340]

An optimal policy for two-stage imperfect production system with random machine unavailability

Shu Hui[1], Li Wei[2], Zhou Xideng[3]*

[1]Research Center of Cluster and Enterprise Development, School of Business Administration, Jiangxi University of Finance and Economics, Nanchang, (CHINA)

[2]School of Business Administration, Jiangxi University of Finance and Economics, Nanchang, (CHINA)

[3]Business Administration College, Nanchang Institute of Technology, Nanchang, (CHINA)

E-mail: chowxd@foxmail.com

ABSTRACT

This article studies a two-stage imperfect production system with machine unavailability. The production rates are finite; preventive maintenance of unreliable machines follows some stochastic processes with known probability distributions. Due to different preventive maintenance times for two stages, two cases for shortage cost are investigated, and two cost-minimization models are formulated. The total cost includes production cost, shortage cost, rework cost, maintenance cost and inventory carrying cost. Numerical examples are provided to illustrate the proposed models.

KEYWORDS

EPQ; Imperfect production process; Inventory; Machine unavailability.

INTRODUCTION

In an imperfect manufacturing process, the product quality is usually depends on the state of the production process. Production models with unreliable machines have been considered in many papers. Porteaus, Rosenblatt and Lee proposed models assuming imperfect quality[1,2]. Kim and Hong have done an extension of Rosenblatt and Lee by considering that an elapsed time until process shift is randomly distributed[3]. Chung and Hou further extended Kim and Hong to investigated allowable shortages in the imperfect production system[4]. Lin and Hou developed an imperfect production model by taking the restoration cost into account[5]. Lin and Lin assumed that the defective products are scraped with additional cost incurred[6]. Chiu proposed models with random machine breakdowns to determine production run time[7]. Lo et al. further extended Chung and Hou's model to consider inflation, and developed a manufacturer- retailer integrated inventory model[8]. Biswas and Sarker extended the above research to consider a lean production system with in-cycle rework and scrap[9]. Liao et al. developed an EPQ model by taking scrap, rework and machine breakdown maintenance into account. They introduced two types of preventive maintenance: perfect and non-perfect preventive maintenance[10].

Most of the research above focuses on machine breakdown in the production uptime period, but they do not give attention to machine unavailability. In reality, this circumstance often occurs because of some machine needing to be maintained, while others may breakdown randomly. There have been several intensive studies on production inventory model with preventive maintenance. Abboud et al. assumed random machine unavailability, no machine breakdown in the production period and allowable shortages[11]. Chung et al. extends the work of Abboud et al. by introducing EPQ for a deteriorating item model with stochastic unavailability of machine[12]. Wee and Widyadana further extended Chung et al. to consider rework process, preventive maintenance and shortage[13].

In the other hand, in practice, products are manufactured through multi-stage production process. A few authors have developed various multi-stags models in the literatures and consider that the two-stage models can be also used to approximate more complicated multi-stage production systems. Szendrovits proposed several two-stage production-inventory models where smaller lots are produced at one stage and one larger lot is processed at the other stage[14]. Kim's investigated two-stage production lot sizing problems considering various lot sizing depending on batch transfer and finite production rates between stages[15]. Hill extended Kim's model to provide an alternative way of solving the analysis which is easier to understand[16]. Darwish and Ben-Daya considered the effect of imperfect production processes involving variable the frequency of preventive maintenance[17].

In this paper, we investigate the two-stage production system with imperfect processes and machine unavailability. In a two-stage production system, products move from the first stage 1 to the final stage 2. When demand is greater than the stock during machine maintenance time, shortage will occur. Depending on different machine preventive maintenance time of each stage, we investigate two cases for shortage cost, and develop two cost-minimization models.

NOTATION AND ASSUMPTIONS

In this section, mathematical notation and the relevant assumptions are presented.

Notation

Q production quantity.

K_i setup cost in stage i, i=1,2.

K_i production cost per item in stage i, i=1,2

t_i production time in stage i.

t_{12} time period when there is no production and the inventory depletes in stage 1.

t_{i3} time period when there is no production and the inventory depletes in stage 2.

t_{ri} preventive maintenance time in stage i.

