Int. J. Chem. Sci.: 9(1), 2011, 44-51
ISSN 0972-768X

STUDY OF BCS-BEC CROSSOVER PHYSICS AND EVALUATION OF ENERGY GAP PARAMETERS AND CHEMICAL POTENTIAL FROM BCS-THEORY

L. N. SINGH[*] and L. K. MISHRA

Department of physics, D. N. College, Masaurhi, (Patna) (Bihar) INDIA
Department of Physics, Magadh University, BODH-GAYA – 824234 (Bihar) INDIA

ABSTRACT

Using the theoretical formalism of C. Regal et al. PRL (2004), we have studied the BCS-BEC crossover regime. We have evaluated the value of energy gap parameter and chemical potential for the crossover as a function of dimensionless parameters $|K_F a_s|^{-1}$ from BCS-theory. Our theoretical result indicates that the crossover occurs in the very small region of the parameter-1< $|K_F a_s|^{-1} < 1$.

Key words: BCS-BEC crossover, Feshbach resonance, Pseudo-gap, Chemical potential, Energy gap parameter, Evaporative cooling.

INTRODUCTION

The techniques used to create alkali BEC were applied to other class of quantum particle, Fermions. Earlier alkali atoms such as ^{87}Rb (composite Bosons) were cooled as a gas down to nanokelvin temperature via laser cooling and evaporative cooling.[1-3] At these temperatures, the thermal de-Broglie wavelength of the particles becomes in order of the inter particle spacing in the gas and Bose-Einstein condensate is formed. Experiments also observed that condensate behaves as matter wave[4] and verified the superfluid nature of the condensates.[5,6]

To create a Fermi gas of atoms, experiments applied the same cooling technique as those used to achieve BEC in ^{87}Rb or ^{23}Na. For Fermi gas of atoms, one has two stable alkali atoms ^{40}K and ^{6l}Li with an odd number of electrons, protons, and neutrons. The first gas of Fermionic atoms to enter the quantum degenerate regime was created at JLLA[7] in 1999 using ^{40}K. The observation in this experiment was not a phase transition as in the case of Bose gas but rather the presence of more and more energy that would be expected classically

[*]Author for correspondence; E-mail: muphysicslkm@gmail.com

as the Fermi gas was cooled below the Fermi temperature. Many more Fermi gas experiments using a variety of cooling techniques then followed.[8-16]

The next goal after the creation of a normal Fermi gas of atoms was to form a superfluid in a paired Fermi gas. In conventional superconductors s-wave pairing occurs between spin-up and spin-down electrons. The hope was that s-wave pairing could similarly occur with the creation of two components atomic gas with an equal Fermi energy for each components. Such a two- component gas can be realized using an equal mixture of alkali atoms in two different hyperfine spin state. The idea was that BCS state would appear if the temperature of this two component gas were cold enough and the interaction between Fermions is attractive and large enough. However, for typical interactions, the temperature required to reach a true BCS state were far too low compared to achievable temperature to imagine creating Cooper pairs. Stoof et al.[17] noted that the interaction between ^{6}Li atoms were large compared to typical values of the scattering length ($|a_s|=2000\ a_o$) as well as attractive bringing the BCS transition temperature closer to the realistic temperature.[18,19] It was then realized that a type of scattering resonance known as Feshbach resonance could allow arbitrary changes is the interaction strength. Theories were developed that explicitly treated the case where the interactions were enhanced by Feshbach resonance.[20,21] In a theory, it was proposed that the BCS wave functions was more generally applicable to the weakly interacting limit. As long as the chemical potential is found self consistently (as the interaction is increased the BCS ground state) can describe everything from Cooper pairing to the BEC of composit Boson made up of two Fermions. After nearly a century of ^{4}He and superconductor being considered as separate entities that experimental realization of superfluid in BCS-BEC crossover regime would provide a physical link between the two.[22] More recent interest in crossover theories has also come in response to the possibility that they could apply to high T_c superconductors. These superconductors differ from normal superconductor both; in their high transition temperature and the apperant presence of the psudo gap, which are both characteristics expected to be found in a Fermi gas in the crossover.[23,24]

