



## REVIEW ON EFFLUX TIME

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### ABSTRACT

Different types of storage vessels are in use in chemical industries. The reasons for the choice of the typical shape or geometry may be attributed to convenience, insulation requirements, floor space, material costs, corrosion and safety considerations. The time required to drain these vessels off their liquid contents is known as efflux time and this is of utmost importance in many emergency situations besides productivity considerations. Literature reports theoretical and experimental works for arriving at efflux time. The present review focuses on the literature available on efflux time. The scope for future work is also presented in this paper.

**Key words:** Chemical industry, Storage vessels, Efflux time, Productivity, Geometry.

### INTRODUCTION

Processing and storage vessels in the chemical and related industries appear in a large variety of shapes. The time required to empty these vessels off their liquid contents is known as efflux time<sup>1</sup> and this is of crucial importance in many emergency situations besides productivity considerations. This is of considerable interest in a variety of industries like chemical, food and pharmaceutical<sup>2</sup>.

For organizational purposes, the literature on efflux time is categorized in to

- Through restricted orifice.
- Through an exit pipe when the flow in the exit pipe is turbulent.

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- Through an exit pipe when the flow in the exit pipe is laminar.
- In presence of polymer solutions.

### Through restricted orifice

Mathematical analysis of efflux time for Newtonian liquid (below its bubble point) through restricted orifice located at the bottom of the vessel for annular (both horizontal and vertical) containers is carried out by Hart and Sommerfeld<sup>1</sup>. The authors mentioned that two fundamental equations must be invoked while solving draining problems from open storage tank. The first equation is related to the mass balance and a second equation is to the discharge coefficient and is derived based on Bernoulli equation. The two equations reported were

$$A \frac{dh}{dt} = -VA_0 \quad \dots(1)$$

where  $A$  and  $A_0$  refer to cross sectional area of tank and restricted orifice respectively,  $V$  refers to the linear velocity of liquid in the tank and  $\frac{dh}{dt}$ , the time variation of liquid level in the tank and  $h$  is the height of liquid in the tank and

$$V = C_0 \sqrt{2gh} \quad \dots(2)$$

Where  $C_0$  refers to discharge coefficient. They arrived at a general mathematical expression for efflux time given by

$$t = \frac{1}{C_0 A_0 \sqrt{2g}} \int_0^H \frac{A}{\sqrt{h}} dh \quad \dots(3)$$

Where  $H$  is the height of liquid before draining. If  $C_0 = 1$ , the equation for efflux time becomes Toricelli theorem.

Sommerfeld<sup>3</sup> derived efflux time equation for five new configurations. They are parallelepiped, vertical elliptical cylinder, regular tetrahedron, pyramid and paraboloid. The author mentioned that such shapes may be of use for academic purposes.

When drainage occurs through an orifice drain hole located at the bottom of the vessel, formulas for computing the drainage time required have been summarized by Foster<sup>4</sup> for a number of vessel shapes –vertical, horizontal, cylindrical and spherical. The author

mentioned that the discharge coefficient ( $C_d$ ) is constant for Newtonian fluid in turbulent flow, but it depends on the shape of orifice. The author considered discharge coefficient as 0.61 for sharp edged orifice, 0.8 for short flush mounted tube and 0.98 for rounded orifice.

However, Delozier et al.<sup>5</sup> reported a  $C_d$  value of 0.75 for an undergraduate experiment on efflux time through a drain hole at the bottom of a horizontal cylinder with flat ends. Work is also reported for comparing the efflux times for different geometries of vessels through a circular hole. Comparison of efflux time for cylindrical, spherical, cone and inverse cone, hemispherical shapes of tanks<sup>6</sup> are carried and the author derived the following expression for efflux time (T).

$$T = K \frac{V}{A\alpha\sqrt{gh}} \quad \dots(4)$$

V is the volume of liquid in the tank, A is the cross sectional area of tank, and h is the height of the liquid in the tank,  $\alpha$  is a constant and depends on the physical properties of the liquid and K is the coefficient given by -

$$K = \frac{\int_0^H \frac{A(u)du}{\sqrt{u}}}{\int_0^H A(u)du} \sqrt{H} \quad \dots(5)$$

