International Journal of Chemical Sciences

International Journal of Chemical Sciences
ISSN: 0972-768 X

Research | Vol 15 Iss 4

Structure of M (I): Ternary Gamma-Semigroups

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Received: October 25, 2017; **Accepted:** November 20, 2017; **Published:** November 23, 2017

Abstract

The terms, 'I-dominant', 'left I-divisor', 'right I-divisor', 'I-divisor' elements, 'M (I)-ternary Γ-semigroup' for a ternary Γ-ideal I of a ternary Γ-semigroup are introduced and we characterized M (I)-ternary gamma semigroups.

Keywords: ***Completely prime ternary Γ-ideal; I-dominant element; I-dominant ternary Γ-ideal; I-divisor; M (I)-ternary Γ-semigroup***

Introduction

In [1] introduced the concepts of A-potent elements, A-divisor elements and N (A)-semigroups for a given ideal A in a semigroup and characterized N (A)-semigroups for a pseudo symmetric ideal A. He proved that if M is a maximal ideal containing a pseudo symmetric ideal A, then either M contains all A-dominant elements or M is trivial. In this paper we extent these notions and results to M (I)-ternary Γ-semigroups.

Experimental

Preliminaries

Definition 2.1: Let T and Γ be two non-empty set. Then T is said to be a Ternary **Γ**-*semigroup* if there exist a mapping from T × Γ × T × Γ × T to T which maps $(x_1, \alpha, x_2, \beta, x_3) \rightarrow [x_1\alpha x_2\beta x_3]$ satisfying the condition: $[[x_1\alpha x_2\beta x_3]\gamma x_4\delta x_5] = [x_1\alpha[x_2\beta x_3\gamma x_4]\delta x_5] = [x_1\alpha x_2\beta[x_3\gamma x_4\delta x_5]]$ $\forall x_i \in$ T, $1 \le i \le 5$ and $\alpha, \beta, \gamma, \delta \in \Gamma$. A nonempty subset A of a ternary Γ-semigroup T is said to be ternary Γ-ideal of T if *b, c,* ∈ T, *α, β* ∈ Γ, *a* ∈ A implies b*α*c*β*a ∈ A, b*α*a*β*c ∈ A, a*α*b*β*c ∈ A. A is said to be a completely prime **Γ**-ideal of T provided *x, y, z* ∈ T and *x*Γ*y*Γ*z* ⊆A implies either *x* ∈ A or *y* ∈ A or *z* ∈ A. and A is said to be a *prime* **Γ**-*ideal* of T provided X, Y, Z are

Citation: Vasantha M, Madhusudhana Rao D. Structure of M (I): Ternary Gamma-Semigroups. Int J Chem Sci. 2017;15(4):224

Ternary Γ-ideals of T and XΓYΓZ ⊆ A ⇒ X ⊆ A or Y ⊆ A or Z ⊆ A. A ternary Γ-ideal A of a ternary Γ-semigroup T is said to be a completely semiprime **Γ**-ideal provided *x* ∈ T, $(x\Gamma)^{n-1}x \subseteq A$ for some odd natural number *n*>1 implies *x* ∈ A. Similarly, A ternary Γ-ideal A of a ternary Γ-semigroup T is said to be semiprime ternary Γ-ideal provided X is a ternary Γ-ideal of T and $(X\Gamma)^{n-1} X \subseteq A$ for some odd natural number *n* implies X ⊆ A [2-6].

Definition 2.2: A ternary Γ-ideal I of a ternary Γ-semigroup T is said to be pseudo symmetric provided x, y, z ∈ T, xΓyΓz ⊆ I implies xΓsΓyΓtΓz ⊆ I for all s, t ∈ T and I is said to be semi pseudo symmetric provided for any odd natural number n, x ∈ T, $(x\Gamma)^{n-1} x \subseteq I \Rightarrow (< x >\Gamma)^{n-1} < x > \subseteq I$.

Theorem 2.3: Let I be a semi-pseudo symmetric ternary **Γ**-ideal of a ternary **Γ**-semigroup T. Then the following are equivalent.

