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Research on damage identification of tower structure based on analysis of key component

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ABSTRACT

In order to enhance the pertinence of the damage identification, a damage identification of tower structure based on the analysis of key component is proposed, since it is difficult and less significant to carry out a comprehensive damage identification for all components of the tower structure because of numerous tower structure components, most of which is the secondary member, and only a few key components have a big damage probability and severe impact of damage on the structure. Firstly, the components that the main failure modes should experience when the structure is intact and under different initial damage by searching for main failure modes of the tower structure through the global branch-and-bound method are found, which are ports leading to invalidation of the structure and the key components of the tower structure system. Then, based on these key components, by accelerating response signals with fluctuating winds, WVD damage identification indictor DI of time-frequency statistical characteristics is proposed, to achieve the diagnose of the damage and corresponding location of key components. Finally, the proposed method is verified by a communication tower.

KEYWORDS

Key components; Tower structure; Damage identification; Failure mode; Time-frequency statistical characteristics.

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INTRODUCTION

As a common structural form, tower structure has been widely used in the fields of construction, communication, and electric power and so on. Tower structure is predisposed to produce damage and damage accumulation in the service period, thus leading to collapse accident. Therefore, the identification research on damage of tower structure provides the basis for timely maintenance and remedial measures, and preventing the tower collapses has a great practical significance for guaranteeing the security of national energy, communications and other important fields.

The security and reliability of the tower structure have been studied widely, but researches mainly focus on the antiseismic and anti- breeze effect of the system^[1-3], and damage identification research based on the tower structure is still relatively few. At present, the damage identification research on tower structure falls into two major identification methods, one is based on modal parameter and the other is based on wavelet transform (WT). Among them, the method based on modal parameter mainly includes dynamic condensation method^[4-6], element modal strain energy method^[7] and modal double-indictor method^[8] and so on, which all need to identify the modal parameters, and also need to arrange many measure points and certain precision of modal parameter identification; and the identification method based on wavelet transform^[9] cannot identify effectively the damage occurred in tower's stiffness mutation position.

The tower structure is a large complex spatial frame, which includes abundant secondary components(such as secondary web component and local horizontal bar etc.), the damage of these components less affect the structure, damage identification research on these components would be time-consuming, strenuous, less significant and difficult. On the contrary, damages are easy to occur in some key components, which can greatly affect the structure. Therefore, it has a special significance for improving the pertinence of damage identification of tower structure and guaranteeing the security of tower structure to find out these key components and carry out the damage identification research on them.

Therefore, the components that the main failure modes should experience, namely the key components in structural system, when the structure is intact and under different initial damage by searching for main failure modes of the tower structure through the global branch-and-bound method and reliability analysis are found. Aimed at key components, the damage identification indictor of time-frequency statistical characteristics based on acceleration response signals under random wind loads is proposed, the physical significance of this indictor is explored, and the method to identify the damage location of key components is put forward through the rules of The average statistical indictors, to achieve the purpose of the damage identification of tower structure based on the analysis of key components. This method transforms the complex damage identification of tower structure into the identification to vulnerable key components, improving the pertinence to the damage identification of tower structure. Finally, the proposed method is verified through a communication tower.

Analysis of key components

There are numerous tower structure components and high redundancy, so the failure probability of structural systems under vertical loads has a certain process of evolution, namely the failure mode, among which the components that the main failure modes shall experience under the different initial damage is the basic failure element, which control the reliability performance of structure with low degree of reliability, poor safety allowance, and big failure probability^[12]. They are important components of the structural system on the failure path, the source resulting in the failure of other components, the port resulting in the final failure of the structure, the most important and the most vulnerable key components in the structural system. And the analysis of key components is to find out the main failure modes produced by the structure under normal load and in the accident after the different initial damage and the major failure components experienced, which cannot achieve appropriate load model, ultimate bearing capacity of pole element and appropriate search method^[10].

Load model

The wind load is a main lateral load of tower structure, an important factor controlling the structure design. The along-wind static equivalent wind load of tower structure is built according to the load code of the architectural structure:

$$W = \beta_z \mu_s \mu_z \omega_0 A \tag{1}$$

In the formula, β_z is a gust response coefficient, μ_s is a shape coefficient, μ_z is a variation coefficient of wind pressure height, ω_0 is a reference wind pressure, and A is a windward area.

