Shot throwing best mechanical parameters optimization research and application based on mechanical model

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ABSTRACT
This paper analyzes shot force status, utilizes release height, release speed, release angle and throwing distance relations, it analyzes shot movement status from geometric perspective, and establishes the three and shot throwing distance functional relationship shot projectile mathematical model. To make model more reasonable, it further optimize the model, and state model hypothesis rationality. By further calculation on throwing model, it fixes release height and then solves shot throwing best release angle is \( \theta \pm 2k\pi \in (0, \pi/4), k \in N \), from which \( \theta = 1/2 \arccos(gh/(gh + v^2)) \) and furthest throwing distance is \( S = v_t \sqrt{v^2 + 2gh/g} \) and that is verified by numerical simulation.

KEYWORDS
Release speed; Release angle; Release height; Throwing distance.

INTRODUCTION

Throwing events are favored by mass, therefore in early period; Chinese shot was still at the top in international. But in recent years, due to Li Mei-Su, Huang Zhi-Hong and others the group of senior athletes’ retirement, Chinese shot performance has appeared standstill even backward phenomenon. For example, in Sydney Olympic Games in 2000, Chinese women shot putters hadn’t achieved any medals, and even men shot kept standstill in Asian level for a long term, due to Chinese status as well as domestic and foreign pressure, it urgently needs us should research from techniques and training as well as others as soon as possible; besides, due to sports competitions are going more intense, athletes scores gap gets smaller, slight score gap will suffer a sudden decline in rank, as difference between the bronze and champion was 0.1m in the 8th national games, the phenomenon showed contemporary competition required us should more deeply excavate technical potential in training so as to give athletes maximum potentials into play and get satisfied result.

In shot putting, lots of problems should be taken into account so as to improve techniques, such as, release angle, release height and release speed and others relationships, so that let shot arrive at best release angle and furthest distance; for the problem, there are lots of scholars have made research at home and abroad, such as: Liu Ben-Liang (1984) proposed that shot flight trajectory was described by ground oblique angle. Ground oblique angle was shot release point and landing two points connection straight line and ground included angle. According to author research, it was clear
that best release angle is 45°, thereupon “ground oblique angle p and O is not the same”, so best release angle should less than 45° that would be more reasonable; Cureton(1939) made lots of analyses on shot putters technical motions movies, finally he got that best angle was between 40° and 42°; However, James. G. Hay(1978) according to actual measurement, he put forward that best release angle should be between 38° and 42°, in addition, he got release speed and best release angle relations, with speed increasing, best release angle just slightly increased, though it got closer to 45°, it could not arrive at 45°; Miao Wen-Ke etc.(1984) pointed out, “for throwing release angle problem, to arrive at maximum distance and best release angle, it needs coaches and shot putters work together to change and revise shot throwing angle”\cite{1}. The paper on the basis of previous research results, with regard to how to choose maximum release speed and best release angle so that let distance be the maximum, by establishing mathematical model, it gets shot best throwing model, and verifies that it is reasonable to ignore shot throwing moment air motion suffered air resistance and human body factors influence.

**SHOT THROWING MODEL**

Model symbols are as TABLE 1 show, to shot throwing problems, at first establish throwing distance and V, h, \( \theta \) relationship, and on condition that V, h is certain, solve best release angle and furthest distance. (1) Shot movement process air resistance influence is very little, therefore it can ignore\cite{3}. (2) Thrower to shot throwing force \( f \) and height \( h \) are fixed values.

<table>
<thead>
<tr>
<th>Variable parameters</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h )</td>
<td>People height</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Speed direction and throwing horizontal direction formed angle ((0 \leq \theta \leq 90^\circ))</td>
</tr>
<tr>
<td>( f )</td>
<td>Shot throwing release strength</td>
</tr>
<tr>
<td>( S )</td>
<td>Shot put landing point and people’s distance</td>
</tr>
<tr>
<td>( t_1 )</td>
<td>After throwing for time ( t_1 ), shot arrives at top point</td>
</tr>
<tr>
<td>( t_2 )</td>
<td>In ( t_2 ) time, shot lands.</td>
</tr>
<tr>
<td>( t_3 )</td>
<td>Hand and ball acting time(When ( t_3 \leq 1s ), it can be ignored)</td>
</tr>
<tr>
<td>( g )</td>
<td>Local gravity accelerated speed</td>
</tr>
<tr>
<td>( v )</td>
<td>Shot throwing initial speed</td>
</tr>
<tr>
<td>( v_1 )</td>
<td>Vertical component speed when ball is released</td>
</tr>
<tr>
<td>( v_2 )</td>
<td>Horizontal component speed when ball is released</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Coefficient</td>
</tr>
<tr>
<td>( A )</td>
<td>is object sectional area</td>
</tr>
<tr>
<td>( \rho_0 )</td>
<td>Fluid density</td>
</tr>
<tr>
<td>( V )</td>
<td>Object relative to fluid speed</td>
</tr>
</tbody>
</table>
Shot throwing mathematical model

