Research on multi objective planning rental pricing mode of the public rental housing based on tripartite satisfaction

Shan Xueqin*, Teng Yue, Li Xiao, Chen Jiajia
Department of Construction Management and Real Estate, Chongqing University, (CHINA)
E-mail : 895648944@qq.com; 604758622@qq.com; 364693126@qq.com; 794410443@qq.com

ABSTRACT
Public rental housing is provided for urban low-income hierarchy and is an important part of affordable housing system. The third plenary session of the 18 “decision” points out that the system of affordable housing allocation need be optimized and the construction of low-rent public housing and public rentals will be gradually merged. At this point, the importance of the public rental house has further increased. In 2013, China plans to start the construction of 6.3 million suites of new affordable housing, including 4.7 million suites to be completed basically. With the completion of the public rental housing project, Community for public rental project research is gradually moving from the construction phase to management. As an important part of the late-stage management, the reasonable of rent-setting determines the acceptability of tenants and the sustainability of subsequent funds of public rental housing. However, the standard of pricing is disordered at present and thus hinder the development of Housing Social Security System. The paper is in order to find an Interest-balancing system under the overall objective of satisfying the government, the constructors and the tenements at the same time to solve this problem. On the basis of considering the time value of money, the basic method of Multi-Objective Planning was used to build the objective function and the constraints. Using the expert grading method, the weight of the objective function was obtained. Then Utility Optimization Model was used to solve the objective function. Combined with the objectives and the model of pricing, the Rental Pricing Mode of the public rental housing based on tripartite satisfaction was built. Then the models was tested and verified to see if it is feasible by using the data of “Min Xin Jia Yuan” which will help to institute a thorough mechanism of public rental housing in the period of late-stage management.

KEYWORDS
Public rental housing; Rent-setting; Multi-objective planning; Tripartite satisfaction.

INTRODUCTION
Public rental housing means the indemnification housing that invested by the government. They provide policy support and offer specific person with preferential rental price. With the completion of the public rental housing project, the pricing of public rental housing seems to be more and more important. The high rent will make
an excessive burden to the tenants. But the low rent will still lead to an inequitable distribution of resources about social welfare and cause the pressure of social capital debt service which is harmful to the sustainable development of public rental projects. Therefore, under the basic objective that satisfying the government, the constructors and the tenements at the same time, the multiple objective dynamic Rental pricing model on the basis of cash flow will be made in the paper.

DOMESTIC AND INTERNATIONAL LITERATURE REVIEW

In western countries, public housing means the affordable housing which has no property provided by non-profit institutions or government to low-income people. As for rent-setting of public rental housing, Gyourko and Lin Lemneman thought that ability to pay should be considered and that people with differential income should have differential rents\(^1\). For the same reason, Ngai Ming and Yip Kwok Yu Lau suggested that Hong Kong integrate rent pricing mechanism with affordability\(^2\). When considering the factors affecting the rent, Ibem E O pointed out that the location of the house, the environment, the facility and the management might have huge effects on residents according to the questionnaire survey in Nigeria\(^3\).

In 2009, China began to build public rental housing, and lots of scholars began to study the rent-setting of public rental housing. Meng Weidong and Liu Xin put forward that we should follow the principle based on social average burden and implement differential pricing\(^4\). Feng Hui presented similar ideas and further explained that government should support it by making rules\(^5\). Cheng Rongrong come up with a "fundamental rent and adjustment rent" model. However, it is not perfect because of lacking enough adjustment factors\(^6\).

Views mentioned above generally fall into two broad types. One contends the importance of guarding the benefits of all tenants. The other focuses on the sustainability of the project, but few of them integrate both of sides to consider. Therefore, the article puts forward the multiple objective dynamic Rental Pricing Model which synthesize the objects of tenants, government and the constructors, and fully reflects the social security and sustainability of public rental housing and forms a reasonable rent mechanism.

THE OBJECTIVE OF RENT-SETTING THAT SATISFYING TRIPARTITE PARTS

The minimize of the living costs

The living costs of tenants include not only the housing expenditure, but a series of non- housing expenditure, such as property management fees, transportation fees, water and electricity gas fees etc. These factors were used in the model.

