Research on mechanical model-based billiards stroking technique affecting cue ball movement status after collision

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ABSTRACT

According to object movements corresponding physical laws, using rigid body rotational and translational principles, through establishing mechanical model, it carries out detailed research on cue ball each kind of movement statuses when cue stroking cue ball from different parts, and makes quantitative calculation of cue stroking billiards different heights influence on cue ball movement states, as well as qualitative analyzes cue ball movement statuses after it colliding with object ball. So that it plays certain technical guiding roles in athletes’ billiards technique improvement.

INTRODUCTION

In the beginning of 20th century, billiards game started gradually becoming a competitive sports event. As one of its symbols, billiards competition administrative organizations were successively founded in different countries and regions in the world. For example, in 1919, Britain founded its top billiards organization – Britain billiards union; In 1940, world billiards federation were founded, it was in charge of international billiards competitions, headquarters was set on Brussels in Belgium, administrative center in Barcelona, Spain. In 1948, America founded American billiards association, organization was set in Chicago. These organizations founding has propelled to billiards competitive sports development. During 30s, 40s in this century, billiards had ever been popular among China metropolitan upper class. Since 1980s, billiards has become increasingly prevalent in civil.

For a sports event, 20 years time though it is far from long, it is sufficient for experiencing from rising to downturn then to revival the whole process, such as billiards. Unconsciously, in China, the sports event well-known with “elegant”, “gentlemen” has already had 20 years history. Now, billiards has just become one of China most popular and most participants sports events. Relative department statistics data show that at present, there are 50 million people had ever played billiards, 25 million people often play billiards, and 0.5 to 1 million people are playing billiards every day in China. That is to say, billiards popularization degree even has gone beyond China national ball table tennis.

With the rise of China billiards new star in international competition court, billiards increasingly becomes important contents of mass sports. When playing the ball, reasonable using cue ways, meanwhile grasping strength is quite important. Strength grasping ultimately is to let cue ball has good positions that can be helpful...
for next stroking. Therefore, carry out relative mechanical analysis of billiards movement, finds out its mechanical laws is very necessary. This paper, for billiards movement process some phases force statuses, it mainly utilize theoretical mechanics knowledge to make theoretical analysis, analyze cue ball and object ball their respective movement status after the two colliding, implement calculating and analyzing with theoretical mechanics knowledge as basis, which provides theoretical guiding and practical suggestions on billiards movement popularization and improvement.[2]

BILLIARDS BASIC TECHNIQUES AND MODEL ESTABLISHMENT

Billiards basic techniques including stop shot, top spin, back spin, side spin, jump shot and so on, while many techniques are evolved through these basic techniques; this paper mainly discusses stop shot, top spin and back spin. Analysis is as Figure 1.

1. When cue strokes cue ball center position point C, at this time player strokes stop shot
2. When cue strokes cue ball upside position semi-circle area A, at this time player strokes top spin and middle-high spin
3. When cue strokes cue ball upside position semi-circle area B, at this time player strokes back spin
4. When cue strokes cue ball area between circle 1 and circle 2, at this time player strokes out of cue.

At first analyze cue ball movement. Through resolution billiards movement in the table, it can be divided into translation surrounding mass center and rotation surrounding mass center.

Given billiards mass to be M, radius to be R, it statistics on horizontal table, player uses cue collides sphere along horizontal direction, given impact force to be \( F \), horizontal turn right, and its distance from ball center C is \( h \), so billiards suffered impulse \( p = \int_1^2 F dt \), therefore impulsive moment along clockwise direction is \( J = Ph \). Carry out force analyzing of billiards, it is known that billiards suffered gravity force \( Mg \) (direction is vertical and turn downwards), positive pressure \( F_n \) (direction is vertical and upwards), as Figure 2 shows. According to theorem of moment of momentum, it is clear that ball has along \( x \) positive direction speed \( u_0 \) and angular speed \( \omega_0 \) (turning clockwise). Expression is as following formula (1):

\[
M \omega_0 = P, I \omega_0 = J \quad (1)
\]

Stipulate that positive direction is along clockwise direction, it gets rotational inertia \( I = \frac{2}{5} MR^2 \), ball linear speed \( V_0 \) is formula (2):

\[
V_0 = u_0 - R \omega_0 \quad (2)
\]