Ignoring throwers mechanical process get involved in throwing circle, at first it carries out research on shot releasing moment throwing angle and initial speed. After shot releasing, due to it moves in a plane, shot in releasing point area vertical direction is movement height \( H(t) \), takes time \( t \) as \( x \) axis to construct rectangular plane coordinate system. In this way, shot after leaving out of hand, its movement path can be expressed by rectangular plane coordinate system, as Figure 1, when shot moves to \( t_1 \) time, then shot arrives at top point, and its speed in vertical direction is 0\(^4\).

\[ g^2 \sin^2 \theta + h + \frac{v^2 \sin^2 \theta}{2g} = H(0) \]  
(6)

When time is \( t = 0 \):

\[ H(0) = a \frac{v^2 \sin^2 \theta}{g^2} + h + \frac{v^2 \sin^2 \theta}{2g} = h \]

So it can get:

\[ a = -\frac{g}{2} \]  
(7)

Input formula (7) into formula (6), it can get:

\[ H(t) = -\frac{g}{2} \left( t - \frac{v \sin \theta}{g} \right)^2 + h + \frac{v^2 \sin^2 \theta}{2g} \]  
(8)

Assume \( H(t_2) = 0 \), it gets:

\[ t_2 = \sqrt{\frac{2h}{g} + \frac{v^2 \sin^2 \theta}{g^2} + \frac{v \sin \theta}{g}} \]  
(9)

Due to:

\[ S = v \cos \theta \cdot t_2 \]  
(10)

In case release height is known, thrower and shot landing point distance:

\[ S = \sqrt{\frac{2hv^2 \cos^2 \theta}{g} + \left( \frac{v^2 \sin^2 \theta}{2g} \right)^2 + \frac{v^2 \sin 2\theta}{2g}} \]  
(11)

By above formula (11), it is clear that shot throwing distance \( S \) has connections with release angle \( \theta \), release speed \( v \) and release height \( h \).

Optimize throwing model

Above model is established on the condition that we ignore shot movement process air resistance and height influence factors, in the following we will consider the two factors to verify whether the factors ignorance is reasonable or not.

Consider human body factor

In real life, to athlete height, the factor should be taken into consideration. When athlete throws shot, put the shot between neck and shoulder, so may as well set shot throwing is 150cm far from ground, \( g \) takes 9.8m/s\(^2\), \( V_0 = 12.5 \text{m/s} \), Chinese men average height is around 170cm, draw functional Figure 2 for formula (11). From above Figure 2, it is clear that B point available extract maximum value \( S = 23.55 \text{m} \), corresponding \( \theta = 0.83 \approx 47.5^\circ \), which is equal to world record, but by experience, it is clear that shot throwing best angle should be around 40\(^\circ\), calculated throwing angle is obvious larger than the angle here,
which indicates during shot throwing researching process, athlete height can be ignored.

**Consider air resistance**

In the initial hypothesis, we ignore air resistance, but in precise analysis, it perhaps is an indispensible factor, in the following we will make analysis of air resistance to shot throwing influence.

If men shot radius is around 120mm, then $A = 0.045 \text{m}^2$, it is known earth surface area $\alpha = 0.45$, air density $\rho_0 = 1.25 \text{kg/m}^3$. According to fluid mechanics relative knowledge, object acting force in fluid:

$$f = \alpha \rho_0 AV$$  \hspace{1cm} (12)

Input above parameters, and then it can get:

$$f = 0.025V^2$$  \hspace{1cm} (13)

And according to $V_0 = 12.5 \text{m/s}$, it can calculate acting force when throwing:

$$f = 4.3\text{N}$$  \hspace{1cm} (14)

So it can get:

$$a = \frac{f}{m} = 0.60 \text{ m/s}^2$$  \hspace{1cm} (15)

By experience, it can know that all shot flight is 1.5 seconds, therefore shot flying speed under air influence its variable is less than $1 \text{ m/s}$, but to $g$ comparing with its speed variable, it cannot reach, it may as well set $V_0 = 2.7 \text{ m/s}$ to offset air to throwing influence, takes $h = 1.7$, draw function Figure 3 for formula (11) under the circumstance.