Where A is the cross sectional area of tank and V is the volume of the liquid in the tank given by  $V = \int_0^H A(u) du$

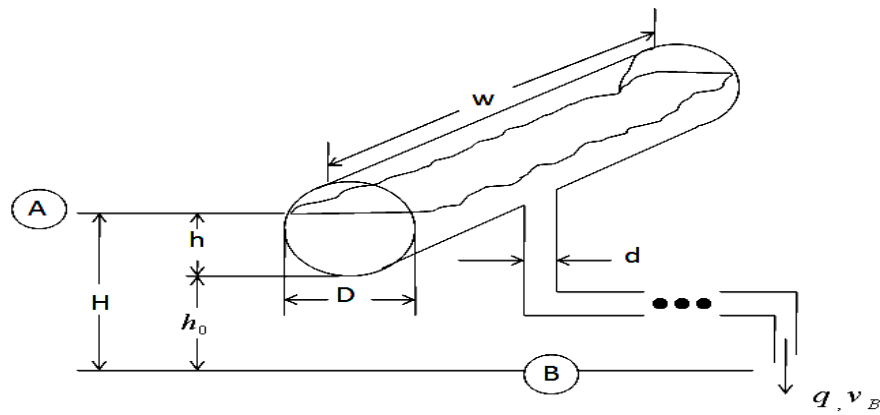
Higher values of K suggest higher draining time. The reported values of K are 3.2 for inverse cone, 2 for cylinder, 1.6 for sphere, 1.4 for hemisphere.

Libii<sup>7</sup> carried out the mathematical analysis of efflux time for draining a liquid from a cylindrical tank through restricted orifices of different diameters. The mathematical analysis is based on pseudo steady state assumption. The author mentioned that this assumption is valid for cross sectional ratio of tank to orifice as low as 100. It is also mentioned that, unlike a free falling particle which travels at constant acceleration during its fall, the free surface of a liquid decelerates continuously while draining. It is also highlighted that during draining of a liquid from a cylindrical tank through restricted orifice, Froude number remains constant.

Some authors<sup>8</sup> measured the diameter of jet of water exiting a hole near the base of the cylindrical container without knowing the efflux time. Their results suggested that contraction of the jet is not constant during draining and it increases with the height of liquid in the tank. They reported a contraction coefficient of 0.47 for the case of water drained through an exit pipe when the ratio of tank cross section to pipe cross section is 500.

**Through a single exit pipe when the flow in the exit pipe is turbulent**

Somemrfield and Stallybrass<sup>2</sup> derived expression for efflux time for the case of a horizontal cylindrical vessel with associated drain piping for the case of turbulent flow in the exit pipe. The configuration they considered is shown in Fig. 1.



**Fig. 1: Horizontal cylindrical tank with drain piping**

They reported the following equation for time ( $\tau$ ) required to drain the tank from some initial level ( $H_1 = h_1 + h_0$ ) to some final level  $H_2 (= h_2 + h_0)$   $\tau = \frac{G(H_1) - G(H_2)}{\alpha}$

Where the function G is defined as -

$$G = \int_{h_0}^{H_i} \left[ \frac{(2R + h_0 - H)(H - h_0)}{H} \right]^{1/2} dH \quad \dots(6)$$

$i = 1$  or  $2$   $R$  is the radius of the vessel and  $\alpha$  is a constant given by -

and 
$$\alpha = \frac{s}{2W} \left( \frac{2g}{1 + f \frac{L}{d}} \right)^{1/2} \quad \dots(7)$$

where  $s$  is cross sectional area of piping,  $f$  is friction factor and  $W$  is length of the tank,  $L$  is the length of piping,  $d$  is diameter of exit pipe.