The balance of the construction costs

Public housing is a “people’s livelihood” project, so the rent is unfavorable too high. But low rent cannot guarantee the constructors to pay off the debts on time. So the objective of constructors should be ensuring that the construction costs can be balanced.

The maximization of social benefits

The social benefit of government investment project is based on benefiting the whole society, which means the public interest is satisfied. Therefore, public housing project should be regarded to be social beneficial, the government’s goal is to achieve maximum social benefits under the condition of the limited resources.

CONSTRUCTION OF RENT-SETTING MODEL

The above analysis shows the rent-setting model of Public rental housing needs to consider the interests of the three parties, it is a typical multi-objective decision-making problem. The multi-objective planning method is used to solve such problems.

The basic model of multi-objective pricing

Suppose the objective function is \( F(X) \), constraint condition is \( C(X) \), \( G \) is a constant vector, and \( X \) is decision variables of plan. The general model for multi-objective planning problem:

\[
\begin{align*}
\max \ / \ min \ Z = & \ F(x) \\
& A(x) \leq G
\end{align*}
\]  

(1)

The Utility Optimization Model is used to solve the equation when they reach the extreme:
\[
\max Z = \sum_{i=1}^{n} \sigma_i f_i(x) \quad \left( \sum_{i=1}^{n} \sigma_i = 1 \right) \tag{2}
\]

Construction of the dynamic multi-objective rental pricing mode

Construction of the objective function

(1) The minimization of living cost

Living costs includes public rental housing rents \( P_i \), and cost of living \( U_t \). \( U_i \) includes expenses of property management fees \( U_{1t} \), transportation fees \( U_{2t} \), utility and gas costs \( U_{3t} \) in the t year of the family, \( U_t = U_{1t} + U_{2t} + U_{3t} \). \( Z \) is the time value of money factor.

Equation:

\[
\min U(P) = (P \times A \times 12 + U_t) \times Z_t \tag{3}
\]

(2) The balance of the construction costs

Suppose the government will sell a part of the public rental housing and shops after a certain number of years. It can be divided into three stages to create the function: construction period \( V_1 \), the period before selling property rights \( V_2 \) and the period after selling property rights \( V_3 \) (if not sold, \( V_3 = 0 \)). \( \pi \) is (revenues - costs) surplus, \( \pi \geq 0 \)

According to TABLE 1:

\[
\begin{align*}
V_1 : \pi_1 &= S_q - (C_t - W_t) - R \\
V_2 : \pi_2 &= S_q - R \\
V_3 : \pi_3 &= (S_q + S_q)^R - R \\
S_q &= 12 \times (P \times Q + P \times Q + P \times Q) \\
\end{align*}
\]

In consideration of time of money, the paper discount and allocate it to each year. Suppose \( \pi_1, \pi_2, \pi_3 \) be the present value of cost; \( T_1, T_2, T \) is the period; \( i \) is the cost of capital of social funds. According to formula (4), (5), (6):

\[
\begin{align*}
V(P) &= \sum_{t=1}^{T} \frac{\pi_t}{(1+i)^{T-t}} \\
V_1(P) &= \sum_{t=1}^{T} \frac{\pi_1}{(1+i)^{T-t}} \\
V_2(P) &= \sum_{t=1}^{T} \frac{\pi_2}{(1+i)^{T-t}} \\
V_3(P) &= \sum_{t=1}^{T} \frac{\pi_3}{(1+i)^{T-t}} \\
\end{align*}
\]

(3) Maximization of social benefits

According to the relevant Principles of Economics, Social benefits mainly consist of the consumer surplus, producer surplus and external negative effects. The consumer surplus is the difference between the price which the consumer is willing to pay and the actual price that is paid about a commodity. The costs that tenants are willing to pay is the market rent \( P' \), the actual price that need to be paid is the public rental housing rent \( P_i \). So consumer surplus is \( P' - P_i \). Producer surplus means economic surplus that is obtained by producers. Singer producer surplus \( \nu(P) \) refers the revenues of selling a commodity minus the cost of it. As quasi-public goods, Public rental housing has positive externalities, thus, suppose \(-E = 0 \).

