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# SELF-FOCUSING OF LASER BEAMS IN THE PARAXIAL RAY APPROXIMATION IN COLLISIONAL INHOMOGENEOUS PLASMAS FOR ARBITRARY LARGE MAGNITUDE OF NONLINEARITY

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## ABSTRACT

This paper presents an investigation of the self-focusing behaviour of radially symmetrical Gaussian laser beam propagating in an axially inhomogeneous collisional plasma. Considering the non-linearity to arise from the redistribution of electrons, due to thermal conduction across the cross-section of the beam and following the extended version of Sodha et al. theory based on the WKB and paraxial – ray approximation, the self-focusing behaviour has been investigated in some detail. The effect of different types of axial inhomogeneities in plasma, on the self-focusing of laser beam has been studied for arbitrary large magnitude of nonlinearity. The self-focusing is found to depend on type of axial inhomogeneity as well as characteristic scale length of axial inhomogeneity. When thermal conduction is the dominant mechanism of nonlinearity of dielectric constant, the critical power $P_{cr}$ of the beam is seen to be the same as that given by the small nonlinearity theory. When power of the beam $P > P_{cr}$, the medium behaves as an oscillatory wave-guide.

**Key words:** Paranial ray approximation, Laser beam, Colloisional inhomogeneous plasma, Self - focusing.

## INTRODUCTION

The understanding of self-focusing and filamentation of laser light may be important to the success of laser fusion. In laser-induced fusion, the most important problem is the efficient coupling of the energy of the laser beam to plasma to heat the latter[1]. In this coupling process, many nonlinear phenomena such as self-focusing,

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filamentation instabilities, stimulated Raman scattering (SRS) and stimulated Brillouin scattering (SBS) play a crucial role[2,3]. The laser light absorption, penetration, and conversion to x-rays could also be affected by self-focusing and filamentation.

But most of these studies (theoretical and numerical) are limited to various approximations such as homogeneous medium, non-linear part of the dielectric constant much smaller than the linear part etc. These approximations are rather restrictive and limit the applicability of the theory to many real life situations.

There are number of mechanisms that can degrade the uniformity of a laser beam. At very high intensities, the relativistic mass variation of the electrons oscillating in the laser electric field can increase the index of refraction in the center of a beam or in a hot spot in the beam[4, 5]. This leads to focusing that increases the mass variation further, causing the system to go unstable. At high intensities, the ponderomotive force of the laser can drive plasma from the interior of a beam; thus, raising the index of refraction there, leading to focusing and instability[6]. Finally, at lower intensities, if the beam or hot spot width is large compared with an electron mean free path $\lambda_m$, inverse Bremsstrahlung heating can raise the pressure in the interior of the beam. The increased pressure drives plasma out of the beam, once again raising the index of refraction there, leading to instability[7]. The first mechanism, relativistic self-focusing is not of interest to laser fusion. With the intensities, wavelengths, and plasma scale lengths envisioned for reactor targets, little self-focusing is expected from this mechanism. Ponderomotive self-focusing could be important for small-scale hot spots. The third mechanism, thermal self-focusing, is important in the focusing of whole beams and large scale hot spots. In collisional plasmas, the nonlinearity can also arise from the redistribution of electrons, due to thermal conduction across the cross-section of the beam. The relative importance of energy relaxation due to collisions and thermal conduction mechanism is given by ratio[8].

$$R = \left(\frac{\text{Rate of energy loss due to collisions}}{\text{Rate of energy loss due to thermal conduction}}\right)$$

$$= \left(\frac{2m}{M}\right)\left(\frac{\text{width of the beam}}{\text{mean free path of electrons}}\right)^2 \qquad \ldots(1)$$

m and M are the mass of the electron and the scatterer (atoms/molecules in case of weakly ionized plasma and ions in strongly ionized plasma), respectively. In a strongly ionized plasma, mean free path $\lambda_m$, of electrons is sufficiently large and the ratio R is less than unity. In this paper, we have discussed the self-focusing of laser beams under such conditions.

Sodha et al.[8] had developed a steady state paraxial theory of self-focusing of laser beam in a non-linear, non-absorbing homogeneous medium. The extended version of this theory has been used in the present study of self-focusing of laser beam in an inhomogeneous plasma in the paraxial ray approximation, taking energy loss due to thermal conduction.

