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Study of Cosmological Bias of Galaxies in Quasi Linear Regime

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Abstract

Power spectrum of density variation of matter dominated era is calculated to prove that structure formation in the universe starts in the quasi-linear regime. Dimensionless power spectra for 1-halo and 2-halo for baryonic matter are calculated and compared with the 1-halo and 2-halo power spectra for dark matter. The higher values obtained for baryonic matter indicates over dense regions of baryonic matter leading to collapse to form large scale structures. The power spectrum values lead to cosmological bias values in unperturbed and perturbed universe. The obtained bias values being approaching unity indicate formation of large scale structure of galaxies starts in the quasi-linear regimes.

***Keywords:** Cosmological bias; Large scale structure; Quasi linear regime; Power spectrum; Dark matter; Baryonic matter*

Introduction

The concept of galaxies being biased tracers with respect to dark matter was introduced by Kaiser Nick [1]. Bias, which depends on the length scale, refers to the ratio of the baryonic matter density to the underlying dark matter density. Dark matter is invisible, as it does not emit radiation and its presence is inferred by its gravitational effect. In order to study bias, knowledge of the inhomogeneity in density is required. Models such as the spherical collapse model, halo model, local bias model and linear and non-linear perturbation models are required to obtain density contrast of baryonic matter and dark matter. On large scale, the universe is homogeneous and isotropic and hence does not lead to bias. But bias is prominent in small scale. Gravity equally affect both baryonic matter and dark matter. Baryonic matter is acted upon by some other forces like radiation pressure, magnetic field etc. While dark matter only affected by gravity and behaves like a potential well inside the halo into which the baryonic matter is going on accumulating to form small structures first further leading to formation of large scale structures. So the distribution of galaxies and the dark matter within a given halo can be used to estimate the statistical properties of the dark matter density in the quasi-linear regime, where the density fluctuation (δ) approaches unity. These galaxies are the biased tracers of the dark matter in the halo. Under these conditions the matter is in virial equilibrium, where the desired physical properties of the halo can be estimated [2]. The spherical collapse model was first studied by Gunn and Gott [3]. This is the simplest non-trivial model for the way an object like galaxy or cluster of galaxies breaks away from the general expansion. In the model, the universe is spherically symmetric about one spot and the matter is an ideal fluid with zero pressure. At the initial epoch everything is expanding smoothly as the universe is expanding though gravity acts there. In this paper, dimensionless power spectra for 1-halo and 2-halo for baryonic matter are calculated and compared with the 1-halo and 2-halo power spectra for dark matter. The higher values obtained for baryonic matter indicate

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over dense regions of baryonic matter leading to form large scale structures. The power spectrum values lead to cosmological bias values in unperturbed and perturbed universe. The obtained bias value being approaching unity indicates the start of formation of large scale structures of galaxies in the quasi-linear regime.

Methods

Analysis of halo power spectrum

Fry and Gatzanaga [4] assume that the density contrast in halo distribution, δ_h can be expressedas a non-linear function of the local density contrast of dark matter (DM) , δ_m:

$$\delta_h = b_0 + b_1\delta_m + \frac{b_2}{2!}\,{\delta_m}^2 + \cdots \text{ (1)}$$

In this section we evaluate the bias values (b_0, b_1, b_2) under certain assumptions, which follow and show using these values that in the quasi-linear regime, the clustering of galaxies is more due to many forces like gravity, magnetic field, radiation pressure etc., than clustering of dark matter which is only affected by gravity.

We assume an initial spectrum of the density fluctuation field as

$$P_0(\mathrm{k}) = \frac{\mathrm{A}}{\mathrm{k}^{3/2}} \text{ (2)}$$

In accordance with [3], where the authors have considered this spectrum as a possible shape of the power spectrum on cluster-like scales in cold dark matter (CDM) models. In terms of variance $\sigma^2(R)$ on scale R, the dimension less power spectrum as per [3]

$${\Delta_0}^2(\mathrm{k}) = \frac{15\delta_{sc}^2}{16\sqrt{\pi}}(kR_*)^{\frac{3}{2}} = \frac{\mathrm{k}^3}{2\pi^2}P_0(\mathrm{k}) \text{ (3)}$$

Where, $\delta_{sc}^2 = \sigma^2(R)$. δ_{sc} is the critical density for an object to collapse.