T_i total replenishment time.

p_i average production rate for stage i in producing Q units.

d demand rate.
C_{hi} holding cost per unit per unit time in stage i.
C_{0i} maintenance cost per unit time in stage i.
C_s shortage cost per unit per unit time.
R_{ci} rework cost per unit in stage i.
τ_i a random variable which is the elapsed time for the production process to shift to ‘‘out- of-control’’ in stage i.
$f_i(\tau_i)$ probability density function for τ_i and is assumed to be exponentially distributed with the parameter u_i.
N_i number of nonconforming items in i stage.
θ_1 probability of nonconforming items when the production process is in “in-control” state in stage 1; and $0<\theta_1<1$.
θ_2 probability of nonconforming items when the production process is in “out-of-control” state in stage 1; and $0<\theta_1<\theta_2<1$.
θ_3 probability of nonconforming items when the production process is in “in-control” state in stage 2; and $0<\theta_3<1$.
θ_4 probability of nonconforming items when the production process is in “out-of-control” state in stage 2; and $0<\theta_3<\theta_4<1$.
t_{ri} time required for repairing the mach ine in stage i.
$\phi_i(t_{ri})$ probability density function for t_{ri} and is assumed to be exponentially distributed with the parameter λ_i.

Assumptions

In addition, the following assumptions are used throughout this paper:

1. Imperfect (non-conforming) items are reworked immediately in a parallel manufacturing system.

2. The production system is separated into two stages. The average production rates of the two stages satisfy the condition $p_1 > p_2 > d$.

3. The process quality of two stages is independent.4. Shortages are allowed, but the backlogging of unsatisfied demand is not permitted.

Under the above assumptions and notations, the graphic representation of the inventory behavior for the two-stage imperfect production system can be shown as in Figure 1 and Figure 2.

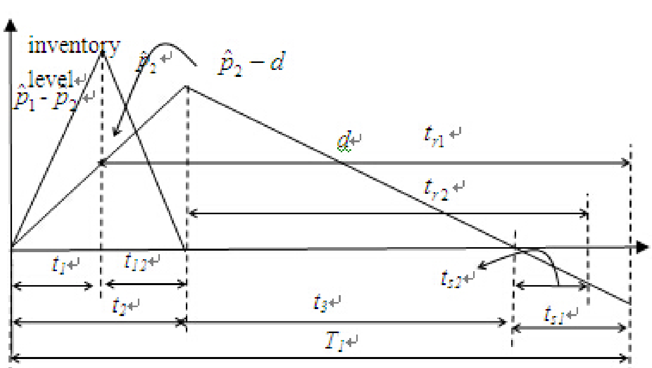

Figure 1 : Inventory level for $E(t_{r1}) \geq E(t_{r2})$

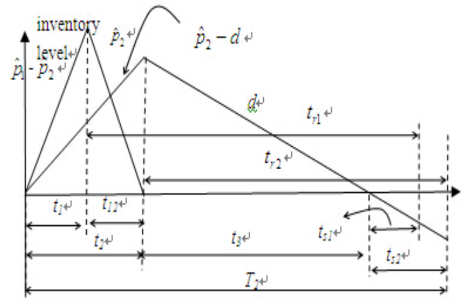

Figure 2 : Inventory level for $E(t_{r1}) < E(t_{r2})$.

FORMULATION OF THE MATHEMATICAL MODEL

Under the assumptions in the previous section, we can formulate the objective function as a cost minimization problem. The total cost for the two-stage imperfect process discussed in this paper includes production cost, inventory holding cost, maintenance cost, shortage cost and reworks cost. The formulations of these four costs are described in detail as follows.