1) I_1=The intersection of all completely prime ternary **Γ**-ideals of T containing I.
2) I_1^1 =The intersection of all minimal completely prime ternary **Γ**-ideals of T containing I.
3) I_1^{11} =The minimal completely semiprime ternary **Γ**-ideal of T relative to containing I.
4) $I_2=\{x \in T: (x\Gamma)^{n-1} x \subseteq I$ for some odd natural number $n\}$
5) I_3=The intersection of all prime ternary **Γ**-ideals of T containing I.
6) I_3^1 =The intersection of all minimal prime ternary **Γ**-ideals of T containing I.
7) I_3^{11} =The minimal semiprime ternary **Γ**-ideal of T relative to containing I.
8) $I_4=\{x \in T: (<x>\Gamma)^{n-1}< x > \subseteq I$ for some odd natural number $n\}$.

Theorem 2.4: If I is a ternary **Γ**-ideal of a semi simple ternary **Γ**-semigroup T, then the following are equivalent.
1) I is completely semiprime.
2) I is pseudo symmetric.
3) I is semi-pseudo symmetric.

Results and Discussion

M (i)-ternary gamma-semigroup

We now introduce the terms I-dominant element and I-dominant ternary Γ-ideal for a ternary Γ-ideal of a ternary Γ-semigroup [7].

Definition 3.1: Let I be a ternary Γ-ideal in a Ternary Γ-semigroup T. An element *x* ∈ T is said to be I-dominant provided there exists an odd natural number *n* such that $(x\Gamma)^{n-1}x \subseteq I$. A ternary Γ-ideal J of T is said to be I-dominant ternary Γ-ideal provided there exists an odd natural number *n* such that $(J\Gamma)^{n-1}J \subseteq I$.

Note 3.2: If I is a ternary Γ-ideal of a ternary Γ-semigroup T, then every element of I is a I-dominant element of T and I itself an I-dominant ternary Γ-ideal of T.

Definition 3.3: Let I be a ternary Γ-ideal of a ternary Γ-semigroup T. An I-dominant element x is said to be a nontrivial I-dominant element of T if $x \notin$ I.

Notation 3.4: M_o (I)=The set of all I-dominant elements in T.
M_1 (I)=The largest ternary Γ-ideal contained in M_o (I).
M_2 (I)=The union of all I-dominant ternary Γ-ideals.

Theorem 3.5: If I is a ternary Γ-ideal of a ternary Γ-semigroup T, then $I \subseteq M_2(I) \subseteq M_1(I) \subseteq M_0(I)$.

Proof: Since I is itself an I-dominant ternary Γ-ideal, and M_2 (I) is the union of all I-dominant ternary Γ-ideals. Therefore, I $\subseteq M_2$ (I). Let $x \in M_2$ (I) $\Rightarrow x$ belongs to at least one I-dominant ternary Γ-ideals $\Rightarrow x$ is an I-dominant element. Hence, $x \in M_0$ (I). Therefore, M_2 (I) $\subseteq M_0$ (I). Clearly M_2 (I) is a ternary Γ-ideal of T. Since M_1 (I) is the largest ternary Γ-ideal contained in M_o (I), we have $M_2(I) \subseteq M_1(I) \subseteq M_0(I)$. Hence, $I \subseteq M_2(I) \subseteq M_1(I) \subseteq M_0(I)$.

Theorem 3.6: If I is a ternary Γ-ideal in a ternary Γ-semigroup T, then the following are true.

1. M_0 (I)=I_2.
2. M_1 (I) is a semiprime ternary Γ-ideal of T containing I.
3. M_2 (I)=I_4.

Proof: (1) M_o (I)=The set of all I-dominant elements={$x \in$ T: $(x\Gamma)^{n-1}x \subseteq I$ for some odd natural number n}=I_2.

(2) Suppose that $(<x>\Gamma)^{n-1}<x> \subseteq M_1$ (I) for some odd natural number n. Suppose, if possible $x \notin M_1$ (I). M_1 (I), $< x >$ are the ternary Γ-ideals implies M_1 (I) $\cup < x >$ is a ternary Γ-ideal. Since M_1 (I) is the largest ternary Γ-ideal in M_0 (I),
We have M_1 (I) $\cup < x > \not\subset M_0$ (I) $\Rightarrow < x > \not\subset M_0$ (I). Hence, there exists an element y such that $y \in < x > \setminus N_0$ (I). Now $(y\Gamma)^2 y \subseteq (<x>\Gamma)^2 <x> \subseteq M_1(I) \subseteq M_0(I) \Rightarrow (y\Gamma)^2 y \subseteq M_0(I) \Rightarrow (y\Gamma)^2 y\Gamma)^{n-1}(y\Gamma)^2 y \subseteq I$ for some odd natural number $n \Rightarrow ((y\Gamma)^2 y\Gamma)^{n-1}(y\Gamma)^2 y \subseteq I \Rightarrow y \in M_0(I)$. It is a contradiction. Therefore, $x \in M_1$ (I). Hence, M_1 (I) is a semiprime ternary Γ-ideal of T containing I.