Ultimate bearing capacity of dimension pole element

Pull bars in the structural system are different from failure modes of pressure bars under the action of ultimate load., the mode of pull bars is strength failure generally, and the mode of pressure bars is stability failure in most cases^[11]. The failure mode is strength failure when the element is tensioned, and its ultimate bearing capacity is:

$$N_{y} = A_{n} f_{y}$$
⁽²⁾

In the formula, A_n and f_y are the net cross-sectional area of the element and the yield strength of the material respectively. The failure mode is buckling failure when the element is pressured, and its ultimate bearing capacity is:

$$N_{cr} = \varphi A_n f_y \tag{3}$$

In the formula, φ is the stability coefficient of the axial pressure bar, which values according to the design code for steel structures.

Search for failure modes and key components

In this paper, the global fatigue life branch-and-bound method is mainly used to search for failure modes and key components, which searches for the main failure modes to select the candidate failure components at current stage based on the real stress state of structural components, with a guiding principle of "minimizing the load increment of failure probability of structural systems", fully considering the failure state and its evolutionary process of structural systems^[12-13].

Considering there are $r_1, r_2, \cdots, r_{k-1}$ in the structural system composed of *n* elements, among which, k-1 elements have lost effectiveness in succession, the corresponding load incremental factors are $\Delta F_{r_1}^{(1)}, \Delta F_{r_2}^{(2)}, \ldots$. ΔF_r^{k-1} respectively. At the *k* stage of the failure process, the critical strength $R_{S(r_k)}^{(k)}$ of failure at system stage between the effective strength $R_{r_k}^{(k)}$ of elements $r_k \left[r_k \in (1, 2, \cdots, n), r_k \notin (r_1, r_2, \cdots, r_{k-1}) \right]$ and its corresponding element r_k is:

$$I_{r_{k}} = sign\left[a_{r_{k}}^{(k)}\right]$$

$$R_{r_{k}}^{(k)} = R_{r_{k}}^{I} - I_{r_{k}} \times \sum_{i=1}^{k-1} a_{r_{k}}^{(i)} \Delta F_{r_{i}}^{(i)} m_{r_{i}}$$

$$\Delta F_{r_{k}}^{(k)} = \frac{R_{r_{k}}^{(k)}}{a_{r_{k}}^{(k)}}$$

$$R_{S(r_{k})}^{(k)} = \Delta F_{r_{k}}^{(k)} + \sum_{i=1}^{k-1} \Delta F_{r_{i}}^{(i)} m_{r_{i}}$$

$$R_{S(\min)}^{(k)} = \min\left[R_{g(r_{k})}^{k} \times c_{k}, R_{S}^{*} \times c_{s}\right]$$
(4)

In the formula: $a_{r_k}^{(k)}$ is an internal stress of the element r_k obtained when generalized load is applied to external loads in the structural system composed of (n-1+k) residual elements; $R_{r_k}^{(k)}$ is an effective strength that the element r_k can use to bear external load increment at the k stage of the failure process; I_{r_k} is a sign function; $R_{r_k}^I$ is a strength of the element r_k when tension & compression difference exists; $R_{S(r_k)}^{(k)}$ is a critical strength of failure at system stage corresponding to the element r_k at the k stage of the failure process; c_k is a branch-and-bound parameter, c_s is walking control width of the critical strength, c_k is generally valued around 1.2 in actual projects; m_{r_i} is a selection parameter of materials. If the failure element r_k is composed of elastic-perfectly plastic materials, $m_{r_i} = 1$; if the failure element r_k is composed of ideal brittle materials, $m_{r_i} = 0$.

In this paper, the effective strength $R_{r_k}^{(k)}$ is a strength that can be used to bear external applied loads after deducting feeder weight, ladder and structure deadweight and other dead load stresses. Load changes in load increment factors generated by the basic wind pressure increment at the *p* stage shall be considered:

$$\Delta F_r^{\ p} = (\Delta \omega_0)_p \beta_z \mu_s \mu_z A \tag{5}$$

Reliability and failure probability of component

Key components whose structure can be damaged easily and the damage can greatly affect the structure safety are obtained by calculating the reliability and failure probability of components that the main failure path shall experience through the first-order second-moment method, based on component resistance and safety margin equation:

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$$\beta = \frac{\mu_m}{\sigma_m}, P_f = (M < 0) = \Phi(-\frac{\mu_m}{\sigma_m}) = \Phi(-\beta)$$

DAMAGE IDENTIFICATION INDICTOR

WVD distribution

Wigner-Ville distribution (WVD) is one of the most fundamental and the most widely-used time-frequency distributions. As a binary function of time and frequency, WVD has almost all the mathematical properties that the damage detection expects^[14]. The WVD definition for the signal s(t) is as follows:

$$W_{z}(t,f) = \int_{-\infty}^{\infty} z(t+\frac{\tau}{2}) \cdot z^{*}(t-\frac{\tau}{2}) e^{-i\cdot 2\pi\tau \cdot f} d\tau$$
(7)

In the formula, z(t) is a analytic signal corresponding to s(t) obtained by Hilbert transform, $z^*(t)$ stands for a complex conjugate of z(t), t and f represent time and frequency respectively, τ stands for time offset, and $i^2 = 1$.