## Inhomogeneous plasma medium

Inhomogeneous plasma means that charge density is not uniform throughout the space where laser plasma interactions are considered. For the study of self-focusing of laser beam in plasma, some simple models for variations of charge density are devised and considered here in the present analysis. The inhomogeneity in charge density of the plasma at any time in space can be represented by the relation

$$N(x, y, z, t) = N_o W(x, y, z, t), \quad ...(2)$$

where $N_o$ is the density of the plasma at x = 0, y = 0, z = 0 and t = 0. Here, W (x, y, z, t) is the density profile function and may have different shapes for different types of inhomgeneities.

Let the laser beam, whose effect is to be studied, is propagating in z-direction in plasma. In axially inhomogeneous plasma, the electron density varies along the z-direction only i.e. the non-uniformity in charge density is present in the propagation direction only and system is supposed to be under steady-state i.e. time-independent. For such type of inhomogeneity (axial only), the eq. (2) can be rewritten as

$$N(z) = N_o W(z), \quad ...(3)$$

where $N_o$ is a constant (density of plasma medium at the boundary, where wave is incident on it i.e. at z = 0) and density profile function W(z) is only z-dependent. This function W(z) can have different shapes corresponding to different types of axially inhomogeneous plasma. In the present study, few shapes are considered which are founded to be of practical importance.

## Linearly increasing axial inhomogeneity

The charge density is supposed to increase linearly with the propagation distance. For such type of axially inhomogeneous plasma, density profile function which is of practical importance can be written as

$$W(z) = 1 + z/L \qquad ...(4)$$

Here, z is the propagation distance in the plasma medium and L is the characteristics scale length of axial inhomogeneity.

**Exponentially varying axial inhomogeneities**

The electron charge density functions for such type of axially inhomogeneous plasma which are considered in the present study, can be written as

$$W(z) = 1 + \frac{z^2}{L^2} \exp\left(-\frac{z^2}{L^2}\right) \qquad ...(5)$$

and

$$W(z) = 1 + \exp\left(1 - \frac{z}{L}\right)^2. \qquad ...(6)$$

The value of plasma frequency depends on the plasma charge density. Therefore, it is noticed that the plasma frequency is not a constant (as in case of homogeneous plasma) but varies in inhomogeneous plasma as –

$$\omega_p^2 = \frac{4\pi N e^2}{m} \qquad ...(7)$$

Substitution of N from eq. (3) for axially inhomogeneous plasma, gives

$$\omega_p^2 = \frac{4\pi N_o e^2 W(z)}{m} \qquad ...(8)$$

$$= \omega_{po}^2 \; W(z),$$

where $\omega_{po}^2 = \dfrac{4\pi N_o e^2}{m}$ is the homogeneous plasma frequency or the plasma frequency at the boundary of inhomogeneous plasma. Eq. (8) gives the z-dependence of plasma frequency in case of axially inhomogeneous plasma. For different shapes of W(z) i.e. for different type of inhomogeneities, this dependence is going to be different.

**Self-focusing equation with arbitrany large non-linearity**

The intensity distribution of a linearly polarised Gaussian laser beam can be written as –

$$E\,E^{*} = E_o^2 \exp\left(-r^2/r_o^2\right) \qquad ...(9)$$

where r is the radial coordinate of the cylindrical coordinate system and $r_o$ is the initial beam width. $E_o$ represents the amplitude of the electric field due to propagating laser beam. For the study of self-focussing phenomena, the nonlinear dielectric constant of the medium can be written as –

$$\in(\langle E\,E^{*}\rangle) = \in_o + \phi\,(\langle EE^{*}\rangle) \qquad ...(10)$$

In the paraxial – ray approximation, one generaly expands $\phi$ around $\phi \cong 0$. However with such an expansion, one can study only those cases where $\phi << \in_o$. To study self-focusing for arbitrary large non-linearity, one should expand $\phi$ around an arbitrary large value at r = 0. In order to do this, the non-linear dielectric constant of the medium may be rewritten as

$$\in[\langle EE^{*}\rangle] = \in_o + \phi\left[\left\langle \frac{k(o)E_o^2}{2k(f)f^2}\right\rangle\right] + \phi\,[\langle EE^{*}\rangle] - \phi\left[\left\langle \frac{k(o)\,E_o^2}{2k(f)\,f^2}\right\rangle\right] \qquad ...(11)$$

$$\text{or, } \in(\langle EE^{*}\rangle) = \in_0^{'}(f) + \psi(r, f) \qquad ...(12)$$

$$\text{where } \in_0^{'}(f) = \in_o + \phi\left(\left\langle \frac{k(o)E_o^2}{2k(f)f^2}\right\rangle\right), \qquad ...(13)$$

$$\psi(r,f) = \phi\,[\langle EE^{*}\rangle] - \phi\left(\left\langle \frac{k(o)E_o^2}{2k(f)f^2}\right\rangle\right) << \in_0^{'}(f) \qquad ...(14)$$

Here, f is the dimensionless beam-width parameter, defined below in eq. (15) and k is the propagation constant defined below in eq. (16).

