# Research on numerical model-based volleyball best spiking angle application in competition tactics 

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#### Abstract

Spiking is a power way to gain scores in volleyball competition, the paper establishes mathematical model for volleyball spiking, analyzes ball drop points after spiking volleyball in different orientations and angles in competitions, gets hitting angle range that drops into difficult defensive regions after volleyball being hit by players, in solving process $v$ size is also considered as fixed. For established mathematical model, use Lagrange differential equation extremum method to solve, due to compute sign computed results are very complex, formula is very long, we use Matlab simulation data, then use computer to solve, and solve the range of hit angle $\theta_{x}, \theta_{y}$. Make comprehensive comparison, it gets that ball optimal hitting direction is in cased optimal hitting angle in the range of $\theta_{x}=\left[0^{\circ}, 41.34^{\circ}\right], \theta_{y}=\left[71.82^{\circ}, 80.10^{\circ}\right]$, which can spike good shot to difficult defensive regions, from which $\theta_{x}, \theta_{y}$ are respectively angles with $x, y$ axis during hitting instant.


## KEYWORDS

Spiking speed; Difficult defensive region; Angle range; Differential equation; Numerical modeling.

## INTRODUCTION

Volleyball competition is a competition that fights for high and quick, viewed from the side, it is composed of field left and right two teams with six person each, a set is 25 points, the one that first gains the 25 points will win the set, two parties in competition process generally gain scores by spiking.

Lots of scholars have studies on athlete spiking motion and mechanical aspects, Huang Jin-Ping (2009) studied on volleyball spiking technical motions, especially for running-up, taking-off, hitting in the air and dropping, Lv Pin (2003) studied on volleyball spiking instant taking-off technique, improved spiking hit rate, Cheng Zhan-Ming and Chen Ya-Xian (2002) got best arm swinging motion from member spiking arm swinging kinematics researches. Ren Ran mainly studied on spiking effects from technical motions during spiking and mechanical aspects.

The paper researches on player instantaneous spiking status mainly from players spiking instantaneous leaping ability and instant hitting ball strength as well as orientations, defines how player hit ball so that can succeed in spiking good shot, so-called spiking good shot is the ball can succeed in dropping into opponent field after hitting. Due to field six players are arranged in symmetry, relatively middle defensive ability is stronger, which requires ball to be succeeded in dropping into opponent difficult defensive regions after player spiking, generally is the region in the back row.

## VOLLEYBALL HITTING MECHANICAL ANALYSIS

## (1) Ideal model establishment

To simplify research process, regard volleyball as particle and during volleyball motion process, and don't consider volleyball rotation, air resistance to volleyball motion influences.

## (2) Volleyball motion physical process

Regard volleyball motion after volleyball competition close spiking, far spiking and jump service as throwing. After athletes hitting, volleyball is thrown at horizontal speedv ${ }_{0}$, volleyball flight motions meet relative mechanics principles.

Volleyball flight motion time is up to volleyball heighth, volleyball flight motion horizontal distance is up to volleyball heighth and horizontal speedvo.

## (3) Volleyball motion rules in different cases

Front row attacking extreme (maximum) speed studies
When ball is horizontally thrown from the height of 2.43m,drop point is just on the baseline.
Similarly it can also solve ball's maximum speeds in different heights. So when make close spiking in the front row of men's volleyball, maximum speed is less than $12.78 \mathrm{~m} / \mathrm{s}$.