Equation:

\[
\max S(P) = 12 \times (P - P_i) \times Q \times Z_t + \nu(P) - E \tag{9}
\]

Constraint conditions

(1) Rent/income ratio refers to the percentage of individuals’ or families’ rent in total income. The higher the ratio is, the lower the tenants’ affordability is. In consideration of affordability of tenants, constraint conditions 1:

\[
P_i \leq r \times t \text{entan's income} \tag{10}
\]

(2) In view of social security of public rental housing, \( \pi \) should be limited in a maximum value. Suppose this value is \( h \) times as much as the cost, let \( Y_1, Y_2, Y_3 \) be the present value of cost in the period of \( V_1, V_2 \) and \( V_3 \) respectively. The other indicators are same as above.

Constraint conditions 2:

\[
\begin{align*}
0 \leq V(P) &\leq h \times (Y_1 + Y_2 + Y_3) \\
Y_i &\leq \sum_{t=1}^{T} \frac{\pi_t}{(1+i)^{T-t}} \\
Y_i &\leq \sum_{t=1}^{T} \frac{R}{(1+i)^{T-t}} \\
Y_i &\leq \sum_{t=1}^{T} \frac{R}{(1+i)^{T-t}} \\
\end{align*}
\]

(3) The rent of public rental housing can’t exceed the rent of commercial buildings at the same location; well it can’t below the rent of low-rent housing, or else it will be unfair for other more low-income earners.

Constraint conditions 3:

\[
P_0 \leq P_i \leq P' \tag{12}
\]

Establishment of the model

According to formula (1), (3), (8), (9) we can get the objective function as follows:

Objective function:

\[
\begin{align*}
\max U(P) &= (P \times A \times 12 + U_t) \times Z_t \\
\max V(P) &= [V_1(P) + V_2(P) + V_3(P)] \times Z_t \\
\max S(P) &= 12 \times (P - P_i) \times Q \times Z_t + \nu(P) - E
\end{align*}
\]
According to formula (10), (11), (12) we can get objective function as follows:

\[
\begin{align*}
\max \quad & \left[-\sigma_1 U(P) + \sigma_2 M(P) + \sigma_3 S(P)\right] \\
\text{Constraint conditions:} & \quad \begin{cases} 
P \leq r \times \text{收入} \\
0 \leq V(P) \leq h \times (Y_1 + Y_2 + Y_3) \\
P_0 \leq P_1 \leq P
\end{cases}
\end{align*}
\]

**Solution method for model**

Use the Utility Optimization Model to solve the model after normalization. In the model, the weight can be determined by Expert Marketing Method. According to formula (2), objective function can be transferred to:

\[
\max \left[-\sigma_1 U(P) + \sigma_2 M(P) + \sigma_3 S(P)\right] (13)
\]

**EMPIRICAL RESEARCH**

“Min Xin Jia Yuan” is the first project of public rental housing which was constructed in Chongqing and now it has been completed. It has all been allocated now and the commercial facades and garages have all achieved market operation. The total investment of this project is 34.02 billion, which conclude 30 percent of financial fund, 23.8 percent of housing provident fund loans and 46.2 percent of bank loans. The cost of per square meter of the public rental housing is 3150 yuan.

**Objective function**

(1) Chongqing’s average household size is 2.91, suppose each household has 3 person. The gross leasable area of each person is 20 square meters in every month. According to questionnaires, property management fees is 1.03 yuan per square meters, Household transportation fees is 200.09 yuan per square meters, household water and electricity gas is 170.15 yuan per month. Suppose other expenditures rise 5% each year, the rent of public housing will be adjusted once every two years and each time will rise 5%.

According to formula (3):

\[
U(P) = 780.21P + 6212.44
\]

(2) Public rental housing that has been constructed should meet the demand of 2020 for Chongqing at least. Suppose the period is 10 years and annual interest is equal. Public Housing Fund (PHF) mortgage rates are 4.257%, bank loans rate are 5.94%. According to the proportion, the composite interest rate can be calculated: 5.37%. “Min Xin Jia Yuan” project begun to allot in 2011, with an area of about 400,000 square meters, and the occupancy rate is 43.48%.