The production cost (*PC*) per cycle is composed of the fixed setup cost *K* and the variable unit manufacturing cost $C_{pi}P_i t_i$. Namely,

$$PC = K_i + C_{pi}P_i t_i \tag{1}$$

From Figure 1 and Figure 2, the following parameters can be obtained directly.

$$t_2 = t_1 P_1 / P_2 \tag{2}$$

$$t_{12} = (P_1 - P_2)t_1 / P_2 \tag{3}$$

$$t_3 = (P_2 - d)P_1 t_1 / dP_2 \tag{4}$$

Now, we proceed to formulate the inventory holding cost each stage for our model.
The maximum inventory level during stage 1 is

$$(p_1 - p_2)t_1 \tag{5}$$

Then, inventory holding cost in stage1 is

$$EHC_1 = \frac{1}{2}C_{h1}(p_1 - p_2)t_1(t_1 + t_{12}) \tag{6}$$

Similarly, the maximum inventory level during stage 2 is equal to

$$(p_2 - d)t_2 \tag{7}$$

Then, inventory holding cost in stage 2 is

$$EHC_2 = \frac{1}{2}C_{h2}(p_2 - d)t_2(t_2 + t_3) \tag{8}$$

Therefore, the total inventory holding cost (EHC) in two-stage is obtained as

$$EHC = \frac{1}{2}C_{h1}(p_1 - p_2)t_1(t_1 + t_{12}) + \frac{1}{2}C_{h2}(p_2 - d)t_2(t_2 + t_3) \quad \textbf{(9)}$$

Before formulating the rework cost for two stage process (ERC), we have to find out the expected number of non-conforming items two-stage production process. The exponentially distributed random variable τ_i denotes the elapsed time for the production process to shift to the 'out-of- control' state in two-stage production. The number of non-conforming items N can be expressed as follows:

$$\sum_{i=1}^{2} E[N_i] = \theta_2 p_1 t_1 + (\theta_1 - \theta_2) p_1 \frac{1}{u_1}(1 - e^{-u_1 t_1}) + \theta_4 p_2 t_2 + (\theta_3 - \theta_4) p_2 \frac{1}{u_2}(1 - e^{-u_2 t_2}) \quad \textbf{(10)}$$

Therefore rework cost can be obtained as follows:

$$ERC = R_{c1}\theta_2\theta_2 p_1 t_1 + R_{c1}(\theta_1 - \theta_2) p_1 \frac{1}{u_1}(1 - e^{-u_1 t_1}) + R_{c2}\theta_4 p_2 t_2 + R_{c2}(\theta_3 - \theta_4) p_2 \frac{1}{u_2}(1 - e^{-u_2 t_2}) \quad \textbf{(11)}$$

From Figure.1, when the machine unavailability time t_{r1} for first stage is longer than the production down-time period t_{12}+ t_3, shortage occurs and the duration with shortage is t_{s1}; or when the machine unavailability time t_{r2} for second stage is longer than the production down-time period t_3, shortage takes place and shortage period is t_{s2}. Hence, comparing excepted shortage period $E[t_{s1}]$ and $E[t_{s2}]$, there are two cases to be discussed. Excepted shortage period $E[t_{s1}]$ and $E[t_{s2}]$ can be obtained as:

$$E(t_{s1}) = \frac{1}{\lambda_1} e^{-\lambda_1 (t_{12} + t_3)} \quad \textbf{(12)}$$

$$E(t_{s2}) = \frac{1}{\lambda_2} e^{-\lambda_2 t_3} \quad \textbf{(13)}$$

Case I : $E(t_{s1}) \geq E(t_{s2})$.

The shortage cost (ESC_1) for two-stage imperfect production system is dependent on $E(t_{s1})$, and can be obtained as

$$ESC_1 = \frac{1}{\lambda_1} e^{-\lambda_1 (t_{12} + t_3)} c_1 \quad \textbf{(14)}$$

The expected maintenance cost EMC can be expressed as

$$EMC = C_{01}\frac{1}{\lambda_1} + C_{02}\frac{1}{\lambda_2}. \tag{15}$$

By summing up the production cost, inventory holding cost, shortage cost maintenance cost and rework cost, we can have the following formulation for expected total cost:

$$ETC = PC + EHC + ESC_1 + ERC + EMC \tag{16}$$

The total replenishment time includes the positive inventory time $t_1 + t_2$ in stage 1, production down time t_3 in stage 2 and machine unavailability probability time in stage 1. The expected total replenishment time can be formulated as:

$$E(T_1) = t_1 + t_{12} + t_3 + \int_{t_{12}+t_3}^{\infty}(t_{r1} - t_{12} - t_3)\phi_1(t)dt_{r1} \tag{17}$$