Vandongen and Roche Jr.<sup>9</sup> carried out efflux time analysis from cylindrical tanks with exit pipes and fittings in the Reynolds number range of 40,000-60,000. The authors mentioned that under turbulent flow conditions in the exit pipe, the efflux time can be related to the height of the liquid ( $H_1$ ) relative to the bottom of the exit pipe  $H$  by  $t_{eff} = K_1 * (H_1^{3/7} - H^{3/7})$  where  $K_1$  is a constant given by

$$K_1 = \frac{7}{3} * \left(\frac{r}{r_0}\right)^2 \left[ \frac{0.0791 L_e \mu^{1/4}}{2^{1/4} g \rho r_0^{5/4}} \right]^{4/7} \quad \dots(8)$$

where  $r$  is radius of cylindrical tank,  $r_0$  is the radius of exit pipe,  $L_e$  is the total equivalent length of the exit pipe and fittings. They also mentioned that the goal of the experiment is that an abstract term  $L_e$  does have physical significance, a term that can be directly measured and observed through proper data analysis. They further stated that the analysis of set of data from different runs where the length of exit pipe has been changed can clearly demonstrate the concept of 'entrance effect'. The equivalent length of the pipe can be calculated that would have the same pressure drop or flow resistance caused by entrance effect.

When change in friction factor is ignored and an average value is used that that represents the average of flow regimes and pipe roughness, they used the following equation for efflux time

$$t_{eff} = K_2 * (H_1^{1/2} - H^{1/2}) \quad \dots(9)$$

where  $K_2 = 2 \left(\frac{r}{r_0}\right)^2 \left[ \frac{f_{avg} L_e}{g r_0} \right]^{1/2}$   $r$  is radius of cylindrical tank,  $r_0$  is radius of exit

pipe,  $f_{avg}$  is average friction factor and  $L_e$  is equivalent length. They used the following equation for friction factor for calculating the efflux time.

$$f = \frac{0.0316}{Re^{0.25}} \quad \dots(10)$$

They also mentioned that under such high Reynolds numbers, there is a possibility of existence of vena-contracta immediately downstream of contraction or piping that might contain trapped air or vapor.

Morrison<sup>10</sup> also modeled the efflux time equation for using computational techniques through an exit pipe for turbulent flow in the exit pipe. The Reynolds number considered is around 6,400. The author considered a contraction coefficient of 3.8 while arriving at efflux time. The maximum efflux time reported is 35 seconds. The tank to pipe cross sectional area in the work is 228.

The tank drainage problem in a cylindrical tank is studied in detail by Joye and Barret<sup>11</sup>. They derived the following equation for efflux time for draining the contents of a storage vessel through exit pipe for turbulent flow in the exit pipe.

$$t_{eff} = \frac{D^2}{d^2} \sqrt{\frac{2(4fL/d + \sum K)}{g}} (\sqrt{H_i + L_v} - \sqrt{H_f + L_v}) \quad \dots(11)$$

$t_{eff}$  is the efflux time to drain the tank from fluid height  $H_i$  to  $H_f$ ,  $L_v$  is the vertical drop of the exit pipe,  $D$  is the diameter of tank,  $d$  is diameter of exit pipe,  $\sum K$  is the resistance coefficient to account for fittings in the line, entrance and exit losses,  $L$  is the length of straight pipe. While deriving the above expression, they assumed the friction factor  $f$  to remain constant.  $\sum K$  include sum of exit kinetic energy loss ( $K = 1$ ) and the entrance loss ( $K = 0.5$ ) from tank to pipe. They reported an average deviation of 8% between experimental values and their model for turbulent flow in the exit pipe. They also stated that as the length of the exit pipe increases, efflux time decreases.