We assume that the density around a virialized halo is given by [4]

$$\frac{\rho(r|m)}{\bar{\rho}} = \frac{2\Delta_{\mathrm{nl}}}{3\pi}\mathrm{C}^3(m)\frac{y^{-2}}{1+y^2} \text{ (4)}$$

Where $\mathrm{y} = \frac{r}{r_s}$, r_s is the core radius of the halo profile, C is the halo concentration parameter, Δ_{nl} is the non-linear dimensionless power spectrum. For which the normalized Fourier transform is

$$u(k|m) = \frac{1-e^{-kr_s}}{kr_s} \text{ (5)}$$

Where $r_s = \left(\frac{m}{m_*}\right)^{\gamma}$, γ is a numerical coefficient independent of scale. The dimensionless 1-halo and 2-halo power spectrum for massive and less concentrated halo, for $\gamma = \frac{1}{6}$, are given by

$$\Delta_{1h}^2(\mathrm{k}) = \frac{2\Delta_{\mathrm{nl}}}{3\pi}{\mathrm{C}_*}^3\boldsymbol{\kappa}\left(1 + \frac{1}{\sqrt{1+4\boldsymbol{\kappa}}} - \frac{2}{\sqrt{1+2\boldsymbol{\kappa}}}\right) \text{ (6)}$$

$$\Delta_{2h}^2(\mathrm{k}) = B^2(k)\Delta_0^2(\mathrm{k}) = B^2(k)\frac{15\delta_{sc}^2}{16\sqrt{\pi}}\left(\Delta_{nl}^{1/3}\mathrm{C}_*\right)^{3/2}\kappa^{3/2} \text{ (7)}$$

B(k) is known as bi-spectrum which is given by

$$B(k) = \frac{1}{\boldsymbol{\kappa}}\left[\frac{2}{\delta_{sc}} - 1 + \sqrt{2\boldsymbol{\kappa}}\left(1 - \frac{1}{\delta_{sc}}\right) - \frac{1}{\delta_{sc}\sqrt{1+2\boldsymbol{\kappa}}}\right] \text{ (8)}$$

Where $\delta_{sc}^2 = \sigma^2(R)$. δ_{sc} is the critical density for an object to collapse. Dimensionless power spectrum for 1-halo term and 2-halo term for more massive and more concentrated halo have been derived analytically [4] which are as follows:

$$\Delta_{1h}^2(\mathrm{k}) = \frac{2\Delta_{\mathrm{nl}}}{\pi} \mathrm{C}_*{}^3 \boldsymbol{\kappa}^3 \left(\frac{\mathbf{1-e^{-\kappa}}}{\boldsymbol{\kappa}}\right)^{\mathbf{2}} \quad (9)$$

$$\Delta_{2\mathrm{h}}{}^2(\mathrm{k}) = \left(\frac{1-e^{-\kappa}}{\boldsymbol{\kappa}}\right)^2 \Delta_0{}^2(\mathrm{k}) \quad (10)$$

Equation-4 and 5 explains the variation of concentration parameter (C) with halo mass (m). Which further contributes to the total power. At small scale most of the power comes from 2-halo terms. At large k 1-halo term dominates the power.

Where $\boldsymbol{\kappa}$ is called as kappa and is given by

$$\boldsymbol{\kappa} = kr_s = \frac{kR_*}{\mathrm{C}_* \Delta_{nl}^{\frac{1}{3}}} \left(\frac{\mathrm{m}}{\mathrm{m}_*}\right)^{\gamma+\frac{1}{3}} \quad (11)$$

Kappa is a distance in the unit of scale radius. The non-linear dimensionless power spectrum $\Delta_{nl} = \left(\frac{R_{ta}}{r_{vir}}\right)^3$=8. Assuming halos to be virialized at half the turn around time $r_{vir} = \frac{R_{ta}}{2}$. R and m are the initial size and mass of the halo. R_* *and* m_* are the characteristic size and mass of the halo respectively [5-7]. In Einstein de-Sitter Cosmology, $\delta_{\mathrm{sc}}(\mathrm{z}) = 1.686$, and for initial power spectrum $\sigma^2(m)$=0.5, at the turn around epoch, since

$$\nu = \frac{\delta_{sc}{}^2}{\sigma^2(m)} = \left(\frac{m}{m_*}\right)^{\frac{1}{2}} \quad (12)$$