Input formula (1) into formula (2), and then it gets formula (3):

\[
V_0 = \frac{P}{M} - \frac{PhR}{I} = \frac{P}{M} \left[ 1 - \frac{5h}{2R} \right] \quad (3)
\]

(1) Cue collides cue ball along center position

When \( h = 0 \) that cue collides cue ball along center position, from above formula it is clear:

\[
u_0 = V_0 = \frac{P}{M} \quad (4)
\]

\[
\omega_0 = 0 \quad (5)
\]

In the moment that cue collides cue ball, angular speed \( \omega_0 = 0 \), cue ball movement is linear speed \( V_0 \) translation (here impulse is bigger, ignoring friction influence), when cue ball collides with object ball, ac-
According to momentum conservation law, object ball gets speed \( V_0 \), cue ball keeps static on spot, it strokes stop shot at this time.

(2) Cue strokes cue ball in ball upper position

When cue ball collides with cue ball along upper position, cue ball movement status can be analyzed according to following three types

1) On the condition that \( h=0.4R \)

From formula (3), it is clear \( V_0 = 0 \), from formula (6), it gives sphere rotational angle \( \theta \):

\[
\theta = \omega_0 t
\]

Billiards forward movement speed is \( v_0 \), its angular speed \( \omega_0 = \frac{v_0}{R} \), (direction is anticlockwise), \( V_0 \) speed direction is just opposite to table friction force direction. When cue ball collides with object ball, it transfers energy to object ball, its angular speed decreases. The kind of cue method is called push stroke, after playing cue ball and object ball distance are reduced.

2) On the condition that \( h>0.4R \)

From above formula, it gets billiards and table contact point speed \( V_0 < 0 \), so it gets friction force \( F_f \) as:

\[
F_f = \mu F_N = \mu Mg
\]

Billiards torque along anticlockwise direction is \( F_f R \). According to theorem of moment of momentum, it gets ball accelerated speed \( \dot{u} \), angular accelerated speed \( \dot{\omega} \) as:

\[
\begin{align*}
\dot{u} &= u_0 + \frac{\mu Mgt}{M} \\
\dot{\omega} &= \omega_0 - \frac{\mu M R \dot{t}}{I}
\end{align*}
\]

Then at \( t \) time, billiards speed and angular speed are:

\[
\begin{align*}
\dot{u}(t) &= \frac{P}{M} + \mu gt, \\
\dot{\omega}(t) &= \omega_0 - \frac{\mu M R t}{I}
\end{align*}
\]

Then it has \( t \) time spherule speed \( \dot{V}(t) \) as:

\[
\dot{V}(t) = \dot{u}(t) - R \dot{\omega}(t) = \frac{7}{2} \mu gt + \frac{P}{M} \left[ 1 - \frac{5h}{2R} \right]
\]

Contact point speed \( \dot{V}(t) \) changes from negative to zero with \( t \) increasing, from formula (10) it can solve \( \dot{V} = 0 \) time \( t_0 \) as:

\[
t_0 = \frac{2}{7} \frac{P}{\mu Mg} \left[ \frac{5h}{2R} - 1 \right] > 0
\]

When \( t \geq t_0 \), billiards rolls clockwise. Billiards speed is:

\[
u = \mu gt + u_0 = \frac{5}{7} \frac{P}{M} \left[ 1 + \frac{h}{R} \right]
\]

Angular speed along clockwise direction is:

\[
\omega = \frac{u}{R} = \frac{5}{7} \frac{P}{M R} \left[ 1 + \frac{h}{R} \right]
\]

At this time cue ball friction force direction is the same as speed direction, so cue ball speed increases, after it collides with object ball, cue ball still has certain angular speed. The cue method is called punch; cue ball and object ball distance is quite farther from playing push stroking.