Mathematical equation efflux time from a cylindrical tank (Where the flow in the tank is essentially laminar) for turbulent flow in the exit pipe is reported by Subbarao and co-researchers<sup>12</sup>. Their analysis is based on macroscopic balances. They mentioned that macroscopic balances are useful for making preliminary estimate of an engineering problem. They are useful for developing approximate relations which will be then verified with the experimental data. They also made an assumption of constant friction factor for deriving the expression. They simplified the efflux time equation to the following form -

$$t = \sqrt{\frac{2}{g_m}} (\sqrt{H + L} - \sqrt{L}) \quad \dots(12)$$

The equation is named it as modified form of Torricelli equation.  $g_m$  is the modified form of acceleration due to gravity and is given by  $g_m = \frac{g}{\left(1 + 4f \frac{L}{d}\right) \left(\frac{A_t}{A_p}\right)^2}$ ,  $f$  is the friction

factor in the pipe line,  $L$  is the length of exit pipe,  $d$  is its diameter,  $A_t$  and  $A_p$  are cross

sectional areas of tank and exit pipe respectively. They mentioned that  $\frac{g_m}{g}$  is proportional to  $(Fr)^2$  where  $(Fr)$  denotes the Froude number. They further stated that the equation so developed will be of use for finding the minimum time required for draining the contents of the storage vessel. While deriving the above equation, the authors have not considered the contraction coefficient, friction in the tank, flow within the tank and roughness of the walls. They performed experiments for a tank of 0.27 m dia and exit pipe of dia.  $4 \times 10^{-3}$  m. They used the following friction factor equation to verify the validity of the model with experimental work

$$f = 0.0014 + \frac{0.125}{Re^{0.32}} \quad (\text{known as Drew correlation}) \quad \dots(13)$$

They mentioned that the advantage of using the equation is that it is valid in the range of Reynolds number starting from 3000 to  $3 \times 10^6$ . Even though the mathematical equation developed suggests that complete draining can be achieved, they could not achieve complete draining. The authors fine tuned the above friction factor equation and developed the following equation to validate the experimental data.

$$f = 0.0014 + \frac{0.125}{Re^{0.25}} \quad \dots(14)$$

The equation so developed took into account the contraction coefficient, the flow within the cylindrical tank and the friction in the pipe line. They verified the validity of the fine-tuned friction factor equation for 0.32 m and 0.34 m dia tanks<sup>13,14</sup> respectively; while keeping the exit pipe dia at  $4 \times 10^{-3}$  m. They also performed experiments for 0.25 m, 0.5 m, 0.75 m and 1 m lengths of exit pipes. The deviations between theoretical and experimental efflux times with fine tuned friction factor equation is observed to be less for 0.75 m and 1 m length of exit pipes possibly due to establishment of fully developed flow. The deviation is more for 0.5 m dia and 0.25 m dia exit pipes. They also noted that as the diameter of the tank increased, the deviation between theoretical and experimental values of efflux time also reduced possibly due to prevalence of pseudo steady state conditions. They observed that when a Newtonian liquid is drained from a cylindrical tank through an exit pipe, Froude number remained constant and only influenced by length and diameter of the exit pipe.

The authors also developed the following equation for efflux time accounting for contraction coefficient<sup>15</sup>.

$$t = \sqrt{\frac{2}{g_m}} (\sqrt{H+L} - \sqrt{L})$$

where

$$g_m = \frac{g}{\left(1 + 4f \frac{L}{d} + K_c\right) \left(\frac{A_t}{A_p}\right)^2} \quad \dots(15)$$

where  $K_c$  is the contraction coefficient.

The authors considered the contraction coefficient values of 1.5 reported by Joye and Barret and 3.8 reported by Morrison and mentioned that their experimental values were close to theoretical values for a contraction coefficient value of 3.8. They used the following friction factor equation reported by Bird et al.<sup>16</sup> for calculating the friction factor which in turn is used for calculating the efflux time.

$$f = \frac{0.046}{\text{Re}^{0.25}} \quad \dots(16)$$

They noticed that the error in efflux time equation using the friction factor equation reported by Bird et al.<sup>16</sup>, is much more than the friction factor equation reported by Drew.