By above Figure 3, it is known that when $\theta = 0.83 = 47.6^\circ$, it can get $S_{\text{max}} = 22.59$, Due to resistance introduced, the data error becomes bigger with regard to the case considering air resistance, so we get conclusion: Ignore air resistance and human body factor situations are reasonable, and it can know formula (11) can solve throwing best distance and angle.

**Define best throwing mode**

After given release angle, to different release speeds, we should define best release angle. Obviously, it is extreme value problem, according to calculus knowledge; it should firstly solve stagnation point.

We know by formula (11) that $S(v, h, \theta)$ is $v$ and $h$ monotonic function, so for $\theta$ maximum point, by differential, it can get:

$$\frac{\partial S}{\partial \theta} = 0$$  \hspace{1cm} (16)

So:
Due to $\theta$ period is $2\pi$, when

$$\theta \pm 2k\pi \in \left[0, \frac{\pi}{4}\right], k \in \mathbb{N}, \theta \pm 2k\pi$$

is best throwing shot release angle, especially when $h = 0$, at this time $\theta = 45^o$

Model test

In the following, by numerical simulation, we explore release angle $\theta$ and release speed $v$ to throwing distance $S$ influence. For professional athletes throwing moment, if it is expected that their release height $h$ keeps relative stable state is mainly up to athletes professional technique mastery degree, arms length and height, here we let release height $h = 2.0m$, gravity accelerated speed $g = 10m/s^2$,release speed is changing between $10m/s - 15m/s$, release angle $\theta$ is changing between $37^o - 43^o$. By calculating, we get TABLE 2:

<table>
<thead>
<tr>
<th>Speed (m/s)</th>
<th>43°</th>
<th>38°</th>
<th>39°</th>
<th>40°</th>
<th>41°</th>
<th>42°</th>
<th>43°</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 m/s</td>
<td>11.78</td>
<td>11.81</td>
<td>11.82</td>
<td>11.83</td>
<td>11.83</td>
<td>11.82</td>
<td>11.79</td>
<td>0.05</td>
</tr>
<tr>
<td>11 m/s</td>
<td>13.86</td>
<td>13.90</td>
<td>13.93</td>
<td>13.95</td>
<td>13.96</td>
<td>13.95</td>
<td>13.93</td>
<td>0.10</td>
</tr>
<tr>
<td>12 m/s</td>
<td>16.12</td>
<td>16.18</td>
<td>16.23</td>
<td>16.26</td>
<td>16.28</td>
<td>16.28</td>
<td>16.26</td>
<td>0.16</td>
</tr>
<tr>
<td>13 m/s</td>
<td>18.57</td>
<td>18.65</td>
<td>18.71</td>
<td>18.76</td>
<td>18.79</td>
<td>18.79</td>
<td>18.78</td>
<td>0.22</td>
</tr>
<tr>
<td>14 m/s</td>
<td>21.20</td>
<td>21.30</td>
<td>21.39</td>
<td>21.45</td>
<td>21.49</td>
<td>21.51</td>
<td>21.50</td>
<td>0.31</td>
</tr>
<tr>
<td>15 m/s</td>
<td>21.89</td>
<td>22.08</td>
<td>22.25</td>
<td>22.39</td>
<td>22.51</td>
<td>22.60</td>
<td>22.66</td>
<td>0.77</td>
</tr>
<tr>
<td>Range</td>
<td>10.11</td>
<td>10.27</td>
<td>10.43</td>
<td>10.56</td>
<td>10.68</td>
<td>10.78</td>
<td>10.87</td>
<td></td>
</tr>
</tbody>
</table>
shot putters, they should place primary emphasis on increasing throwing release speed at ordinary times training so as to get better results in competition. And by TABLE 2, we can also get athletes shot throwing moment, their release angles generally are between $38^\circ - 42^\circ$.

**CONCLUSIONS**

From above model, it is clear that ignore shot throwing moment air motion suffered air resistance and human body factors influence is reasonable. It gets release angle $\alpha$, release speed $v$, factor to throwing distance $S$ influences sizes, therefore it has certain guiding values in future athletes competition and training as well as coaches guiding aspects. It gets best release angle and furthest distance main influence factors—release speed, therefore at ordinary times, it should strengthen explosive power training, increase throwing release speed, so that would beneficial to improve shot throwing distance and let athletes get more ideal performances.

**REFERENCES**