(3) Let $x \in M_2$ (I). Then there exists an I-dominant ternary Γ-ideal J such that $x \in$ J.
J is I-dominant ternary Γ-ideal implies there exists an odd natural number n such that $(J\Gamma)^{n-1} J \subseteq I \Rightarrow (<x>\Gamma)^{n-1} <x> \subseteq (J\Gamma)^{n-1} J \subseteq I$ for some odd $n \in$ N $\Rightarrow x \in I_4$. Therefore, M_2 (I) $\subseteq I_4$. Let $x \in I_4$

$x \in I_4 \Rightarrow (< x > \Gamma)^{n-1} < x > \subseteq I$ for some odd $n \in$ N. So $< x >$ is an I-dominant ternary Γ-ideal in T and hence, $< x > \subseteq$ M_2 (I) $\Rightarrow x \in$ M_2 (I). Therefore, $I_4 \subseteq M_2$ (I). Hence, M_2 (I)=I_4. It is natural to ask whether M_1 (I)=I_3. This is not true.

Example 3.7: In the free ternary Γ-semigroup T over the alphabet *x, y, z*. For the ternary Γ-ideal I=TΓ*x*Γ*x*Γ*x*ΓT, M_0 (I)={*x*} ∪ T^1 Γ*x*Γ*x*Γ*x*ΓT^1 and M_1 (I)={*x*Γ*x*Γ*x*} ∪ TΓ*x*Γ*x*Γ*x*ΓT=T^1 Γ*x*Γ*x*Γ*x*ΓT^1. But TΓ*x*Γ*x*Γ*x*ΓT is a prime ternary Γ-ideal, let I, J, K are three ternary Γ-ideals of T such that IΓJΓK ⊆ TΓ*x*Γ*x*Γ*x*ΓT, implies all words containing *x*Γ*x*Γ*x* ⊆ I or all words containing *x*Γ*x*Γ*x* ⊆ J or all words containing *x*Γ*x*Γ*x* ⊆ K ⇒ I ⊆ TΓ*x*Γ*x*Γ*x*ΓT or J ⊆ TΓ*x*Γ*x*Γ*x*ΓT or K ⊆ TΓ*x*Γ*x*Γ*x*ΓT. Therefore, TΓ*x*Γ*x*Γ*x*ΓT is a prime ternary Γ-ideal. We have I_3=TΓ*x*Γ*x*Γ*x*ΓT, so M_1 (I) ≠ I_3. Therefore, we can remark that the inclusions in $I_3 \subseteq M_1$ (I) $\subseteq M_0$ (I)=I_2 may be proper in an arbitrary ternary Γ-semigroup [8-11].

Theorem 3.8: If I is a semi pseudo symmetric ternary Γ-ideal in a ternary Γ-semigroup T, then M_0 (I)=M_1 (I)=M_2 (I).
Proof: Suppose I is a semi pseudo symmetric ternary Γ-ideal in a ternary Γ-semigroup T. By theorem 3.7, M_0 (I)=I_2 and M_2 (I)=I_4. Also by theorem 2.10, we have I_2=I_4. Hence, M_0 (I)=M_2 (I). By the theorem 3.5, $I \subseteq M_2(I) \subseteq M_1(I) \subseteq M_0(I)$. We have M_2 (I) ⊆ M_1 (I). Now let x ∈ M_1 (I) ⇒ x ∈ M_0 (I) ⇒ x ∈ M_2 (I). Therefore, M_1 (I) ⊆ M_2 (I). Hence, M_1 (I)=M_2 (I). Therefore, M_0 (I)=M_1 (I)=M_2 (I).

Theorem 3.9: For any semi pseudo symmetric ternary Γ-ideal I in a ternary Γ-semigroup T, a nontrivial I-dominant element x (x ∉ I) cannot be semi simple [12,13].