Damage Indictor

Aimed at the acceleration response signal of the structure under pulsating wind loads, the damage identification (DI) based on WVD is calculated by extracting the damage effect of useful features according to various statistics, and the concrete form is shown as follows:

$$DI_f^a = \sum_{\upsilon} \int_{-\infty}^{\infty} W_z(t, f) e^{i\cdot 2\pi \cdot t \cdot \upsilon} dt$$
(8)

In the formula, a represents a measure point, f stands for a frequency scale.

In order to improve the comparability among measurement conditions, normalization processing is carried out to the statistics of each measure point by the maximum points of the condition, and we get:

$$DI^{a} = \frac{DI_{f}^{a}}{\left(DI_{f}^{a}\right)_{\max}} \tag{9}$$

Principles of Indictor Identification

Considering there are *n* degrees of freedom structural system, its differential equation of motion can be expressed as:

$$M\ddot{\mathbf{x}} + C\dot{\mathbf{x}} + K\mathbf{x} = \mathbf{g}(\mathbf{t}) \tag{10}$$

In the formula, M, C and K represent mass matrix, damping matrix and stiffness matrix respectively, and g(t) stands for a load vector. Its acceleration response signal of j measure point in time domain analysis can be expressed as follows:

$$\ddot{x}_{j}(t) = (2\pi f')^{2} \sum_{l=1}^{n} \varphi_{jl} \varphi_{l} \mathbf{g}(\mathbf{t}) e^{-\xi_{l} 2\pi f t} \cos(2\pi f t + \theta)$$
¹¹

In the formula, ϕ_{ji} is the component j of the l mode, ϕ_l is the l mode, ξ_i is modal damping ratio, f and f' are modal frequency and damped frequency respectively, and θ is starting phase angle. Through the Hilbert transform of the signal at the j measure point, the analytic signal is obtained as follows:

$$z_{j}(t) = \left(2\pi f'\right)^{2} \sum_{l=1}^{n} \varphi_{jl} \varphi_{l} \mathbf{g}(\mathbf{t}) e^{-\xi_{l} 2\pi f t} e^{i(2\pi f^{\dagger} + \theta)}$$
(12)

The kernel function:

$$z_{j}(t+\frac{\tau}{2}) \cdot z_{j}^{*}(t-\frac{\tau}{2}) = \left(2\pi f'\right)^{2} \left(\sum_{l=1}^{n} \varphi_{jl} \varphi_{l}\right)^{2} g(t+\frac{\tau}{2}) \cdot g(t-\frac{\tau}{2}) e^{-2\xi_{l} \cdot 2\pi \cdot f \cdot t} e^{i2\pi \cdot f' \cdot t}$$
(13)

The damage identification $DI_f^a = \sum_{\upsilon} \int_{-\infty}^{\infty} W_z(t, f) e^{i \cdot 2\pi \cdot t \cdot \upsilon} dt$

$$= \left(\sum_{l=1}^{n} \varphi_{jl} \varphi_{l}\right)^{2} \left(2\pi f'\right)^{2} \sum_{\nu} \int_{-\infty}^{\infty} g(t + \frac{\tau}{2}) \cdot g(t - \frac{\tau}{2}) e^{-2\xi_{l} \cdot 2\pi \cdot f \cdot t} \cdot e^{i2\pi \cdot f' \cdot t} e^{i\cdot 2\pi \cdot t \cdot t} d\tau \cdot dt$$

$$= \left(\sum_{l=1}^{n} \phi_{jl} \varphi_{l}\right)^{2} \cdot y(f)$$
(14)

In the formula, $f' \approx f$ and y(f) stand for implicit functions related to the frequency f, but without any relation with the measure point information:

$$y(f) = \left(2\pi f'\right)^2 \sum_{\nu} \int_{-\infty}^{\infty} g(t + \frac{\tau}{2}) \cdot g(t - \frac{\tau}{2}) e^{-2\xi_l \cdot 2\pi \cdot f \cdot t} \cdot e^{i2\pi \cdot f' \cdot t} e^{i\cdot 2\pi \cdot t \cdot \tau} d\tau \cdot dt$$
(15)