## (4) Back row attacking height and speed range studies

Spiking in the back row 3 m line 2.70 m height area at maximum speed $v_{\text {max }}$, volleyball drops in baseline. After up casting ball in service point, make jumping service in the area of 3.5 m , when ball falling height (3.5-2.43)1.07m just gets through the net, volleyball possessed speed is:

$$
v_{\text {sever }}=\frac{s}{t}=9 \sqrt{\frac{9.8}{2 \times 1.07}}
$$

Horizontal distance between dropping point and service point is :

$$
\begin{aligned}
& s_{\text {sever }}=v_{\text {sever }} \cdot t_{\text {sever }} \\
& s_{\text {serve }}=19.26 \sqrt{\frac{2 \times 3.5}{9.8}}
\end{aligned}
$$

Set jumping service extreme (maximum) speed is $v_{\max }$. when make jumping service,volleyball drop point is just in the baseline, volleyball horizontal displacement is 18 m ,it has $v_{\text {max }}=s / t=18 / 0.845=21.3 \mathrm{~m} / \mathrm{s}$.

When jumping service minimum height is $h_{\text {min }}$, after service, volleyball drops in baseline.
Due to volleyball net two sides’ sports horizontal displacements are the same, corresponding time is also the same. Therefore ratio between volleyball falling height in the right side of net and volleyball falling height in the left side of net is $1: 3$, jumping service minimum height is:
$h_{\text {min }}=\frac{4}{3} h_{\text {net }}=3.24 \mathrm{~m}$
Similarly, it can solve different heights jumping service speed range.
Back athletes run in the back row and jump into the area of front row 1.5 m A point to attack,if hitting height is 2.8 m . hitting maximum speed $v_{\max }$ is: $v_{\max }=13.89 \mathrm{~m} / \mathrm{s}$.
Change 1. If it wants to drop in 1.5 m Bpoint far from bottom edge, hitting instant speed is $11.9 \mathrm{~m} / \mathrm{s}$. Change 2. When athlete hits in A point, and meanwhile another athlete runs and saves in Cpoint, ball still drops in B point,distance between B, C is 4.0 m , then save athlete minimum speed $v_{\text {min }}$ is $5.29 \mathrm{~m} / \mathrm{s}$.

Of course when spiking in volleyball competition, it can carry on cross spiking; when serving, make cross service to improve attacking speed and reduce opponent net interception opportunities. Such cases will not be further analyzed.

## ESTABLISH MODELS

Establish coordinate axis in hitting point, during ball hit instant, assume sphere center is in the origin (as following Figure 1 shows), then it gets speed components in $x, y, z$ axis $v_{x}, v_{y}, v_{z}$ :


Figure 1 : Hitting model

$$
\begin{align*}
& v_{x}=V \cdot \cos \theta_{x}  \tag{1}\\
& v_{y}=V \cdot \cos \theta_{y}  \tag{2}\\
& v_{z}=\sqrt{V^{2}-v_{x}^{2}-v_{y}^{2}} \tag{3}
\end{align*}
$$

Success lies in the three components speeds sizes, and the three speeds components sizes is up to ball possessed speeds (that hitters' hitting strength and hitting orientation) after player hitting the ball, the problem requires to let ball to drop in difficulty defensive regions by defining $\theta_{x}, \theta_{y}$ angle range, so that problems solution can be summarized as objective functions when considering to succeed in dropping in difficult defensive regions each kinds of status:
$\min \theta_{x}, \max \theta_{x}, \min \theta_{y}, \max \theta_{y}$.
s.t
$\left(L-l_{3}+l_{1}\right) \leq L_{x} \leq\left(L+l_{1}\right)$
$L_{y} \geq H$
$-l_{b} \leq L_{z} \leq\left(L-l_{b}\right)$
$x$ axis direction volleyball can succeed in dropping into regions of back row, ball values range in $x, y, z$ axis direction (use origin as 0 to define values sizes), as Figure 2.