We have $\frac{\mathrm{m}}{\mathrm{m}_*} = 32.312$. Where ν defines the characteristic mas of the halo. Now straightforwardly we have

$$\frac{R}{R_*} = \left(\frac{\mathrm{m}}{\mathrm{m}_*}\right)^{\frac{1}{3}} \quad (13)$$

Or, $R_* = 0.313R$. The halo concentration parameter in NFW models are taken to be in the range 4-40 [8]. Choosing the value of $\mathrm{C}(m)$=4 [1]

$$\mathrm{C}_* = \frac{\mathrm{C}(m)}{\left(\frac{m_*}{m}\right)^{\gamma}} = 1.255 \quad (14)$$

Where γ is a numerical coefficient independent of scale. The value of $\gamma = -\frac{1}{3}$ for more massive and more concentrated halo. From equation 6, Kappa κ = 0.783 . The variance $\sigma^2(R)$ is given by [4]

$$\sigma^2(R) = \frac{16\sqrt{\pi}}{15} \frac{\mathrm{A}}{2\pi^2} \mathrm{R}^{-\frac{3}{2}} \quad (15)$$

The value of A can be obtained as

$$A = \frac{0.5\times 15}{16} \frac{2\pi^2}{\sqrt{\pi}} \mathrm{R}^{\frac{3}{2}} \quad (16)$$

$\Delta_0^2(\mathrm{k})$ from equation 3 can evaluated by using $k = \frac{2\pi}{R}$ as $\Delta_0^2(\mathrm{k}) = 4.165$. $\Delta_{1h}^2(\mathrm{k})$=2.323 using equation-4 and $\Delta_{2h}^2(k)$=2.002 using equation-5. We have

$$\Delta_h^2 = \Delta_{1h}^2 + \Delta_{2h}^2 = 4.325 \quad (17)$$

The density contrast in halo matter power spectrum is then

$$\delta_h = \sqrt{(2\pi^2 \times \Delta_h{}^2)} = 9.237 \quad (18)$$

Analysis of dark matter power spectrum

For an Einstein de-sitter universe the solution to the density field

$\delta(x)$ can be expressed in terms of the Fourier transform of the initial fluctuation $\delta(k)$ [5]

$$\delta^{(1)}(x) = \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}}\, \delta(k) e^{ik.x} \quad (19)$$

$$\langle\delta^{(1)}(x)\rangle^2 = \int \frac{d^3k_1}{(2\pi)^{\frac{3}{2}}} \frac{d^3k_2}{(2\pi)^{\frac{3}{2}}} \delta(k_1)\delta(k_2) e^{ik_1.x_1}\, e^{ik_2.x_2} \quad (20)$$

$$= \int d^3k\ \ P(k) = \int d^3k\ \ P(k)\, W^2(kR) = \sigma^2(R) \quad (21)$$

The window function is $W^2(kR) = 1$ for kR$\ll 1$, and the delta function $\delta_D(k_1 + k_2) = 1$ for $k_1 = -k_2$. For R=8$h^{-1}MPC$, k=1$h\ MPC^{-1}$, $\langle\delta^{(1)}(x)\rangle^2 = 0.5$. The first order density spectrum for matter is $P_{1m}(k) = |\delta^1(k)|^2 = 0.5$. The dimensionless power spectrum for matter is

$$\Delta^2_{1m}(k) = \frac{k^3}{2\pi^2} P_{1m}(k) = 0.025 \quad (22)$$

The unit of power spectrum is $(h^{-1}MPC)^3$.

The second order density field of matter is

$$\delta^{(2)}(x) = \int \frac{d^3k_1}{(2\pi)^{\frac{3}{2}}} \frac{d^3k_2}{(2\pi)^{\frac{3}{2}}} \delta(k_1)\, \delta(k_2) \times\ N^{(p)}(k_1, k_2)\, e^{i(k_1+k_2).x} \quad (23)\ \text{Where}$$

$$N^{(p)}(k_1, k_2) = \frac{k_1.k_2}{(k_2)^2} = \frac{k_1.k_2 cos\theta}{(k_2)^2} = cos\theta$$