Supposed the DI value based on WVD of measure point *m* is the largest one, namely $(DI_f^a)_m = (DI_f^a)_{max}$, the following is obtained in the same frequency value:

$$DI^{a} = \frac{(\sum_{l=1}^{n} \phi_{jl} \varphi_{l})^{2} \cdot y(f)}{(\sum_{l=1}^{n} \phi_{ml} \varphi_{l})^{2} \cdot y(f)} = \frac{(\sum_{l=1}^{n} \phi_{jl} \varphi_{l})^{2}}{(\sum_{l=1}^{n} \phi_{ml} \varphi_{l})^{2}}$$
(16)

From the above formula, it is obvious that, DI^{a} is a function related to mode shapes, and structural damage will cause the change of mode shapes, which will certainly result in the change of DI^a , therefore, it is feasible to identify the damage by using the change of DI^a before and after the structural damage.

ANALYSIS OF EXAMPLES

Taking a communications tower in Chongqing as an example, the height of tower is 30m, composed of steel angles with 5 kinds of cross sections, all using the space bar element simulation. See Figure 1 for the t model. The site is B class, the fundamental wind pressure is $0.45 KN/m^2$, the variation coefficients of the load and member strength are 0.25 and 0.15 respectively, see TABLE 1 for failure probability and load increment of bar element for the 1st grade structure under 90° maximum design wind load, see Figure 2 for S.N. and location of failure bar element, and see TABLE 2 for the first 16 main failure modes in the structure system obtained by applying the critical strength branch-and-bound method.





Figure 1 : T model of tower structure

Figure 2 : Layout of failure bar element and measure points

It can be seen from TABLE 1 and 2: (1) the failure bar elements are symmetrical distributed, mainly concentrated in the deformation position of the tower leg and the tower structure section, and all are chord damage due to the instability of the pressure bar, which conforms to engineering practice, such as the waist fracture, tower leg damage and chord deformation of tower structure. (2) The correlation between failure modes is robust, that is to say, most of the basic failure elements which constitute these failure modes are the same, which only differ in the order and the compound mode they appear. (3) Seeing from the failure components and failure modes, the load increment for the first component failure accounts for more than 90%.

Component S.N.	Load Increment	Reliability Indictor eta	Failure Probability (×10 ⁻⁴)
68	2.4787	3.30044	4.827
67	2.4788	3.30062	4.824
64	2.4894	3.31445	4.591
63	2.4896	3.31463	4.588
60	2.5682	3.41473	3.192
59	2.5683	3.41491	3.190
72	2.5837	3.43397	2.974
71	2.5839	3.43415	2.972
80	2.6506	3.51446	2.203
79	2.6506	3.51449	2.203
76	2.7014	3.57348	1.761
75	2.7016	3.57365	1.760

TABLE 1 : Failure probability of bar element

TABLE 2 : Main failure modes of structure

Sequence of Failure Element	Bounded Parameter C _s	Total Load Increment
68—71—59—72—79	1.1	2.64680
67—72—60—71—80	1.1	2.64682
64—71—59—72—79	1.1	2.64709
63—72—60—71—80	1.1	2.64711
68—63—71—72—79	1.1	2.64830
67—64—72—71—80	1.1	2.64833
60—59—72—71—79	1.1	2.64857
60—59—72—71—80	1.1	2.64859
64—63—72—71—79	1.1	2.64862
68—67—72—71—79	1.1	2.64862
68—63—72—71—80	1.1	2.64864
68—67—72—71—80	1.1	2.64864
67—64—72—71—79	1.1	2.64893
68—63—71—72—80	1.1	2.64895
63—72—60—71—79	1.1	2.65009
64—71—59—72—80	1.1	2.65011

When the initial damage at different levels occurs in some components because the structure is subjected to accidental load, the example considers 40% and 50% of the initial damage happen to the components 96 and 195 respectively, and use the critical strength branch-and-bound method to search and get the load increment and failure probability of components in the structural system, as shown in TABLE 3 and 4.

The tower structure is statically indeterminate, therefore, the initial damage of the component will cause the internal force redistribution between components, and its bearing load will decrease as the stiffness reduces, namely "unloading". So, it can be seen from TABLE 1-4 that, when 40% and 50% of the initial damage happen to the components 96 and 195 respectively, the load increment of the first grade component failure changes slightly comparing with the one of well-preserved structure, and the paths that the structural failure shall experience are similar, therefore, failure elements are basically the same, which are the key components of this communication tower, to judge critical defects and weaknesses inherent in the structure by section forms and the degree of safety and prosperity of the component during design largely determines the safety performance of the structure.