Figure 2 : Spiking dropping region

## Solve model

$\theta_{\mathrm{x}}, \theta_{\mathrm{y}}$ range sizes changes must conform to following conditions:
$\left(\mathrm{L}-\mathrm{l}_{3}+\mathrm{l}_{1}\right) \leq \mathrm{L}_{\mathrm{x}} \leq\left(\mathrm{L}+\mathrm{l}_{1}\right)$
$L_{y} \geq H$

$$
\begin{equation*}
-l_{b} \leq L_{z} \leq\left(L-l_{b}\right) \tag{5}
\end{equation*}
$$

The process of volleyball from hit to drop, by vertical direction ( $y$ axis), it can get:

$$
v_{y}=\frac{d_{y}}{d_{t}} \Rightarrow d_{y}=v_{y} d_{t}
$$

Input $y_{0}=h, y_{t 1}=0$ and (2); $y_{0}=h, y_{t 2}=H$ and (2), and can get :

$$
\int_{h}^{0} d_{y}=v_{y} t+\frac{1}{2} g t^{2}=V t \cos \theta_{y}+\frac{1}{2} g t^{2}
$$

$$
\Rightarrow\left\{\begin{array}{l}
t_{1}=\frac{-V \cos \theta_{y}+\sqrt{\left(V \cdot \cos \theta_{y}\right)^{2}+2 g h}}{g} \\
t_{2}=\frac{-V \cos \theta_{y}+\sqrt{\left(V \cdot \cos \theta_{y}\right)^{2}+2 g(h-H)}}{g}
\end{array}\right.
$$

Then, volleyball can succeed in dropping in back row difficulty defensive region, then it must conform to following conditions:
$\left\{\begin{array}{l}\left(L-l_{3}+l_{1}\right) \leq v_{x} t_{1} \leq\left(L+l_{1}\right) \\ -l_{b} \leq v_{z} t_{1} \leq\left(L-l_{b}\right) \\ v_{x} t_{2} \geq l_{1}\end{array}\right.$

## Cases that ball drops to boundary difficult defensive region (divide into two cases here)

(1) Ball drops into boundary difficult defensive regions that further from hitting positions

If it wants the ball to drop into the region, it must conform to following conditions:
$\left\{\begin{array}{l}l_{1} \leq v_{x} t_{1} \leq\left(L+l_{1}\right) \\ v_{x} t_{2} \geq l_{1} \\ \left(L-l_{2}-l_{b}\right) \leq v_{z} t_{1} \leq\left(L-l_{b}\right)\end{array}\right.$
(2) Drop into boundary difficult defensive region that is near to hitting position

Similarly the ball needs to drop into the region, it must conform to following conditions:

$$
\left\{\begin{array}{l}
l_{1} \leq v_{x} t_{1} \leq\left(L+l_{1}\right) \\
v_{x} t_{2} \geq l_{1} \\
\left|l_{b}-l_{2}\right| \leq v_{z} t_{1} \leq l_{b}
\end{array}\right.
$$

To the same player, leaping ability is fixed, it might as well adopt maximum heights that he can arrive at to solve spiking problems, and meanwhile to a same player, generally he hits with his strongest strength to spike, so in solving process, V size is also thought to be fixed, in this way by above established model, use Lagrange differential equation extremum method to solve, due to compute signs computed results are very complex, formulas are very long, we use simulation data and then use computer to solve, and solve hitting ball angle $\theta_{x}, \theta_{y}$ range :
Due to:

$$
\left\{\begin{array}{l}
\left(L-l_{3}+l_{1}\right) \leq V t_{1} \cos \theta_{x} \leq\left(L+l_{1}\right) \\
-l_{b} \leq t_{1} \sqrt{V^{2}-\left(V \cos \theta_{x}\right)^{2}-\left(V \cos \theta_{y}\right)^{2}} \leq\left(L-l_{b}\right) \\
V t_{2} \cos \theta_{x} \geq l_{1} \\
t_{1}=\frac{-V \cos \theta_{y}+\sqrt{\left(V \cos \theta_{y}\right)^{2}+2 g h}}{g} \\
t_{2}=\frac{-V \cos \theta_{y}+\sqrt{\left(V \cos \theta_{y}\right)^{2}+2 g(h-H)}}{g}
\end{array}\right.
$$