In terms of the loop function [9-11], $\delta^{(2)}(x)$ can be written as

$$= \int d^3k\ \ P(k)$$

$$= \int d^3k\ \ P(k)\, W^2(kR) = \sigma^2(R) = 0.5$$

$$\langle\delta^{(2)}(x)\rangle^2 = (0.5)^2 = 0.25 \quad (24)$$

Now 2nd order density power spectrum for matter is $P_{2m}(k) = |\delta^2(k)|^2 = 0.25$ and hence dimensionless power spectrum for matter is $\Delta^2_{2m}(k) = \frac{k^3}{2\pi^2} P_{2m}(k)(h^{-1}MPC)^3$

$$= \frac{0.25}{2\pi^2} = 0.012 \quad (25)$$

Calculation of bias

It is the square root of ratio of halo matter power spectrum to the dark matter power spectrum.

$$\text{Bias } b_1 = \sqrt{\frac{\Delta_{1h}{}^2}{\Delta_{1m}{}^2}} = 9.639 \quad (26)$$

$$\text{Bias } b_2 = \sqrt{\frac{\Delta_{2h}{}^2}{\Delta_{2m}{}^2}} = 12.916 \quad (27)$$

Using equation (1), b_0 can evaluated as

$$b_0 = 0.809 \quad (28)$$

TABLE 1. **$[\Delta\_h]^2$ represents dimensionless power spectrum of Halo profile and $[\Delta\_m]^2$ represents dimensionless matter power Spectrum, b represents bias values.**

Sl. No.	Δ_h^2	Δ_m^2	b	b_0
1	2.323	0.025	9.639	0.809
2	2.002	0.012	12.916	

We calculated bias in small scales as most of the power comes from both the halo terms **(TABLE 1 and FIG.1)**.

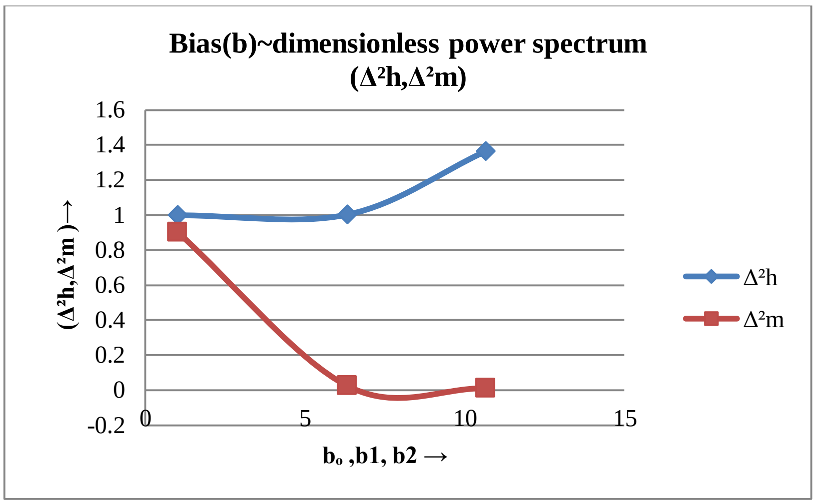

FIG. 1. **It represents the graph of power spectrum versus bias. It compares the value of bias of power spectrum of halo profile and that of dark matter.**

Result and Discussion

The values of bias has been studied analytically and found as $b_0 = 0.809$, $b_1 = 9.639$, $b_2 = 12.916$. The value of b_0 clearly indicates that clustering of baryonic matter and clustering of dark matter are almost equal. That means this is the quasi-linear regime where large scale structure formation starts and being transformed from linear to nonlinear regime which is indicated by the increased value of bias values b_1 and b_2. From the graph one can observe that with increase of bias, the halo matter power spectrum increases. But the dark matter power spectrum decreases with increase of bias. That means the clustering of galaxies are more under the action of different forces including gravity than the clustering of dark matter under the force of gravity only to form the large scale structures [12].

Conclusion

We have calculated the bias values b_0, b_1, b_2 for baryonic matter and dark matter using the 1-halo and 2-halo power spectra. From our study, we find that the dimensionless power spectrum for baryonic matter increases at a faster rate than that for dark matter. In equation (1), in Taylor expansion of density perturbation the cosmological bias values increases. The lowest value b_0 approaching 1, indicates the quasi-linear regime where the baryonic matter (under the action of gravity as well as radiation force and magnetic field etc) accumulates into the potential well of dark matter (affected by gravity only) to form small structures first. Which further leads to large scale structures we see today. The largest values of bias indicate the non-linearity domain where the formation of large scale structure formation is more complex and can't studied by analytic method. It can be obtained by simulation only.

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