Proof: Since x is a nontrivial I-dominant element, there exists an odd natural number n such that $(x\Gamma)^{n-1} x \subseteq I$. Since I is semi pseudo symmetric ternary Γ-ideal, we have $(< x > \Gamma)^{n-1} < x > \subseteq I$. If x is semi simple, then $< x > = (< x > \Gamma)^2 < x >$ and hence, $< x > = (< x > \Gamma)^{n-1} < x > \subseteq I$, this is a contradiction. Thus, x is not semi simple.

Theorem 3.10: If I is a ternary Γ-ideal in a ternary Γ-semigroup T, such that M_0 (I)=I, then I is a completely semiprime ternary Γ-ideal and I is a pseudo symmetric ternary Γ-ideal.

Proof: Let x ∈ T and $(x\Gamma)^2 x \subseteq I$. Since M_0 (I)=I, $(x\Gamma)^2 x \subseteq M_0(I)$. Thus, there exists an odd natural number n such that $((x\Gamma)^3)^{n-1}(x\Gamma)^2 x \subseteq I \Rightarrow x \in M_0(I) = I.$ Therefore, I is a completely semiprime ternary Γ-ideal. By corollary 2.11, A is pseudo symmetric ternary Γ-ideal. Hence, I is completely semiprime and pseudo symmetric ternary Γ-ideal.

Theorem 3.11: If I is a semi pseudo symmetric ternary Γ-ideal of a ternary semi simple Γ-semigroup then I=M_0 (I).

Proof: Clearly, I ⊆ M_0 (I). Let x ∈ M_0 (I). If x ∉ I then x is a nontrivial I-dominant element. By theorem 3.9, x cannot be semi simple. It is a contradiction. Therefore, x ∈ I and hence, M_0 (I) ⊆ I. Thus M_0 (I)=I.

We now introduce a left I-divisor element, lateral I-divisor element, right, I-divisor element and I-divisor element corresponding to a ternary Γ-ideal A in a ternary Γ-semigroup.

Definition 3.12: Let I be a ternary Γ-ideal in a ternary Γ-semigroup T. An element $x \in$ T is said to be a left I-divisor (a lateral I-divisor, right I-divisor) provided there exist two elements y, $z \in$ T\I such that $x\Gamma y\Gamma z \subseteq I$ $(y\Gamma x\Gamma z \subseteq I, y\Gamma z\Gamma x \subseteq I)$.

Definition 3.13: Let I be a ternary Γ-ideal in a ternary Γ-semigroup T. An element $x \in$ T is said to be two-sided A-divisor if x is both a left I-divisor and a right, I-divisor element.

Definition 3.14: Let I be a ternary Γ-ideal in a ternary Γ-semigroup T. An element $x \in$ T is said to be I-divisor if a is a left I-divisor, a lateral I-divisor and a right, I-divisor element.

We now introduce a left I-divisor ternary Γ-ideal, lateral I-divisor ternary Γ-ideal, right I-divisor ternary Γ-ideal and I-divisor ternary Γ-ideal corresponding to a ternary Γ-ideal I in a ternary Γ-semigroup.

Definition 3.15: Let I be a ternary Γ-ideal in a ternary Γ-semigroup T. A ternary Γ-ideal J in T is said to be a left I-divisor ternary Γ-ideal (lateral I-divisor ternary Γ-ideal, right I-divisor ternary Γ-ideal, two sided I-divisor ternary Γ-ideal) provided every element of J is a left I-divisor element (a lateral I-divisor element, a right I-divisor element, it is both a left I-divisor ternary Γ-ideal and a right I-divisor ternary Γ-ideal).

Definition 3.16: Let I be a ternary Γ-ideal in a ternary Γ-semigroup T. A ternary Γ-ideal J in T is said to be I-divisor ternary Γ-ideal provided if it is a left I-divisor ternary Γ-ideal, a lateral I-divisor ternary Γ-ideal and a right I-divisor ternary Γ-ideal of a ternary Γ-semigroup T.

Notation 3.17: R_l (I)=The union of all left I-divisor ternary Γ-ideals in T.
R_r (I)=The union of all right I-divisor ternary Γ-ideals in T.
R_m (I)=The union of all lateral I-divisor ternary Γ-ideals in T.
R (I)=R_l (I) $\cap$ R_m (I) $\cap$ R_r (I). We call R (I), the divisor radical of T.

Theorem 3.18: If I is any ternary Γ-ideal of a ternary Γ-semigroup T, then M_1 (I) $\subseteq$ R (I).