3) On the condition \( h<0.4R \)

From formula (3), it gets speed \( V_0 > 0 \), spherule suffered table friction force towards \( x \) axis negative direction, that \( F_f = -\mu Mg \), therefore spherule speed decreases, ball center torque (along clockwise direction) is \( F_f R \), accelerated speed and angular accelerated speed are:

\[
\begin{align*}
\dot{u} &= u_0 - \mu gt, \\
\dot{\omega} &= \omega_0 - \frac{\mu M R \dot{t}}{I}
\end{align*}
\]

Then at \( t \) time, billiards speed and angular speed are:

\[
\begin{align*}
\dot{u}(t) &= u_0 + \frac{\mu Mgt}{M}, \\
\dot{\omega}(t) &= \omega_0 - \frac{\mu M R t}{I}
\end{align*}
\]

Spherule speed is:

\[
\begin{align*}
V(t) &= u(t) - R \dot{\omega}(t) = \frac{7}{2} \mu gt + \frac{P}{M} \left[ 1 - \frac{5h}{2R} \right]
\end{align*}
\]

After a moment \( t_0 \), \( V=0 \), from formula (9), it can get \( t_0 \) as:

\[
t_0 = \frac{2}{7} \frac{P}{\mu Mg} \left[ 1 - \frac{5h}{2R} \right] > 0
\]

When \( t \geq t_0 \), it solves:

\[
u = \mu gt_0 + u_0 = \frac{5}{7} \frac{P}{M R} \left[ 1 + \frac{h}{R} \right]
\]

\[
\dot{w}(t_0) = \frac{5}{7} \frac{P}{M R} \left[ 1 - \frac{h}{R} \right]
\]
When cue ball simultaneously owns sliding and rolling, at this time cue ball suffered friction force is opposite to $V_0$ direction. After cue ball collides with object ball, generated friction force will let cue ball continue to move forward a small distance. Cue ball and object ball distance is smaller than push trod. At this time it plays middle high spin, the cue method can be used for short distance and little range cue ball position.

(3) Cue strokes cue ball along lower position

Player use cue strokes billiards lower half, as Figure 3 show:

At this time billiards speed is:

$$V = \mu_0 - R\omega_0 = \frac{P}{M} + \frac{PhR}{I} = \frac{P}{M} \left[1 + \frac{5h}{2R}\right] \tag{21}$$

At this time billiards positive moves, it suffered friction force is in negative direction, friction force size is $F_f = -uMg$, from rigid body movement theorem and theorem of moment of momentum, it gets:

$$M\dot{\mu} = -\mu gM, I\dot{\omega} = \mu gMR \tag{22}$$

$$u(t) = u_0 - u gt, w(t) = w_0 + \frac{ugMRt}{I} \tag{23}$$

$t$ time billiards linear speed, angular speed $\omega(t)$ is:

$$u(t) = \frac{P}{M} - u gt,$$

$$w(t) = -\frac{Ph}{I} + \frac{ugMRt}{I} = -\frac{5}{2} \left[\frac{ph}{MR} - u gt\right] \tag{24}$$

From above formula it can get when $(t)>0$, billiards moves in positive direction, when $t_1 = \frac{P}{Mug}$, cue ball speed $u(t_1) = 0$, when $(t)<0$, cue ball moves in negative direction. In the beginning, $\omega(t) < 0$, cue ball makes anticlockwise rotation, when $t_2 = \frac{Ph}{MRug}$, angular speed $\omega(t) = 0$, cue ball direction is clockwise. It is obvious $t_1 > t_2$, when $t_1 > t > t_2$, billiards moves in positive direction and makes clockwise rotation, and exists some time $t_0$, let ball speed $V = 0$, during the time, it surely has some time $t_0$, billiards translation and rotation mutual cooperated, it makes ball and table contact point speed $V = 0$, it can get:

$$V(t_0) = u(t_0) - R\omega(t_0) = \frac{P}{M} - ug t_0 + \frac{PhR}{I} + \frac{ugMR^2}{I} t_0 = 0 \tag{25}$$