Subbarao and co-researchers performed experiments for understanding the hydrodynamics of a Newtonian liquid while draining through two exit pipe system for turbulent flow in each of the exit pipe<sup>17</sup>. They derived the following equation for efflux time

$$t = \sqrt{\frac{2}{g_m'}} (\sqrt{H+L} - \sqrt{L}) \quad \dots(17)$$

$$g_m' = \frac{g}{\left(1 + 8f \frac{L}{d}\right) \left(\frac{A_t}{A_p}\right)^2} \quad \dots(18)$$

$g_m'$  is modified form of acceleration due to gravity for two exit pipe system. While deriving the above equation, they considered equal dia. of exit pipes and hence made an assumption that the velocity of fluid in each of the pipes is same. However, the authors did not verify this assumption. They used friction factor equation reported by Drew while evaluating the efflux time.

They carried out studies for two exit pipes each of  $4 \times 10^{-3}$  m dia and single exit pipe length of 0.75 m. They observed a maximum deviation of 12.7% between experimental



values and theoretical values of efflux time. The less deviation is due to reduced cross sectional area for flow leading to possibility of eliminating the vortices at the entrance of the exit pipe. They also mentioned that the ratio of efflux times for single exit pipe system with out polymer additions to that of two exit pipe system is at 1.7 for the tank diameters considered.

Subbarao et al.<sup>18</sup> also used the friction factor equation reported by Bird et al.<sup>16</sup> for verifying the mathematical equation for efflux time for two-exit pipe system. They mentioned that the error in arriving at theoretical efflux time using friction factor equation reported by Bird is more than that calculated based on the friction factor equation reported by Drew.

For two-exit pipe system, Santosh Kumar et al.,<sup>19</sup> derived the following equation for efflux time ( $t_{eff}$ ) for the case of turbulent flow in the exit pipe when a liquid is drained from a cylindrical storage tank.

$$\theta = 0.6044 * \left[ \left( 1 + \frac{H}{L} \right)^{3/7} - 1 \right] \frac{A_t}{A_p} \frac{L}{d} \quad \dots(19)$$

where dimensionless time  $\theta$  is given by -

$$\theta = \frac{t_{eff}}{\left( \frac{\mu d^2}{\rho g^4} \right)^{1/7}}$$

$H$  is the initial height of liquid in the tank,  $L$  is the length of the exit pipe,  $A_t$  is the cross sectional area of tank,  $A_p$  is the cross sectional area of exit pipe and  $d$  &  $L$  are diameter and length of the exit pipe respectively. The authors stated that the equation even though is derived for variable friction factor can also be used for constant friction factor in the exit pipe.

They performed experiments for fixed exit pipe lengths and reported a maximum deviation of 16% between theoretical and experimental values. They also mentioned that the variation of Reynolds number (and hence the friction factor) with initial height of liquid is marginal and hence the assumption of constant friction factor is justified. They also mentioned that during draining, the Froude number remains constant and influenced by diameter and length of the exit pipe.

Subbarao<sup>20</sup> reported efflux time models for draining a Newtonian liquid from a cylindrical storage vessel (Where the flow is laminar) through an exit piping system (When the flow is turbulent) without assuming constant friction factor. The efflux time equation is written in terms of dimensionless groups as shown below

where

$$\theta_1 = 0.8133 * \left[ \left( 1 + \frac{H_1}{L} \right)^{3/7} - 1 \right] \frac{D_1^2}{d^2} \frac{L}{d} \quad \dots(20)$$

where dimensionless time  $\theta_1$  is given by -

$$\theta_1 = \frac{t_1}{\left( \frac{\mu d^2}{\rho g^4} \right)^{1/7}}$$

$D_1$  is the dia. of cylindrical tank,  $d$  is dia. of exit pipe,  $H_1$  is initial height of liquid in the tank,  $L$  is the length of the exit pipe,  $t_1$  is efflux time and  $\rho$  is the density of liquid,  $\mu$  is the viscosity of liquid.