Proof: Let $x \in M_1$ (I). Since M_1 (I) $\subseteq M_0$ (I), we have $x \in M_0(I) \Rightarrow (x\Gamma)^{n-1}x \subseteq I$ I for some odd natural number n. Let n be the least odd natural number such that $(x\Gamma)^{n-1}x \subseteq I$. If n=1 then $x \in$ I and hence, $x \in$ R (I).

If n >1, then $(x\Gamma)^{n-1}x = (x\Gamma)^{n-4}x\Gamma x\Gamma x \subseteq I$, where $(x\Gamma)^{n-4}x \subseteq T / I$.

Hence, x is an I-divisor element. Thus, $x \in$ R (I). Therefore, M_1 (I) $\subseteq$ R (I).

Theorem 3.19: If I is a ternary Γ-ideal in a ternary Γ-semigroup T, then R (I) is the union of all I-divisor ternary Γ-ideals in T.

Proof: Suppose I is a ternary Γ-ideal in a ternary Γ-semigroup T.

Let J be I-divisor ternary Γ-ideal in T. Then J is a left I-divisor, a lateral I-divisor and a right I-divisor ternary Γ-ideal in T. Thus J $\subseteq R_l$ (I), J $\subseteq R_m$ (I) and J $\subseteq R_r$ (I)

$\Rightarrow$ I $\subseteq R_l$ (I) $\cap R_m$ (I) $\cap R_r$ (I)=R (I) $\Rightarrow$ B $\subseteq$ R (I).

Therefore, R (I) contains the union of all I-divisor ternary Γ-ideals in T. Let $x \in$ R (I). Then $x \in R_l$ (I) $\cap R_m$ (I) $\cap R_r$ (I). So $< x > \subseteq R_l$ (I) $\cap R_m$ (I) $\cap R_r$ (I).

Hence, $< x >$ is I-divisor ternary Γ-ideal. So, R (I) is contained in the union of all divisor ternary Γ-ideals in T. Thus R (I) is the union of all divisor ternary Γ-ideals of T.

Corollary 3.20: If I is a pseudo symmetric ternary Γ-ideal in a ternary Γ-semigroup T, then R (I) is the set of all I-divisor elements in T.

Proof: Suppose I is a pseudo symmetric ternary Γ-ideal in T. Let x be I-divisor element in T. Then $x\Gamma y\Gamma z \subseteq I$, where $y, z \in$ T\I. $x\Gamma y\Gamma z \subseteq I$, I is pseudo symmetric

$\Rightarrow < x >\Gamma< y >\Gamma < z > \subseteq I \Rightarrow < x >$ is I-divisor ternary Γ-ideal $\Rightarrow < x > \subseteq$ R (I)

$\Rightarrow x \in$ R (I). Hence, R (I) is the set of all I-divisor elements in T. We now introduce the notion of M (I)-ternary Γ-semigroup.

Definition 3.21: Let I be a ternary Γ-ideal in a ternary Γ-semigroup T. T is said to be a M (I)-ternary Γ-semigroup provided every I-divisor is I-dominant.

Notation 3.22: Let T be a ternary Γ-semigroup with zero. If I={*0*}, then we write R for R (I) and M for M_0 (I) and M-ternary Γ-semigroup for M (I)-ternary Γ-semigroup.

Theorem 3.23: If T is an M (I)-ternary Γ-semigroup, then R (I)=M_1 (I).

Proof: Suppose T is an M (I)-ternary Γ-semigroup. By theorem 3.18, M_1 (I) ⊆ R (I).

Let $x \in$ R (I) ⇒ x is an I-divisor ⇒ x is an I-dominant ⇒ $x \in M_1$ (I). ∴ R (I) ⊆ M_1 (I).

Hence, M_1 (I)=R (I).

Theorem 3.24: Let I be a semipseudo symmetric ternary Γ-ideal in a ternary Γ-semigroup T. Then T is an M (I)-ternary Γ-semigroup iff R (I)=M_0 (I).

Proof: Since I is a semi-pseudo symmetric ternary Γ-ideal, by theorem 3.8**,** M_0 (I)=M_1 (I)=M_2 (I). If Tan M (I)-ternary Γ-semigroup, then by theorem 3.23, R (I)=M_1 (I). Hence, R (I)=N_0 (I). Conversely suppose that R (I)=M_0 (I). Then clearly every I-divisor element is an I-dominant element. Hence, T is an M (I)-ternary Γ-semigroup.