The corresponding total cost per unit time $ETCUT_1$, can be obtained from Eq (16) by dividing by the expected cycle length $E_1(T)$. Namely,

$$ETCUT_1(t_1) = ETC / E(T_1) \tag{18}$$

Substituting Equation (2), (3) and (4) and into Equation (18), after simplification we have

$$\begin{aligned} ETCUT_1(t_1) = {} & \frac{1}{M_5}\{K_1 + K_2 + \frac{C_{01}}{\lambda_1} + \frac{C_{02}}{\lambda_2} + P_1(C_{p1} + C_{p2})t_1 + C_S\frac{1}{\lambda_1}e^{-\lambda_1(\frac{P_1}{d}-1)t_1} \\ & + \frac{1}{2P_2}[C_{h1}(p_1 - p_2)P_1 + C_{h2}(p_2 - d)\frac{P_1^2}{d}]t_1^2 + (R_{c1}\theta_2 p_1 + R_{c2}\theta_4 P_1)t_1 \\ & + R_{c1}(\theta_1 - \theta_2)P_1\frac{1}{u_1}(1 - e^{-u_1 t_1}) + R_{c2}(\theta_3 - \theta_4)P_2\frac{1}{u_2}(1 - e^{-\frac{u_2 P_1}{P_2}t_1})\} \end{aligned} \tag{19}$$

Differentiate (19) with respect to t_1, one has:

$$\begin{aligned} \frac{dETCUT_1(t_1)}{dt_1} = {} & \frac{\left[P_1(C_{p1} + C_{p2})t_1 - C_S(\frac{P_1}{d} - 1)e^{-\lambda_1(\frac{P_1}{d}-1)t_1} + M_1 t_1 + M_2 + M_3 u_1 e^{-u_1 t_1} + M_4\frac{u_2 P_1}{P_2}e^{-\frac{u_2 P_1}{P_2}t_1}\right]}{M_5} \\ & - \frac{\left[\begin{array}{l} K_1 + K_2 + \frac{C_{01}}{\lambda_1} + \frac{C_{02}}{\lambda_2} + P_1(C_{p1} + C_{p2})t_1 + C_S\frac{1}{\lambda_1}e^{-\lambda_1(\frac{P_1}{d}-1)t_1} \\ + M_1 t_1^2 + M_2 t_1 + M_3(1 - e^{-u_1 t_1}) + M_4(1 - e^{-\frac{u_2 P_1}{P_2}t_1}) \end{array}\right](\frac{P_1}{d} - (\frac{P_1}{d} - 1)e^{-\lambda_1(\frac{P_1}{d}-1)t_1})}{M_5^2} \end{aligned} \tag{20}$$

where: $M_1 = \frac{1}{2P_2}\left[C_{h1}(p_1 - p_2)P_1 + C_{h2}(p_2 - d)\frac{P_1^2}{d}\right]$; $M_2 = (R_{c1}\theta_2 p_1 + R_{c2}\theta_4 P_1)$;

$M_3 = R_{c1}(\theta_1 - \theta_2)P_1\frac{1}{u_1}$; $M_4 = R_{c2}(\theta_3 - \theta_4)P_2\frac{1}{u_2}$; $M_5 = \frac{P_1}{d}t_1 + \frac{1}{\lambda_1}e^{-\lambda_1(\frac{P_1}{d}-1)t_1}$.

The optimal T_1 value can be derived by solving (20). The detailed calculation is shown in Appendix 1.

Case II: $E(t_{s1}) < E(t_{s2})$.