They also derived the following efflux time equation for a conical tank

$$\theta_2 = 0.35 * \frac{D_2^2}{H_2^2} \left( \frac{L}{d} \right)^3 X_2 \quad \dots(21)$$

Where  $\theta_2$  is dimensionless time and is also given by -

$$\theta_2 = \frac{t_2}{\left( \frac{\mu d^2}{\rho g^4} \right)^{1/7}}$$

$$X_2 = \left\{ \left[ \frac{7}{17} \left( 1 + \frac{H_2}{L} \right)^{17/7} - 1 \right] + \left[ \frac{7}{3} \left( 1 + \frac{H_2}{L} \right)^{3/7} - 1 \right] - \left[ \frac{7}{5} \left( 1 + \frac{H_2}{L} \right)^{10/7} - 1 \right] \right\} \quad \dots(22)$$

Where

Where  $t_2$  is efflux time,  $H_2$  the height of liquid in the tank,  $L$  is the length of the exit pipe,  $D_2$  is maximum diameter of cone and  $d$  is diameter of exit pipe.

For draining the same volume of liquid, the author carried out comparison of efflux times between cylinder and cone (for draining through exit pipe of same length and diameter). The ratio of efflux times is reported to be influenced by the height of liquid to length of the exit pipe. The author concluded that efflux time for cylinder is greater than that of cone.

Reddy and Subbarao<sup>21</sup> derived efflux time equation for sphere as well and obtained the following relation.

$$\theta_3 = 1.393 \left( \frac{L}{d} \right)^3 X_3 \quad \dots(23)$$

Where  $\theta_3$  is dimensionless time given by -

$$\theta_3 = \frac{t_3}{\left( \frac{\mu d^2}{\rho g^4} \right)^{1/7}} \quad \dots(24)$$

$$X_3 = \left\{ \frac{7R_3}{5L} \left[ \left( 1 + \frac{H_3}{L} \right)^{10/7} - 1 \right] - \frac{14R_3}{3L} \left[ \left( 1 + \frac{H_3}{L} \right)^{3/7} - 1 \right] - \left[ \frac{7}{17} \left( 1 + \frac{H_3}{L} \right)^{17/7} - 1 \right] - \left[ \frac{7}{3} \left( 1 + \frac{H_3}{L} \right)^{3/7} - 1 \right] + \left[ \frac{7}{5} \left( 1 + \frac{H_3}{L} \right)^{10/7} - 1 \right] \right\} \quad \dots(25)$$

Where  $t_2$  is the efflux time for spherical tank,  $R_3$  is radius of sphere,  $H_3$  is height of liquid in the sphere,  $d$  is the diameter of exit pipe,  $L$  is the length of the exit pipe.

The authors carried out the theoretical comparison of for spherical tank with that of a cylindrical tank for draining the same volume of liquid through an exit pipe of same length and diameter. The authors concluded that for draining the same volume of liquid, Sphere drains faster than a Cylinder. How faster is the draining time is influenced by height and length of the exit pipe.

### **Through an exit pipe when the flow in the exit pipe is laminar**

Joye and Barret<sup>11</sup> used the following efflux time equation reported by Bird et al.<sup>16</sup> for laminar flow in the exit pipe for verifying the experimental values.

$$t = \frac{32\mu LD^2}{\rho g d^4} \ln\left(1 + \frac{H}{L}\right) \quad \dots(26)$$

$t$  is efflux time,  $D$  is the diameter of tank,  $L$  is the length of exit pipe,  $\mu$  and  $\rho$  are the viscosity and density of liquid respectively.  $H$  is the initial height of liquid in the tank. The authors neglected the exit kinetic energy and other friction losses in the tank.

The authors mentioned that laminar flow equations can't be valid for short pipes.