40% damage of component S.N.	Load Increment	50% damage of component 195 S.N.	Load Increment
68	2.4775	67	2.4772
67	2.4801	68	2.4805
64	2.4882	63	2.4878
63	2.4908	64	2.4912
60	2.5669	59	2.5665
59	2.5696	60	2.5700
71	2.5814	72	2.5832
72	2.5863	71	2.5844
79	2.6416	80	2.6445
80	2.6597	79	2.6567
75	2.699	76	2.7009
76	2.7041	75	2.7022

TABLE 3 : Failure bar element under initial damage



Figure 3 : Wind speed time history and frequency spectrum

Obviously, any damage happening to key components greatly affect the safety and reliability of structure, therefore, improving the damage monitoring and identification to these key components (59, 60, 63, 64, 67, 68, 71, 72, 79 and 80) will be of great importance to ensure safety of tower structure. So, based on the analysis results of key components, identification research on the damage of component 60, 64, 68, 72 and 80 is carried out by using the structural symmetry (see Figure 2 for the measure point layout) in this paper.





Adopting the harmony superposition method, compounded by the linear superposition caused by the harmonic vibration of random amplitude and phase, in 10m height, the design wind speed is 30m/s, the duration is 60s,the time step is 0.002s, the wind speed time history and frequency spectrum is obtained through the pulsating wind load simulation is as shown in Figure 3. Adopting ANSYS Transient, the structural damping ratio is 0.02, the time step is 0.002s, the vibration responses in well-preserved structure and each damage condition under the pulsating wind load are calculated separately, and

the acceleration time history curve of each measure point is extracted. Figure 4 shows the acceleration time history curve and WVD graph of measure point 2 in intact state, while Figure 5 expresses WVD graph of acceleration signal of measure point 2 of the component 60 in damaged state.



Figure 5 : WVD graph

By the comparison of acceleration signals shown in Figure 4&5, it is obvious that, when key components are damaged (damage degrees are all 20%), the WVD graph will change significantly, therefore, the WVD damage identification based on the response signal is feasible. In this paper, WVD DI^a of each measure point at f = 2.684Hz (near the natural vibration frequency of structure) is mainly calculated. Figure 6&7 show the conditions of measure point 5& 7 respectively. It can be seen from the Figure 6&7 that, when key components are damaged, the average value of any measure point will significantly offset, therefore, whether the components are damaged or not can be judged by using the average value of the statistics.



Figure 7 : Measure point 7

Based on the above judge, under the circumstances that each component is damaged, WDI is calculated by using the damage condition and the average value \overline{DI}_s^a and \overline{DI}_w^a of each measure point in good condition, in order to further determine the damage location of key components (see Figure 8 for details).

$$WDI = \frac{S^a - W^a}{W^a} \tag{16}$$

$$\begin{cases} S^{a} = (\overline{DI}_{s}^{a^{2}} - \overline{DI}_{s}^{a+1} \times \overline{DI}_{s}^{a-1}) \\ W^{a} = (\overline{DI}_{w}^{a^{2}} - \overline{DI}_{w}^{a+1} \times \overline{DI}_{w}^{a-1}) \end{cases}$$
(17)



Figure 8 : Damage indictor WDI

Figure 8 shows *WDI* value of each key component differs from its location in damaged state, therefore, the damage location can be identified by using *WDI*.

CONCLUSION

In this paper, the damage identification of the tower structure under the wind load is studied and discussed, which is also analyzed and validated by an example, and the following conclusions are obtained:

(1) The failure mode of structure system is numerous, the correlation is high, only a few key components have a great influence on the structural safety, which are mainly distributed in feeble locations that can easily produce intensive stresses, like the cross section deformation part of tower structure and the column and so on, which can better fit the actual failure mode. Therefore, to strengthen damage monitoring and identification of these key components is of great significance to guarantee the safety of the tower structure;

(2) The method to identify key component damage of tower structure is proposed, the pertinence of damage identification of the tower structure component is improved, and the workload and complexity of all damage identifications is reduced, providing a reference for the research on damage identification of complicated space truss structures;

(3) The damage identification indictor based on time-frequency statistical characteristics is put forward, which is a implicit function of mode shapes, gained from the direct calculation by using the acceleration response signal, avoiding the identification procedure of modal parameters; can effectively find the damaged of key components under non-stationary pulsating wind load; in addition, the damage location of components can be identified by using the average value of statistical indicators. Therefore, it is a prospecting method.

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