Similarly, the expected shortage cost ESC_2 for two-stage production system is obtained as

$$ESC_2 = \frac{1}{\lambda_2} e^{-\lambda_2 t_3} \tag{21}$$

The total replenishment time includes the positive inventory time in stage 2 and machine unavailability probability time in stage 2. The expected total replenishment time can be formulated as:

$$E(T_2) = t_2 + t_3 + \frac{1}{\lambda_2} e^{-\lambda_2 t_3} \tag{22}$$

Similarly, the total expected cost per unit produced is

$$\begin{aligned} ETCUT_2(t_1) = (&PC + EMC + ESC_2 \\ &+ EHC + ERC) / E(T_2) \end{aligned} \tag{23}$$

Substituting Equation (2), (3) and (4) and into Equation (18), after simplification one has

$$\begin{aligned} ETCUT_2(t_1) = \frac{1}{M_6}\{&K_1 + K_2 + \frac{C_{01}}{\lambda_1} + \frac{C_{02}}{\lambda_2} + P_1(C_{p1} + C_{p2})t_1 + C_S \frac{1}{\lambda_2} e^{\frac{-\lambda_2 (P_2-d) P_1 t_1}{dP_2}} + \frac{1}{2P_2}[C_{h1}(p_1 - p_2)P_1 + C_{h2}(p_2 - d)\frac{P_1^2}{d}]t_1^2 \\ &+ (R_{c1}\theta_2 p_1 + R_{c2}\theta_4 P_2)t_1 + R_{c1}(\theta_1 - \theta_2)P_1 \frac{1}{u_1}(1 - e^{-u_1 t_1}) + R_{c2}(\theta_3 - \theta_4)P_2 \frac{1}{u_2}(1 - e^{-\frac{u_2 P_1}{P_2}t_1})\} \end{aligned} \tag{24}$$

Optimal t_1 value can be obtained by the derivative of the total cost per unit time with respect to t_1 and by setting the result equal to zero. One has:

$$\begin{aligned} \frac{dETCUT_2(t_1)}{dt_1} = &\frac{\left[P_1(C_{p1} + C_{p2})t_1 - C_S(\frac{P_1}{d} - 1)e^{-\lambda_1 \frac{(P_2-d)P_1}{dP_2}t_1} + M_1 t_1 + M_2 + M_3 u_1 e^{-u_1 t_1} + M_4 \frac{u_2 P_1}{P_2} e^{-\frac{u_2 P_1}{P_2}t_1}\right]}{M_6} \\ &- \frac{\left[\begin{aligned}&K_1 + K_2 + \frac{C_{01}}{\lambda_1} + \frac{C_{02}}{\lambda_2} + P_1(C_{p1} + C_{p2})t_1 + C_S \frac{1}{\lambda_1} e^{-\lambda_1 \frac{(P_2-d)P_1}{dP_2}t_1} \\ &+ M_1 t_1^2 + M_2 t_1 + M_3(1 - e^{-u_1 t_1}) + M_4(1 - e^{-\frac{u_2 P_1}{P_2}t_1})\end{aligned}\right](\frac{P_1}{d} - (\frac{P_1}{d} - 1)e^{-\lambda_1 \frac{(P_2-d)P_1}{dP_2}t_1})}{M_6^2} = 0 \end{aligned} \tag{25}$$

where: $M_6 = \frac{P_1}{d} t_1 + \frac{1}{\lambda_1} e^{-\lambda_1 \frac{(P_2-d)P_1}{dP_2}t_1}$

The optimal t_1 value can be derived by solving (25) since the total cost per unit time is a convex function. The detailed calculation is shown in Appendix 2.

NUMERICAL EXAMPLE

In this section, we provide two numerical examples to illustrate the features of the proposed model. The values of the parameters are shown in TABLE 1.

TABLE 1 : The values of the parameters.

K_1	**700**	P_2	**60**	C_{01}	**10**	C_{h2}	**0.2**
K_2	500	C_s	0.5	C_{02}	10	θ_1	0.05
C_1	1	R_{c1}	0.2	u_1	0.1	θ_2	0.65
C_2	2	R_{c2}	0.5	u_2	0.2	θ_3	0.05
d	50	C_{h1}	0.2	P_1	70	θ_4	0.65

Example 1. In order to illustrate the above Case I, let λ_1=0.2 and λ_2=0.5, it is obvious $E(t_{s1}) > E(t_{s2})$. Then we can obtain the optimal solution $t_1 = 8.605$. The $ETCUT_1(t_1)$ are 233.9.
Example 2. We consider the Case II, and let λ_1=0.5 and λ_2=0. 2, it is easy to obtain $E(t_{s1}) < E(t_{s2})$. The optimal solutions of t_1,and $ETCUT_2(Q^*)$ are 19.11 and 250 respectively.