Work is also reported for draining a Newtonian liquid through from a storage tank an exit pipe for laminar flow conditions in the exit pipe<sup>22</sup>. It is mentioned that efflux time so obtained will be useful for arriving at the maximum draining time required for draining the contents of the storage vessels. They authors derived expressions for efflux time equations for cylindrical, conical and spherical tanks for laminar flow in the exit pipe. The exit pipe diameter and length in all the cases remained same. The authors arrived at the following equations for efflux time for cylinder, cone and sphere respectively for laminar flow conditions in the exit pipe.

$$t_0 = \frac{128\mu LR^2}{\rho g D^4}$$

Same as Eq. 9

$$t_1 = \frac{32\mu LD^2}{\rho g d^4} \left( \frac{1}{2} - \frac{L}{H} + \frac{L^2}{H^2} \ln\left(1 + \frac{H}{L}\right) \right) \quad \dots(27)$$

$$t_2 = \frac{32\mu LD^2}{\rho g d^4} \left( \frac{1}{2} + \frac{L}{H} \ln\left(1 + \frac{H}{L}\right) - \frac{L^2}{H^2} \ln\left(1 + \frac{H}{L}\right) \right) \quad \dots(28)$$

Where  $t_0$  is the efflux time for cylinder,  $t_1$  is the efflux time for cone,  $t_2$  is the efflux time for sphere,  $L$  is the length of the exit pipe,  $D$  is the diameter (in case of cone maximum diameter) and  $d$  is the dia. of exit pipe.  $\mu$  and  $\rho$  represent the viscosity and density of the liquid. For draining the same volume of liquid through exit pipe of same dia, the authors compared the efflux time equations so developed. The authors stated that the ratio of efflux times of any two tanks is influenced only by ratio of height of liquid in the tank to the length of the exit pipe. The author arrived at the following order of efflux times.

Efflux time for cylinder > sphere > cone

### **In presence of drag reducing polymer solutions**

Subbarao et al.<sup>12-14</sup> reported that while draining a liquid from a large cylindrical storage vessel through an exit pipe, the flow in the tank is essentially laminar and turbulent in the pipe depending on the diameter of the exit pipe and physical properties of the liquid to be drained. During draining, the liquid experiences friction and this friction is a measure of drag. This drag increases drastically when flow transforms from laminar in the tank to turbulent in the exit pipe. Hence, drag reduction options are to be explored. They performed their experiments with water for carrying out efflux time studies, since water is a Newtonian fluid and happens to be used in most of the applications. Besides this, it is a good solvent that offers excellent resistance to shear degradation of polymer additives.

They further studied the effect of water soluble polyacrylamide (PAM) polymer on drag reduction. PAM concentrations considered by the authors are 40, 30, 20 and 10 ppm and arrived at 10 ppm optimum concentration. In the concentration range considered, they assumed the polymer solutions to behave like Newtonian fluids. They also mentioned that polymer solutions decrease the efflux time and hence increase the Froude number.

The authors carried out the analysis of two-exit pipe system and used polyacrylamide polymer solutions of 10, 5, 2.5 and 1 ppm concentration and arrived at an optimum of 10 ppm. They also concluded that maximum drag reduction is 24% for two exit pipe system as against 26% for single exit pipe system.

Subbarao<sup>23,24</sup> performed experiments with polymer solutions for exit pipe diameter of 0.008 m exit pipe and a tank of 0.32 m dia. They noticed no reduction in drag. Hence, they concluded that drag reduction is effective only for ratios of cross section of tank to exit pipe  $> 1600$ . This ratio also establishes the saturation limit of Froude number upon addition of water soluble polyacrylamide solution.

Gopal Singh and co-workers<sup>25</sup> performed experiments for draining a Newtonian liquid from cylindrical tanks of different geometries through exit piping system using polyacrylamide and polythene oxide polymer solutions. They reported that polythene oxide is a better drag reducing agent for laminar flow in the exit pipe where as polyacrylamide is a better drag reducing agent when the flow is turbulent. They observed that optimum concentration using polyacrylamide is 10 ppm for laminar flow and 5 ppm for turbulent flow. The optimum concentration using polythene oxide is 20 ppm for laminar flow and 40 ppm for turbulent flow.

### Scope for future work

Future work can be focused on reduction in efflux time using dilute solutions of different water soluble polymers with different geometries of vessels by maintaining both laminar and turbulent flow conditions in the exit pipe. Some work is initiated in this direction. Work can also be focused on developing models for Non-Newtonian polymer solutions as well. The efflux time equations so developed can be verified with experimental values.

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