Therefore, by computing, it can solve $\theta_{x}, \theta_{y}$ should conform to following formulas:

$$
\left\{\begin{array}{l}
g\left(L-l_{3}+l_{1}\right) \leq V \cdot\left(-V \cdot \cos \theta_{y}+\sqrt{\left.\left(V \cdot \cos \theta_{y}\right)^{2}+2 g h\right)} \cos \theta_{x} \leq g\left(L+l_{1}\right)\right. \\
-g l_{b} \leq V \sqrt{1-\cos ^{2} \theta_{x}-\cos ^{2} \theta_{y}}\left(-V \cdot \cos \theta_{y}+\sqrt{\left.\left(V \cdot \cos \theta_{y}\right)^{2}+2 g h\right) \leq g\left(L-l_{b}\right)}\right. \\
V\left(-V \cdot \cos \theta_{y}+\sqrt{\left(V \cdot \cos \theta_{y}\right)^{2}+2 g(h-H)}\right) \cos \theta_{x} \geq g l_{1}
\end{array}\right.
$$

Compute the inequation, we can use Lagrange differential equation extremum method to solve: We let:

$$
\begin{aligned}
& Q=V\left(-V \cos \theta_{y}+\sqrt{(V \cos \theta)^{2}+2 g h}\right) \cos \theta_{x}-\mathrm{g}\left(\mathrm{~L}-l_{3}+l_{1}\right)+\lambda\left(V \cdot \sqrt { 1 - \operatorname { c o s } ^ { 2 } \theta _ { x } - \operatorname { c o s } ^ { 2 } \theta _ { y } } \text { respectively by solving equation set } \left\{\begin{array}{l}
\frac{\partial Q}{\partial u}=0 \\
\left(-V \cdot \cos \theta_{y}+\sqrt{\left.\left(V \cdot \cos \theta_{y}\right)^{2}+2 g h\right)}+g l_{b}\right) \\
\frac{\partial Q}{\partial V}=0 \\
\frac{\partial Q}{\partial \lambda}=0
\end{array}\right.\right.
\end{aligned}
$$

from which $u=\cos \theta_{x}, v=\cos \theta_{y}$, it solves $u, v$ values, and then substitute values back to (6) to test, if results are incorrect, then adjust $\theta_{x}, \theta_{y}$ range, then substitute back, and solve its range.

Case that ball drops into boundary difficult defensive region that is farther from hitting positions
From context,it can get $\theta_{x}, \theta_{y}$ range size must conform to following conditions:

$$
\left\{\begin{array}{l}
g l_{1} \leq V\left(-V \cos \theta_{y}+\sqrt{\left(V \cos \theta_{y}\right)^{2}+2 g h}\right) \cos \theta_{x} \leq g\left(L+l_{1}\right) \\
g\left(L-l_{2}-l_{b}\right) \leq V \sqrt{1-\cos ^{2} \theta_{x}-\cos ^{2} \theta_{y}}\left(-V \cos \theta_{y}+\sqrt{\left(V \cos \theta_{y}\right)^{2}+2 g h}\right) \leq g\left(L-l_{b}\right) \\
V\left(-V \cos \theta_{y}+\sqrt{\left(V \cos \theta_{y}\right)^{2}+2 g(h-H)}\right) \cos \theta_{x} \geq g l_{1}
\end{array}\right.
$$

## Case that ball drops in boundary difficult defensive region that is closer to hitting positions

 From above, it can get $\theta_{x}, \theta_{y}$ range size that should conform to following conditions:$$
\left\{\begin{array}{l}
g l_{1} \leq V\left(-V \cos \theta_{y}+\sqrt{\left(V \cos \theta_{y}\right)^{2}+2 g h}\right) \cos \theta_{x} \leq g\left(L+l_{1}\right) \\
-g\left|l_{b}-l_{2}\right| \leq V \sqrt{1-\cos ^{2} \theta_{x}-\cos ^{2} \theta_{y}}\left(-V \cos \theta_{y}+\sqrt{\left(V \cos \theta_{y}\right)^{2}+2 g h}\right) \leq-g l_{b} \\
V\left(-V \cos \theta_{y}+\sqrt{\left(V \cos \theta_{y}\right)^{2}+2 g(h-H)}\right) \cos \theta_{x} \geq g l_{1}
\end{array}\right.
$$