In this paper, we have studied the BCS-BEC crossover physics using BCS theory. BCS theory was originally applied in the limit where the interaction energy is extremely small compared to the Fermi energy. In this case, the chemical potential μ can be fixed at E_F and many calculations becomes reasonable simple. Leggett[25] pointed out that if the BCS gap equation is examined allowing μ to vary, the gap equation actually becomes precisely the Schrodinger equation for diatomic molecule in the limit, where μ dominates. The structure of the crossover theory originates with the work of Nozieres and Schmitt-Rink[26] and Randeria et al.[27]

Mathematical formula used in the study

Let us consider a homogeneous Fermi system in three dimensions in an equal mixture of two different states at T = 0 Applications of usual BCS theory[28] results in the gap equation -

$$\Delta_k = -\sum_{k'} U_{kk'} \frac{\Delta_{k'}}{2E_{k'}} \qquad \ldots(1)$$

where

$$E_k = \sqrt{(\varepsilon_k{}^2 + \Delta^2)}$$

$$\varepsilon_k = \in_k - \mu \text{ and } \in_k = \hbar^2 k^2 / 2m$$

$U_{kk'} < 0$ is the attractive interaction for scattering of Fermions with momenta k' and –k' to k and –k. Then one obtains the number equation

$$< N_{tot} >= \sum_k (1 - (\varepsilon_k / \in_k)) \qquad \ldots(2)$$

Where N_{tot} is the total number of Fermions is both states. To solve equation (1) in the BCS limit, the standard approach is to assume that the potential is constant at a value U < 0, which implies that gap is constant as well i.e $\Delta_k = \Delta$. In that case, the gap equation becomes -

$$-\frac{1}{U} = \sum_k \frac{1}{2E_k} \qquad \ldots(3)$$

Now one finds that this equation diverges. For BCS superconductor, this problem is resolved because the interaction can be limited to be within the Debye energy $\hbar\omega_D$ of E_F. This is the result of the nature of the phonon mediated interaction between the electrons that gives rise to the attractive interaction. Further simplification of BCS-limit are that $\mu = E_F$ and that since $\hbar\omega_D << E_F$, the density of states is constant at the value N ($\xi = 0$). The gap equation then becomes -

$$-\frac{1}{N(0)U} = \int_{-\hbar\omega_D}^{\hbar\omega_D} \frac{d\xi}{2[\Delta^2 + \xi^2]^{1/2}} \qquad \ldots(4)$$

Solving equation (4) produces the BCS results.

$$\Delta \cong 2\hbar\omega_D \exp[-\frac{1}{N(0)|U|}] \qquad \ldots(5)$$

To extend this calculation to the crossover in atomic system, one can no longer apply the $\hbar\omega_D$ cut off. The solution to the problem in this case is nontrivial and requires a renormalization procedure. Randeria[23] has obtained the result of such a procedure and obtained the renormalized gap equation.

$$\frac{-m}{4\pi\hbar^2 a_s} = \frac{1}{V}\sum_k \left(\frac{1}{2E_k} - \frac{1}{2\in_k}\right) \quad \ldots(6)$$

Where the interaction U is described by the s-wave scattering length a_s and V is the volume of the system. One can't assume $\mu = E_F$ in the crossover. For this, one solves gap equation given in equation (6) and number equation (2) simultaneously for μ and gap parameter Δ. One solves these parameters as a function of dimensionless parameter ($K_F\, a_s$) where $K_F = (2mE_F)^{1/2}/\hbar$. Marim et al.[29] have done this analytically using the elliptical integrals. We have taken the help of this paper for calculating Δ and μ. The results are shown in Tables 1 and 2 for a homogeneous Fermi gas at T = 0 as determined through NSR theory.[26]

Beyond T = 0

The phase transition temperature Tc is an important parameter for superfluid system. In BCS-BEC crossover, the transition temperature Tc increases as the interaction is increased. It is lowest in the perturbative BCS regime and highest in the BEC limit. In a homogeneous system in the BCS limit[30] -

$$\frac{T_c}{T_F} = \frac{8}{\pi} e^{\gamma-2} \exp\left[\frac{-\pi}{2K_F|a_s|}\right], \qquad \gamma = 0.58 \quad \ldots(7)$$

In the BEC limit[31], $T_c / T_F = 0.22$ …(8)

The BCS transition temperature can be extremely small due to the exponential dependence upon $1/(K_F\, a_s)$. If one puts the interaction strength in the alkali Fermionic gas ($a_s = -100\, a_0$) and a typical K_F ($1/K_F = 200\, a_0$), a_0 is Bohr radius.

$$T_c\,(BCS) = 10^{-14} T_F \quad \ldots(9a)$$

which is completely inaccessible temperature in atomic systems.