Here, in eq. (14), < > represents time average of many cycles. Using the WKB approximation and following the procedure used by Sodha et al.[8] and Akhmanov et al.[9], one can write –

$$E(r,z) = A(r, z)\left[\frac{k(o)}{k(f)}\right]^{1/2} \exp[-ik(f)\,z], \qquad ...(15)$$

$$\text{Where, } k(f) = \frac{\omega}{c}\left[\in_0^{'}(f)\right]^{1/2} \text{ and } k(o) = \frac{\omega}{c}\left[\in_0^{'}(f=1)\right]^{1/2}. \qquad ...(16)$$

In wave equation

$$\nabla^2 E + \frac{\omega^2}{c^2} \in E = 0$$

values of $\in$ and E can be substituted from Eqs. (12) and (15), which leads to parabolic equation as –

$$-2ik\,(f)\,\frac{\partial A}{\partial z} + \nabla_\perp^2 A + \frac{\omega^2}{c^2}\psi\,(r, f)\,A = 0 \qquad ...(17)$$

Putting

$A\,(r,z) = A_o\,(r,z)\exp\left[-i\int k(f)dS\right]$

and separating real and imaginary parts, one gets –

$$2\frac{\partial S}{\partial z} + \left[\frac{\partial S}{\partial r}\right]^2 = \frac{1}{k^2(f)A_o}\left[\frac{\partial^2 A_o}{\partial r^2} + \frac{1}{r}\frac{\partial A_o}{\partial r}\right] + \frac{\omega^2}{k^2(f)c^2}\psi(r, f) \qquad ...(18)$$

and

$$\frac{\partial A_o^2}{\partial z} + \frac{\partial S}{\partial r}\frac{\partial A_o^2}{\partial r} + A_o^2\left[\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r}\right) = 0 \qquad ...(19)$$

The solution of Eqs. (18) and (19) can be written as –

$$A_o^2 = \frac{E_o^2}{f^2}\exp\left[-\frac{r^2}{r_o^2 f^2}\right] \qquad ...\ (20)$$

$$S = \frac{r^2}{z}\beta(z) + \eta(z)$$

$$\beta = \frac{1}{f}\frac{\partial f}{\partial z}$$

where β corresponds to the inverse radius of curvature of the wave front and rf is the width of the main beam in the medium.

**Ohmic nonlinearity**

When the time duration of laser beam is longer than a temperature relaxation time, Ohmic heating of electrons becomes important. Solving the equation of motion for the oscillatory electron velocity due to the electromagnetic wave, we get –

$$\vec{v} = \frac{e\vec{E}}{mi\omega}\left(1 - i\nu/\omega\right) \qquad ...(21)$$

The component of $\vec{v}$ in phase with $\vec{E}$ causes electron heating at the rate $-\frac{e}{2}\vec{E}^{*}.\vec{v}$ $= e^2EE^*\nu / 2m\omega^2$ . In the steady state, this rate is balanced by the power loss via thermal conduction and collisons with ions and neutrals –

$$-\nabla.\left(\frac{\chi}{N}\nabla T_e\right) + \frac{3}{2}\delta\nu\,(T_e - T_o) = \frac{e^2\nu EE^*}{2m\omega^2}\ , \qquad ...(22)$$

where $\chi/N = v_{th}^2/\nu$ , $\delta$ =2 (m/$m_i$) for electron – ion energy exchange collision.

$v_{th}^2 = (2T_o / m)^{\frac{1}{2}}$ is the electron thermal speed and $T_e$ is the nonlinear field-dependent electron temperature and we may define characteristic times for thermal conduction and collisional energy transfer, $\tau_{con}$ and $\tau_{coll}$ as –

$$\tau_{con} \sim \frac{\nu r_o^2}{v_{th}^2} \text{ and } \tau_{coll} \sim (\delta\nu)^{-1}$$

where $r_o$ is the characteristic scale length of variation of $EE^*$. For $\tau_{coll} >> \tau_{con}$ i.e. $\frac{\nu^2 r_o \delta}{v_{th}^2} << 1$, one may ignore the second term in eq. (22).