CONCLUSION

In this paper, we investigate a two-stage imperfect production system with machine unavailability. Two cases for shortage incurred by different machine unavailability have been discussed. We minimize the expected total cost of the production system through optimal determination of the production time. Numerical examples are provided to illustrate the features of the model and sensitivity analyses are performed to examine the impact of the key parameters' changes on the decision variable and objective function. The results show that the expected total cost is highly sensitive with respect to production cost, production cost, rework cost, the maintenance cost, holding cost and probability of nonconforming items in "out-of-control" state.

ACKNOWLEDGMENTS

This work was supported by Social Science Foundation of People's Republic of China under grant numbers (Grant No: 14AGL003).

APPENDIX 1

Setting $t_1 = 0$, and after some simplification, one has:

$$\frac{\partial^2 ETCUT_1(t_1)}{\partial t_1^{\,2}} = (\frac{1}{\lambda_1})^2 \{ [C_S(\frac{P_1}{d}-1)^2 + 2\frac{M_1}{\lambda_1} - \frac{M_3 u_1^{\,2}}{\lambda_1} - \frac{M_4}{\lambda_1}(\frac{u_2 P_1}{P_2})^2] - 2[P_1(C_{p1}+C_{p2}) - C_S(\frac{P_1}{d}-1) + M_2 + M_3 u_1 + \frac{M_4 u_2 P_1}{P_2}] + (2-(\frac{P_1}{d}-1)^2)(\lambda_1 K_1 + \lambda_1 K_2 + C_{01} + C_{02} + C_S) \} \quad \text{(A1.1)}$$

Equation (A1.1) is non-increasing and it is convex when (A1.2) is positive. One can how that (A1.2) is positive when:

$$C_S > \frac{\left[M_3 u_1^{\,2} \frac{1}{\lambda_1} + M_4 \frac{1}{\lambda_1}(\frac{u_2 P_1}{P_2})^2 + 2P_1(C_{p1}+C_{p2}) + 2M_2 + 2M_3 u_1 + 2\frac{M_4 u_2 P_1}{P_2} - \left\{ 2 - (\frac{P_1}{d}-1)^2 \right\} [\lambda_1 K_1 + \lambda_1 K_2 + C_{01} + C_{02}] - 2M_1 \frac{1}{\lambda_1} \right]}{\left[(\frac{P_1}{d}-1)^2 + 2(\frac{P_1}{d}-1) + (2-(\frac{P_1}{d}-1)^2 \right]} \quad \text{(A1.2)}$$

Setting $t_1 = \infty$, Equation (A1.1) is equal to zero. one can conclude that $ETCUT_1(t_1)$ is convex.
Appendix 2
Setting $t_1 = 0$, and after some simplification, one has:

$$\frac{\partial^2 ETCUT_1(t_1)}{\partial t_1^{\,2}} = (\frac{1}{\lambda_1})^2\{[C_S(\frac{(P_2-d)P_1}{dP_2})^2 + 2M_1\frac{1}{\lambda_1} - M_3u_1^{\,2}\frac{1}{\lambda_1} - M_4\frac{1}{\lambda_1}(\frac{u_2P_1}{P_2})^2] - 2[P_1(C_{p1}+C_{p2}) - C_S\frac{(P_2-d)P_1}{dP_2} + M_2 + M_3u_1 + \frac{M_4u_2P_1}{P_2}] + [2-(\frac{(P_2-d)P_1}{dP_2})^2](\lambda_1K_1+\lambda_1K_2+C_{01}+C_{02}+C_S)\} \tag{A2.1}$$