For $1/(K_F\, a_s) = -1$, $T_c\,(BCS) = 0.1\, T_F$ …(9b)

which can be accessible.

In the BCS limit, pairing and the phase transition to a superfluid state occur at the same temperature. However in the BEC limit, this is not the case because the constituent Fermions are very tightly bound pairs and can form far above T_c.

RESULTS AND DISCUSSION

In this paper, we have studied the BCS- BEC crossover physics from BCS theory. We have taken the theoretical formalism of Regal et al.[32,33] in this study. In Tables 1 and 2, we have presented the evaluated results of the gap parameter Δ/E_F and chemical potential μ/E_F as fraction of dimensionless parameter $(K_F\, a_s)^{-1}$ determined through NSR theory for BCS and BEC limit. From our theoretical results, it appears that crossover occurs in a relatively small region of the parameter $(K_F\, a_s)^{-1}$ from $-1 < (K_F\, a_s)^{-1} < 1$. In Table 3, we have given the value and meaning of both; μ and Δ for the BCS and BEC limit. The recent theoretical result,[34,35] also confirm such behavior.

Table 1: Evaluated results of the gap parameter Δ/E_F as a function of dimensionless parameter $(K_F\, a_s)^{-1}$ determined through NSR theory

$(K_F\, a_s)^{-1}$	(Δ/E_F)	
	BCS Limit	**BEC Limit**
2.5	2.85	0.00
2.0	2.53	0.00
1.5	2.10	0.00
1.0	1.82	0.00
0.5	1.02	0.00
0.0	0.00	0.00
-0.5	0.00	1.27
-1.0	0.00	1.13
-1.5	0.00	0.87
-2.0	0.00	0.42
-2.5	0.00	0.32
-3.0	0.00	0.00

Table 2: Evaluated results of the chemical potential (μ/E_F) as a function of dimensionless parameter $(K_F a_s)^{-1}$ determined through NSR theory

$(K_F a_s)^{-1}$	(μ/E_F)	
	BCS Limit	BEC Limit
-2.5	1.122	0.00
-2.0	1.120	0.00
-1.5	1.116	0.00
-1.0	1.115	0.00
-0.5	1.115	0.00
0.0	1.115	0.00
0.5	0.00	0.085
1.0	0.00	-1.052
1.5	0.00	-2.386
2.0	0.00	-3.458
2.5	0.00	-5.687
3.0	0.00	-6.432

Table 3: Crossover experiments have been performed with ^{40}K and ^{6}Li. The regime corresponds to varying a_s from -2000 a_o through ∞ and to 2000 a_o. a_o is Bohr-radius. a_o = 0.0529 nm. Now the value and meaning of both μ and Δ are different in two limits

BCS Limit	BEC Limit
$\mu \sim E_F$	$\mu \sim \frac{-E_b}{2} = -(\frac{1}{K_F a_s})^2 E_F$
$\Delta \sim e^{-\pi/} K_F \lvert a_s \rvert E_F$	$\Delta \sim [\frac{16}{3\pi}\frac{1}{K_F a_s}]^{1/2} E_F$

Δ is the gap parameter but its meaning is the excitation gap i.e. the smallest possible energy that can create a hole (remove fermions) in the superfluid in the BCS limit.

In general, the excitation energy is

$$E_{gap} = m_{\min} E_k = m_{\min}[(\in_k - \mu)^2 + \Delta^2] = m_{\min}[(\frac{\hbar^2 K^2}{2m} - \mu)^2 + \Delta^2]$$

Δ is positive, when μ is positive (BCS limit) but becomes $[\mu^2 + \Delta^2]^{1/2}$, when μ is negative.

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Revised : 18.06.2010 *Accepted : 20.06.2010*