Then

$$-\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\chi}{N}\frac{\partial}{\partial r}T_e\right) = \frac{e^2\nu EE^*}{2m\omega^2} \qquad ...(23)$$

For a beam of finite extent, Eq. (23) can be solved analytically only in the weak nonlinearity approximation : $T_e = T_o + \Delta T$, $\Delta T << Ti$. On integrating Eq. (23) with limits 0 to r and using the paraxial ray approximation, viz.

$$e^{-\frac{r^2}{r_0^2 f^2}} \cong 1 - \frac{r^2}{r_0^2 f^2}$$

Eq. (23) reduces to –

$$T_e - T_o \simeq -\frac{e^2\nu^2\ k(o)E_o^2 r^2}{8m\omega^2 v_{th}^2\ k(f)\,f^2} + c_1. \qquad ...(24)$$

Employing pressure balance, $\Delta N_e = -N\dfrac{\Delta Te}{T_{eo}+T_o}$

$$= -N(T_e - T_o)/2T_o$$ , Eq. (24) can be recasted as –

$$N_e - N = \Delta N_e = \frac{Ne^2\nu^2 k(o)E_o^2 r^2}{16m\omega^2 v_{th}^2\ T_o\ k(f)f^2} - \frac{c_1 N}{2T_o} \qquad ...(25)$$

Usually $c_1$ is not important in the case of small nonlinearity, however, for large nonlinearity and in the phenomenon of penetration of laser beams in overdense plasma, $c_1$ plays significant role. $c_1$ can be evaluated by assuming that for all practical purpose $N_e \approxeq N$ at $r = r_o$; hence –

$$C_1 = +\ 2T_o \frac{e^2\nu^2 k(o)E_o^2 r_o^2}{16m\,\omega^2\ v_{th}^2 T_o k(f)f^2}. \qquad ...(26)$$

On using Eq. (25) for electronic concentration, the expression for the dielectric constant is given by –

$$\in = \in_o + \frac{\omega_p^2(o)}{\omega}\frac{e^2\nu^2 k(o)E_o^2 r_o^2 W(z)}{16m\,\omega^2 v_{th}^2 T_o k(f)f^2}\left(1-\frac{r^2}{r_o^2}\right). \qquad ...(27)$$

Eq. (27) can be easily put in the form of Eq. (12) and using paraxial – ray approximation, $\in_0'(f)$ and $\psi$ (f) can be written as –

$$\in_o^{'}(f) = \in_o + \frac{\omega_p^2(o)}{\omega^2} \frac{e^2 v^2 k(o) E_o^2 r_o^2 W(z)}{16 m \omega^2 v_{th}^2 T_o k(f) f^2}, \quad ...(28)$$

and

$$\psi\,(\mathrm{f}) = -\, \mathrm{r}_2\, \frac{\omega_p^2(o)}{\omega^2} \frac{e^2 v^2 k(o) E_o^2 W(z)}{16 m \omega^2 v_{th}^2 T_o k(f) f^2}, \quad ...(29)$$

$$= -\, \mathrm{r}^2 \frac{\omega_p^2(o) \alpha E_o^2}{4\omega^2 f^2} \frac{k(o)}{k(f)} w(z),$$

Where $\alpha = \dfrac{e^2 \nu^2}{2 m^2 \omega^2 v_{th}^4}$. Let us substitute the value of $A_o^2$ and S from Eq. (20) and $\psi$ (r, f) from Eq. (29) in Eq. (18). Now equating the $r^2$ coefficients of both sides of resulting equation (followings the WKB approximation) and substituting the value of $\beta$, one obtains –

$$\frac{d^2 f}{dz^2} = \frac{1}{k^2(f) r_o^4 f^3} - \frac{\omega_{po}^2}{c^2} \frac{e^2 v^2 k(o) r_o^2 W(z)}{16 m \omega^2 v_{th}^2 T_o k^3(f)} \frac{E_o^2}{f}. \quad ...(30)$$

The self trapping condition may be written as –

$$\frac{\omega_{po}^2}{\omega^2} \frac{e^2 v_o^2 E_{ocr}^2}{m \omega^2 v_{th}^2 T_o} = \frac{c^2}{\omega^2 r_o^4}. \quad ...(31)$$