Corollary 3.25: Let I be a pseudo symmetric ternary Γ-ideal in a ternary Γ-semigroup T. Then T is an M (I)-ternary Γ-semigroup if and only if R (I)=M_0 (I).

Proof: Since every pseudo symmetric ternary Γ-ideal is a semi-pseudo symmetric ternary Γ-ideal, by theorem 3.24, R (I)=M_0 (I).

Corollary 3.26: Let T be a ternary Γ-semigroup with 0 and $< 0 >$ is a pseudo symmetric ternary Γ-ideal. Then R=M iff T is an M-ternary Γ-semigroup.

Proof: The proof follows from the theorem 3.24.

Theorem 3.27: If N is a maximal ternary Γ-ideal in a ternary Γ-semigroup T containing a pseudo symmetric ternary Γ-ideal I, then N contains all I-dominant elements in T or T\N is singleton which is I-dominant.

Proof: Suppose N does not contain all I-dominant elements.
Let $x \in$ T\N be any I-dominant element and y be any element in T\N.
Since N is a maximal ternary Γ-ideal, $N \cup < x > = N \cup < y > \Rightarrow < x > = < y >$.

Since $y \notin$ N, we have $y \in < x >$. Let n be the least positive odd integer such that $(x\Gamma)^{n-1}x \subseteq I$. Since I is a pseudo symmetric ternary Γ-ideal then I is a semipseudo symmetric ternary Γ-ideal and hence, $(< x > \Gamma)^{n-1} < x > \subseteq I$.

Therefore $(y\Gamma)^{n-1}y \subseteq I$ and hence, y is I-dominant element. Thus, every element in T\N is I-dominant.

Similarly, we can show that if m is the least positive odd integer such that $(y\Gamma)^{m-1}y \subseteq I$, then $(x\Gamma)^{m-1}x \subseteq I$. Therefore, there exists an odd natural number p such that $(x\Gamma)^{p-1}x \subseteq I$ and $(x\Gamma)^{p-3}x\,\text{Ú}\,I$ for all $x \in$ T\N.

Let $x, y, z \in$ T\N. Since N is maximal ternary Γ-ideal, we have $< x > = < y > = < z >$.
So $y, z \in < x > \Rightarrow y \in s\Gamma x\Gamma t$, $z \in u\Gamma x\Gamma v$. So $x \in < y >$ and hence, $x \in s\Gamma y\Gamma t$ for some $s, t \in T^1$. Now since I is a pseudo symmetric ternary Γ-ideal,
we have, $(x\Gamma y\Gamma z\Gamma)^{p-3}=(x\Gamma y\Gamma z\Gamma)^{p-4}\, x\Gamma y\Gamma z=(x\Gamma y\Gamma z\Gamma)^{p-4}\, x\Gamma\, (s\Gamma x\Gamma t)\, \Gamma\, (u\Gamma x\Gamma v) \subseteq I \Rightarrow x\Gamma y\Gamma z \subseteq N$. If $y \neq x$ then $s, t \in$ T. If $s, t \in$ N then $s\Gamma x\Gamma t \subseteq N \Rightarrow y \in N$.

Which is not true. In both the cases we have a contradiction. Hence, $x=y$.
Similarly, we show that $z=x$.

Corollary 3.28: If N is a nontrivial maximal ternary Γ-ideal in a ternary Γ-semigroup T containing *a* pseudo symmetric ternary Γ-ideal I. Then M_0 (I) ⊆N.
Proof: Suppose in M_0 (I) ⊈N. Then by above theorem 3.27, N is trivial ternary Γ-ideal. It is a contradiction. Therefore, M_0 (I) ⊆N.

Corollary 3.29: If N is a maximal ternary Γ-ideal in a semi simple ternary Γ-semigroup T containing *a* semipseudo symmetric ternary Γ-ideal I. Then M_0 (I) ⊆N.

Proof: By theorem 3.11, I is pseudo symmetric ternary Γ-ideal. If $x \in T \backslash N$ is I-dominant, then x cannot be semi simple. It is a contradiction. Therefore, M_0 (I) ⊆ N.

Conclusion

According to theorem 3.11, I is pseudo symmetric ternary Γ-ideal. If $x \in T \backslash N$ is I-dominant, then x cannot be semi simple. Hence, is a contradiction. Therefore, M_0 (I) ⊆ N.

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