Equation (A2.1) is non-increasing and it is convex when (A2.2) is positive. One can how that (A2.2) is positive when:

$$C_S > \frac{\left[\begin{array}{l} M_3u_1^{\,2}\frac{1}{\lambda_1} + M_4\frac{1}{\lambda_1}(\frac{u_2P_1}{P_2})^2 + 2P_1(C_{p1}+C_{p2}) + 2M_2 + 2M_3u_1 \\ +2\frac{M_4u_2P_1}{P_2} - \left\{2-(\frac{P_1}{d}-1)^2\right\}[\lambda_1K_1+\lambda_1K_2+C_{01}+C_{02}] - 2M_1\frac{1}{\lambda_1} \end{array}\right]}{\left[(\frac{(P_2-d)P_1}{dP_2})^2 + \frac{2(P_2-d)P_1}{dP_2} + 2 - (\frac{(P_2-d)P_1}{dP_2})^2\right]} \tag{A2.2}$$

Setting $t_1 = \infty$, Equation (A2.1) is equal to zero. one can conclude that $ETCUT_2(t_1)$ is convex.

REFERENCES

[1] E.L.Porteus; Optimal lot sizing, process quality improvement and setup cost reduction. Operations Research, **34**, 137–144 **(1986)**.

[2] M.J.Rosenblat, H.L.Lee; Economic production cycles with imperfect production processes. IIE Transactions, **18**, 48–55 **(1986)**.

[3] C.H.Kim, Y.Hong; An optimal production run length in deteriorating production processes. International Journal of Production Economics, **58**, 183–189 **(1999)**.

[4] K.J.Chung, K.L.Hou; An optimal production run time with imperfect production processes and allowable shortages. Computers and Operations Research, **30**, 483– 490 **(2003)**.

[5] L.C.Lin, K.L.Hou; EMQ model with maintenance actions for deteriorating production system. International journal of information and management, **16**, 53–65 **(2005)**.

[6] G.C.Lin, H.D.Lin; Determining a production run time for an imperfect production-inventory system with scrap. Journal of scientific and industrial research, **66**, 724–735 **(2007)**.

[7] S.W.Chiu; Production run time problem with machine breakdowns under AR control policy and rework. Journal of scientific and industrial research, **66**, 979–988 **(2007)**.

[8] S.T.Lo, H.M.Wee, W.C.Huang; An integrated production-inventory model with imperfect production processes and Weibull distribution deterioration under inflation. International Journal of Production Economics, **106**, 248–260 **(2007)**.

[9] P.Biswas, B.R.Sarker; Optimal batch quantity models for a lean production system with in-cycle rework and scrap. International Journal of Production Research, **46**, 6585–6610 **(2008)**.

[10] G.L.Liao, Y.H.Chen, S.H.Sheu; Optimal economic production quantity policy for imperfect process with imperfect repair and maintenance. European Journal of Operational Research, **195**, 348–357 **(2009)**.

[11] N.E.Abboud, M.Y.Jaber, N.A.Noueihed; Economic lot sizing with the consideration of random machine unavailability time. Computers and Operations Research, **27**, 335–351 **(2000)**.

[12] C.J.Chung, G.A.Widyadana, H.M.Wee; Economic production quantity model for deteriorating inventory with random machine unavailability and shortage. International Journal of Production Research, **49**, 883–902 **(2011)**.

[13] H.M.Wee, G.A.Widyadana; Economic production quantity models for deteriorating items with rework and stochastic preventive maintenance time. International Journal of Production Research, forthcoming, **(2011)**.

[14] A.Z.Szendrovits; Non-integer optimal lot size ratios in two-stage production/inventory systems. International Journal of Production Research, **21**, 323–336 **(1983)**.

[15] D.Kim; Optimal two-stage lot sizing and inventory batching policies. International Journal of Production Economics, **58**, 221–234 **(1999)**.

[16] R.M.Hill; On optimal two-stage lot sizing and inventory batching policies. International Journal of Production Economics, **66**, 149–158 **(2000)**.

[17] M.A.Darwish, M.Ben-Daya; Effect of inspection errors and preventive maintenance on a two-stage productioninventory system. International Journal of Production Economics, **107**, 301–313 **(2007)**.