The critical power of the beam for self-focusing is thus –

$$P_{cr} = \frac{c}{8} r_o^2\, E_{ocr}^2 \left[ \in_o + \frac{\omega_p^2}{\omega^2} \frac{e^2 v_o^2 E_{ocr}^2}{m \omega^2 v_{th}^2 T_o} \right]^{\frac{1}{2}}$$

$$= \frac{2 c^3 m \omega^2 v_{th}^2 T_o}{r_o^2 \omega_{po}^2 e^2 v_o^2} \left( \in_o + \left( \frac{c}{\omega r_o} \right)^2 \right)^{\frac{1}{2}}$$

$$\simeq \frac{2 c^3 m \omega^2 v_{th}^2 T_o \in_o^{\frac{1}{2}}}{r_o^2 \omega_{po}^2 e^2 v_o^2}, \quad ...(32)$$

where $\frac{c}{\omega r_o} << 1$ . The initial condition on f are f(i = ν) = 1 and df/dz |z = 0 corresponding to an initially plane wave front. The first term on the right hand side (RHS) of eq. (30) corresponds to diffraction divergena and second term corresponds to convergence due to nonlinearity.

Equation (29) is valid for all profiles of unperturbed electron density. However, for the safe of explicitness, we have solved it numerically for a linear profile, viz., $W(z) = (1+B_z)$

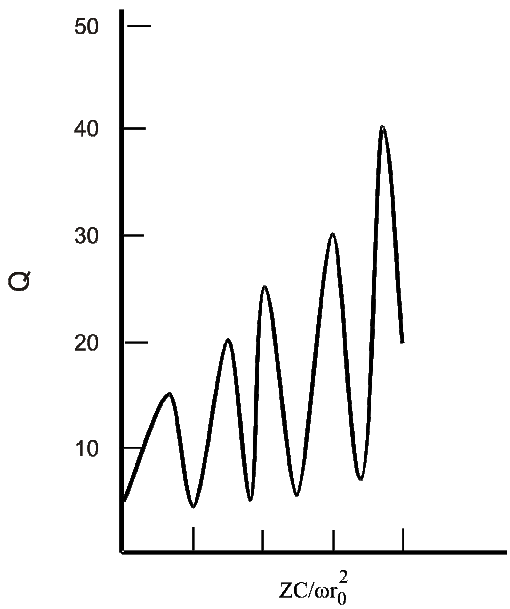


**Fig. 1: Variation of beam width parameter f and axial intensity of the beam as a function of distance of propagation for $r_o$ ω/c = 30, $\frac{\omega_{po}^2}{\omega^2}$ = 0.5, $\alpha E_o^2$ = 4.0, B $\frac{\omega}{c}$ $r_o^2$ = 5.0, $T_o = 10^5$ K, $\nu = 5 \times 10^{10}$ $s^{-1}$ and $r_o$ = 300 μm.**

Fig. 1 shows the variation of axial wave intensity $\left[ Q = \alpha E_o^2 k^{\frac{1}{2}}(o) \Big/ k^{\frac{1}{2}}(f) f^2 \right]$ as a function of distance of penetration into the plasma. As a competition of self-focusing and diffraction effects, the intensity varies in an oscillatory manner. As the beam penetrates in the plasma, the axial dielectric constant decreases, and one expects a turning point where $\in_o^{'}(f) = o$. However, the present treatment is not applicable around this point. A beam of higher power penetrate much deeper in the plasma

## CONCLUSION

A Gaussian laser beam propagating through a collisional plasma causes differential Ohmic heating of electrons. In a collisional plasma, the redistribution of the charge carriers is limited by thermal conduction only when $\delta r_o^2 / \left(\nu_{th}^2 / \nu_{ei}^2\right) << 1$.The electron plasma temperature increases with increasing flux. Moreover, with increasing temperature, the electron-ion collision frequency $\nu_{ei}$ decreases, and thermal – conduction losses become stronger, leading to periodic self – focusing of the main beam. In the case of conduction nonlinearity the critical power of the beam for self-focusing does not change, when power of the beam $p > p_{cr}$, the medium behaves as an oscillatory wave guide. For $p > p_{cr}$ the beam, initially plane, starts converging but after appreciable propagation, the axial inhomogeneity of the medium makes the influence of diffraction divergence so effective as to diverge the beam much before the focus is reached.

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