It solves:

$$t_0 = \frac{2}{7ugM} \frac{P}{M} \left[1 + \frac{5h}{2R}\right] \tag{26}$$

In the time $t_0$, billiards movement direction is posi-
tive, input $t_0^*$ into formula (3.34), and then it can get sphere center $u$ and $w$:

$$u(t_0^*) = \frac{5}{7M} \left[ 1 - \frac{h}{R} \right]$$  \hspace{1cm} (27)$$

$$w(t_0^*) = \frac{5}{7R} \left[ 1 - \frac{h}{R} \right]$$  \hspace{1cm} (28)$$

Assume A ball is stroked by force $F$ on its bottom, when $t < t_2$, rotation direction is anticlockwise, cue ball moves in positive direction. As Figure 4 shows. If in the time $t^*$, A ball and B ball happen to elastic collision, due to mass is the same, according to energy and momentum conservation law, it is known after colliding, two ball speed interconvert, A ball speed is 0, B ball moves with A ball speed $u$. Due to two balls surface are smoothly, it can get friction torque is 0, therefore after collision, two balls’ rotational angular speeds would not change, as Figure 5 show. Then, after colliding, A ball moves reverse along x axis negative direction, rotational direction is anticlockwise, the method is back spin ball. In the following, it analyzes after elastic colliding, ball movement situation. After colliding, A ball speed:

$$u^n = 0$$ \hspace{1cm} (29)$$

From formula (23), it can get A ball angular speed $\omega^*$ as:

$$\omega^* = -\frac{Ph}{I} + \frac{\mu g M R t^*}{I} = -\frac{5}{2R} \left[ \frac{Ph}{MR} - \mu g t^* \right]$$ \hspace{1cm} (30)$$

Then A ball speed is:

$$V = 0 - R \omega^* = \frac{5}{2} \left[ \frac{Ph}{MR} - \mu g t^* \right] > 0$$ \hspace{1cm} (31)$$

Cue ball slides in positive direction, so A ball friction force is in negative direction, $F_f = -\mu g M$, cue ball torque $F_f R$ is in clockwise direction. Given colliding time is 0, after colliding $t^*$ time, A ball linear speed $\dot{\mu}$, angular speed $\omega^*$ is:

$$\dot{\mu} = -\mu g t$$ \hspace{1cm} (32)$$

$$\omega^* + \frac{\mu g M R t}{I} = \omega^* + \frac{5}{2} \mu g t$$ \hspace{1cm} (33)$$

It gets A ball speed as:

$$V' = \dot{\mu} - R \omega^* = -\mu g t - R \omega^* - \frac{5}{2} \mu g t$$ \hspace{1cm} (34)$$

When $V' = 0$, from above formula, it can get:

$$t^* = \frac{5}{7\mu g} \left[ \frac{Ph}{MR} - \mu g t^* \right]$$ \hspace{1cm} (35)$$

When $t \geq t^*$, it gets pure rolling speed $\dot{\mu}$ and $\omega^*$ angular speed as:

$$\dot{\mu} = -\frac{5}{7} \left[ \frac{Ph}{MR} - \mu g t^* \right]$$ \hspace{1cm} (36)$$

$$\omega^* = -\frac{5}{7R} \left[ \frac{Ph}{MR} - \mu g t^* \right]$$ \hspace{1cm} (37)$$

According to above analysis, it can be concluded that: after stroking A ball lower half, the smaller $t^*$ is, when A ball makes negative pure rolling, Cue ball reverse speed would be faster, continuously movement time would also be longer.

**CONCLUSIONS**

Billiards in its movement process, it better followed rigid body movement basic laws, trainers only correctly master and apply billiards movement mechanical prin-
principle, the elegant sports event could make considerable achievement. Billiards movement, it can be calculation and analyzed from translation and rotation two aspects, translation followed rigid body mass center movement theorem of momentum, and rotation followed rigid body translational theorem of moment of momentum. Safety stroking area range is up to table surface and cue tips’ static friction coefficients, for billiards enthusiasts, stroking area selected in the range of billiards envisage circle spherical surface 0.6R range, generally miscue phenomenon would not happen. Reasonable stroking position and appropriate stroking strength, not only can decide object ball movement route and achieve scores, but also create conditions for next stroke through cue ball ideal position. In cue ball and object ball positive collision technique, cue ball position is mainly implemented by changing cue ball rotational speed, while cue ball rotational speed is decided by cue acting cue ball impulse and stroking angle. In cue ball and object ball oblique collision technique, it normally achieves cue ball positions through controlling deviation angles.

In actual billiards technique, billiards kinds of games exploitation, when programming and designing, it can reference this paper achieved result. The research has a certain guiding significance to billiards beginners. It can avoid beginners unquestioning training and have targeted training as well as improve cue methods’ accuracy and proficiency degree, so that potting can be remarkable improved.

REFERENCES