## Model actual data solution

By volleyball competition rules, we can know following data: volleyball court is generally in the length of 18 m ,width of 9 m ,two equal courts areas with the length of 9 m , width of 9 m that are divided by center line ; net height (men's net height is 2.43 m , women's net height is 2.24 m ); Ball specification is with circumference from 65 cm to 67 cm . By international relative staff analyzing, during spiking, ball speed can arrive at $80 \mathrm{~km} / \mathrm{h}$, player spiking height arrives at above 3.5 m ; difficulty defensive region is back court area that nearly 1.2 m , left and right are nearly 0.8 m ; we use player spiking in 1 m area far from center line,ball court gravity accelerated speed $g=9.8 \mathrm{~m} / \mathrm{s}^{2}$ to research on angle range that he can succeed:

We utilize women's volleyball spiking cases, use general ball speed $81 \mathrm{~km} / \mathrm{h}$, hitting point height of 3.5 m to estimate, by above solving, it gets conditions that $\theta_{x}, \theta_{y}$ should conform to, then it can get following three inequation sets, and then utilize computer to compute, use Matlab6.5,it can calculate and get following TABLE 1 :

TABLE 1 : Spiking data

| Angle range | $\boldsymbol{\operatorname { m i n }} \theta_{\mathbf{x}}$ | $\boldsymbol{\operatorname { m a x }} \boldsymbol{\theta}_{\mathbf{x}}$ | $\boldsymbol{\operatorname { m i n }} \boldsymbol{\theta}_{\mathbf{y}}$ | $\boldsymbol{\operatorname { m a x }} \boldsymbol{\theta}_{\mathbf{y}}$ |
| :--- | :---: | :---: | :---: | :---: |
| Back row difficult defensive region | $0^{\circ}$ | $41.34^{\circ}$ | $71.82^{\circ}$ | $80.10^{\circ}$ |
| Longer boundary difficult defensive region | $41.40^{\circ}$ | $84.60^{\circ}$ | $73.80^{\circ}$ | $90.00^{\circ}$ |
| Shorter boundary difficult defensive region | $0^{\circ}$ | $84.60^{\circ}$ | $0^{\circ}$ | $90.00^{\circ}$ |

Then we can get optimal hitting angle is in the range of $\theta_{x}=\left[0^{\circ}, 41.34^{\circ}\right], \theta_{y}=\left[71.82^{\circ}, 80.10^{\circ}\right]$ that can spike good shot to difficult defensive regions.

## CONCLUSION

By reasonable hypothesis and certain simplification, the paper applies qualitative analysis and other methods to establish model, and then puts forward optimal hitting angle, which has certain actual significance in volleyball competition spiking direction, the model can be promoted and applied in differential equation establishment, solve its hitting angles, for service status, it can also solve its optimal hitting plan by establishing the model, but the model has some constraints, model considers ball falling as free fall motion, actually upper ball falling may not always like this, spiking is not surely required to spike into difficult defensive region so that can succeed in gaining scores, competitions should use flexible methods, aim at opponent difficult defensive points to spike, which can easier to get scores, so volleyball competition should use multiple tactics to win, model is just a simple hypothesis spiking method, in actual volleyball training, it has certain effects. From solving process, we can know hitting angle $\theta_{y}$ gets smaller, ball drop point will not prone to out of the bounds, so in volleyball spiking training, it can utilize the point to reduce spiking out-of-bounds rate, the establishment of the model is relative valuable in the aspect.

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