The period of V₁: Chongqing construct 40 million square meters public rental housing in 3 years, T₁ = 3.

According to formula (4):

\[
V_1(P) = 1376.42P - 17735.81 - Z_1 \times 7.938
\]

The period of V₂: In April 2016, the first residents are entitled to buy public rental housing and the period of V₂ is from 2013 to 2016. According to formula (5):

\[
V_2(P) = 2722.06P - 20818.57
\]

The period of V₃: Chongqing’s government will sell a part of public rental housing and all commercial real estate to pay off debts in 2016. Based on actual transaction data about Chongqing’s commercial business from 2007 to 2013, the market price is about 10,000 yuan per square meters in 2013, the annual rate rise more than 10%. Suppose it will rise in a same rate. So in 2016 it will be about 15,000 yuan per square meters, and the cost of public rental is 3,150 yuan per square meters. It can be calculated that the government need to sell 44.32% public rental. According to formula (6):

\[
V_3(P) = 2514.17P - 32839.9 + d \times S_d
\]

(3) According to questionnaire survey and document retrieval, the rent of commercial residential building

\[
\text{Revenues of rents } S_{1t} \text{ include: Public rental housing: rent } P_{1t}, \text{ rentable area } Q_{1t} \\
\text{Commercial supporting facility: rent shops } P_{2t} \text{ and garages } P_{3t}, \text{ rentable area separately } Q_{2t}, Q_{3t}
\]

\[
\begin{array}{|c|c|c|}
\hline
\text{Revenues} & \text{V₁} & \text{V₂} & \text{V₃} \\
\hline
\text{The same to } V₁ & \text{Revenues of rents } s_{1t} \\
\hline
\end{array}
\]
which is located near “Min Xin Jia Yuan” project was 20.54 yuan per square meters in 2011. According to formula (9):

\[
S(P) = 12 \times (20.54 - P) \times Q_t
\]

\[+ V(P) - E = 169.7 \times P + 17903.25
\]

**The constraint conditions**

(1) According to “CHONGQING STATISTICAL YEARBOOK”, the average annual household disposable income of individuals who belong to sandwich stratum group are 43,110.57 yuan per year. According to formula (10): The constraint conditions 1 is \( P \leq 11.98 \)

(2) Wan Changhong, Yang Ming\(^7\) predicted the company’s management fee rate should be 3.47% by the method of linear regression. Suppose \( h \) is 3%.

According to formula (11), the constraint conditions 2 is \( 0 \leq V(P) \leq 2469.643 \)

(3) In 2011, the rent of commercial residential building \( P' \) near “Min Xin Jia Yuan” is about 20.54 yuan per square meters each month, and the low-rent housing rent is 1 yuan per square meters each month. According to formula (12), the constraint conditions 3 is \( 1 \leq P \leq 20.54 \).

**The setting of weights**

Through experts marketing method to determine the weight, 10 experts will be chosen to get the weight of model: \( \sigma_1 = 0.6, \sigma_2 = 0.1, \sigma_3 = 0.3 \)

**Model solution**

According to formula (13) and the data above, the Rent-Setting Model can be established:

Objective function:

\[
\text{Max} \{ -330.044 \times P + 2584.64 \}
\]

\[P \leq 11.98
\]

\[0 \leq V(P) \leq 2469.643
\]

\[1 \leq P \leq 20.54
\]

\[P = 10.796\text{ yuan per square meters}
\]

**CONCLUSION**

At present, the project of public rental housing has been into the period of management, and reasonable rent price is the guarantee of sustainable operation. The paper believed that the pricing mechanism should be put forward under the basic objective that satisfying the government, the constructors and the tenements at the same time. Therefore, the multiple objective dynamic Rental pricing model on the basis of cash flow was made in the paper. At the end, the paper calculated the public rental housing’s reasonable rent of “Min Xin Jia Yuan” in Chongqing is 10.796 per square meters each month based on a certain view of assumption. Such an index not only meet the needs of low-income tenants, but also ensure the sustainability of the public rental housing project.

**ACKNOWLEDGMENT**

The paper is supported by the Key Program for Chongqing Sciences and Technology Commission. The project number (cstc2012cx-rkxA3000).

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