Some new models for strange quark stars with isotropic pressure

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ABSTRACT
We found new class of solutions to the Einstein-Maxwell system of equations for charged quark matter within the framework of MIT Bag Model considering a gravitational potential $Z(x)$ proposed by Thirukkanesh and Ragel (2013), which depends on an adjustable parameter $n$. Variables as the energy density, charge density, pressure and the metric functions are written in terms of elementary and polinominal functions. We show that the form chosen for the gravitational potential allows obtain physically acceptable solutions with any value of the adjustable parameter.

INTRODUCTION
From the development of Einstein’s theory of general relativity, the description of compact objects has been a central issue in relativistic astrophysics in the last few decades\cite{1,2}. Recent experimental observations in binary pulsars\cite{2} suggest that could be quark stars. The existence of quark stars in hydrostatic equilibrium was first by Itoh\cite{3} in a seminal treatment. Recently, the study of strange stars consisting of quark matter has stimulated much interest since could represent the most energetically favourable state of baryon matter.

The physics of ultrahigh densities is not well understood and many of the strange stars studies have been performed within the framework of the MIT bag model\cite{4}. In this model, the strange matter equation of state has a simple linear form given by $p = \frac{1}{3}(\rho - 4B)$, where $\rho$ is the energy density, $p$ is the isotropic pressure and $B$ is the bag constant. However, in theoretical works of realistic stellar models\cite{5-8} it has been suggested that superdense matter may be anisotropic, at least in some density ranges.

Many researchers have used a great variety of mathematical techniques to try to obtain exact solutions for quark stars within the framework of MIT bag model, since it has been demonstrated by Komathiraj and Maharaj\cite{9}, Malaver\cite{10}, Thirukkanesh and Maharaj\cite{11} and Thirukkanesh and Ragel\cite{12}. Feroze and Siddiqui\cite{13} and Malaver\cite{14} consider a quadratic equation of state for the matter distribution and specify particular forms for the gravitational potential and electric field intensity. Mafa Takisa and Maharaj\cite{15} obtained new exact solutions to the Einstein-Maxwell system of equations with a polytropic equation of state. Thirukkanesh and Ragel\cite{16} have obtained particular models of anisotropic fluids with polytropic equation of state which are consistent with the reported experimental observations. More recently, Malaver\cite{17,18} generated new exact so-
Some new models for strange quark stars with isotropic pressure

Our objective in this paper is to generate a new class for charged isotropic matter with the bag equation of state that presents a linear relation between the energy density and the radial pressure in static spherically symmetric spacetime using a gravitational potential \( Z(x) \) of Thirukkanesh and Ragel\[12\] which depends on an adjustable parameter \( n \). We have obtained some new classes of static spherically symmetrical models of charged matter where the variation of the parameter \( n \) modifies the radial pressure, charge density, energy density and the metric functions of the compact objects. This article is organized as follows, in Section 2, we present Einstein’s field equations as an equivalent set of differential equations using a transformations due to Durgapal and Bannerji\[20\]. In Section 3, we make a particular choice of gravitational potential \( Z(x) \) that allows solving the field equations and we have obtained new models for charged isotropic matter. In Section 4, a physical analysis of the new solutions is performed. Finally in Section 5, we conclude.

### EINSTEIN FIELD EQUATIONS

Consider a spherically symmetric four dimensional space time whose line element is given in Schwarzschild coordinates by

\[
ds^2 = -e^{2\omega}dt^2 + e^{2\omega}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)
\]  

Using the transformations, \( x = Cr^2 \), \( Z(x) = e^{-2\omega(x)} \) and \( A^2y^2(x) = e^{2\omega(x)} \) with arbitrary constants \( A \) and \( c \), suggested by Durgapal and Bannerji\[20\], the Einstein field equations as given in (1) are

\[
\frac{1-Z}{x} - 2Z = \frac{\rho}{C} + \frac{E^2}{2c}
\]  

\[
4Z\frac{\dot{y}}{y} - \frac{1-Z}{x} = \frac{p_r}{C} - \frac{E^2}{2C}
\]  

### GENERATING NEW EXACT SOLUTIONS

Using the procedure suggested by Komathiraj and Maharaj\[9\], it is possible to obtain a exact solution of the Einstein-Maxwell system. In this paper, motivated by Thirukkanesh and Ragel\[12\], we take the form of the gravitational potential \( Z(x) \) as \( Z(x) = \left(1 - ax^n\right)^n \), where \( a \) is a real constant and \( n \) is an adjustable parameter. This potential is regular at the origin and well behaved in the interior of the sphere. For the electric field we make the choice

\[
E^2 = \frac{nx}{(1+ax)^{2}}
\]

This electric field is finite at the centre of the star and remains continuous in the interior. We have considered the particular cases for \( n = 1, 2, 3 \).

For the case \( n=1 \), the substitution of \( Z(x) \) and (9) in (8), it allows to obtain the differential equation of

\[
4Zx^2\ddot{y} + 2Zx\dot{y}^2 + \left(Zx - Z + 1 - \frac{E^2}{x}\right)y = 0
\]

Where \( \rho \) is the energy density, \( p_r \) is the radial pressure, \( E \) is electric field intensity, \( \sigma \) is the charge density and dot denote differentiations with respect to \( x \). The equation (4) in the condition of pressure isotropy. We can replace the system of field equations, including the bag equation of state by the system

\[
\rho = 3p + 4B
\]

\[
\frac{p_r}{C} = \frac{Z}{y} - \frac{1}{2} \frac{Z - B}{C}
\]

\[
E^2 = \frac{1-Z}{x} - \frac{3Z\dot{y}}{y} - \frac{1}{2} \frac{Z - B}{C}
\]

\[
\sigma = 2\sqrt{\frac{CZ}{x}} (\dot{x}E + E^2)
\]
the first order

\[
y = -\frac{x}{6C(1+ax)^2} - \frac{D}{3(1-ax)} \tag{10}
\]

Where \( D = B \frac{3}{2} a \)

Integrating (10), we obtain

\[
y(x) = C_1 (1+ax) \frac{\exp(-1)}{12Ca^2(1+ax)} \tag{11}
\]

The equation (11) and \( Z(x) \) allows generate the following analytical model:

\[
e^{2y(r)} = C_1 (1+ax)^\frac{\exp(-1)}{12Ca^2(1+ax)} \tag{12}
\]

With \( n=2 \), the eq. (8) becomes

\[
\frac{\exp \left[ \frac{(F+2a)2Ca'x + 4Ca^2 + 2CFa + 1}{6(-1+ax)a^2c(1+ax)} \right]}{(-1+ax)^{3/2}} \tag{18}
\]

Therefore with eq.(17) and we can generate the analytical model:

\[
e^{2y(r)} = C_2 (1+ax)^\frac{\exp(-1)}{12Ca^2(1+ax)} \tag{19}
\]

\[
e^{2y(r)} = \frac{1}{(1-ax)^2} \tag{20}
\]
Some new models for strange quark stars with isotropic pressure

For the Einstein-Maxwell system which do not present singularities at the origin. We have studied three new types of analytical solutions specifying the form of the gravitational potential \( Z(x) \) and of the electric field \( E \). Three obtained solutions correspond to models which have finite values for energy density, pressure and the charge density at the center of the star. All the solutions present a behavior similar to the solution proposed by Komathiraj and Maharaj\cite{9} and Malaver\cite{10} with isotropic pressure. The method to generate analytical exact solutions will depend on the form of \( Z(x) \) and the electric field intensity \( E \), necessary to determine physically acceptable solutions.

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