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Research on Adomian decomposition method and its application in the fractional order differential equations

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ABSTRACT

This paper mainly studies the basic principle of Adomian decomposition method and uses the principle to solve differential equations of fractional in order to get the approximate analytical solution in series form. The application of this method is extended to fractional order nonlinear convection diffusion equation of time. Using this method, the paper gets the solution of the equation and the solution satisfying the initial conditions. The result is the extension of the standard diffusion equation solved by former scholars. Compared with the standard methods, this numerical solution of this method can obtain approximate precision analytical solutions and the convergence is very fast. It does not require discretization and a large amount of computation.

KEYWORDS

Fractional differential equation; Adomian decomposition method; Fractional order nonlinear convection diffusion equation.

INTRODUCTION

The Adomian decomposition method[1-3] was applied to process linear and nonlinear problems in the fried of science and engineering. Tatari and Dehghan[4] applied Adomian decomposition method to process the multipoint boundary value problem. Wazwaz[5] used Adomian decomposition method to deal with the Bratu-type equations. Daftardar-Gejji and Jafari[6] considered Adomian decomposition method to analyze the Bagley Torvik equation. Larsson[7] presented the solution for Helmholtz equation by using the Adomain decomposition method. Tatari and co-worker[8] investigated solution for the Fokker–Planck equa

To some degree, the traditional control method and the model based on integer order calculus theory have some drawbacks. Along with the increasingly complexity of the actual industrial system and the increasing requirements of control, the control technology of integer order theory is difficult to obtain satisfactory performance under certain circumstances. Fractional order calculus not only provides a new mathematical tool for systematic science, but also offers an effective and feasible way to solve existing problems in practical industrial process. In the cross disciplinary, we can use fractional derivative to establish model[19-22]. Fractional differential equations have been widely applied in different fields of science[23-28], including viscoelasticity, groundwater simulation, financial mathematics, the theory of universal voltage shunt capacitor, guide, telex coefficient of biological system, fractional order models of neurons, data fitting[29,30] etc. Many solution method of integer order differential equation can be developed to solve the fractional differential equations. Sun Yanping and Zhang Xingfang applied the Adomian decomposition method in the solution of nonlinear mathematical model of reaction engineering; Liang Zufeng, Tang Xiaoyan use this method to solve analytic solution of the fractionally Damped Beam; Chen Xin, Li Aiqun, Cheng Wenrang, and Wang Yong used this method to solve the dynamic characteristics of steel chimney. There is a common shortcoming in the application of this method, the complex decomposition and iteration process and the huge amount of computation[31,32].

This paper uses the adomian decomposition method to solve the numerical solution of fractional order linear differential equations, and establishes the fractional nonlinear convection diffusion equation of time with force and absorption in Caputo sense. It is solved by using the Adomian decomposition method. When solving the nonlinear differential equations of fractional order, the Adomian decomposition method provides a series solution which is fast convergence. The results can be written in both closed form and sum form. And the prime indicators of sum is a very good approximation. The Adomian decomposition method with polynomial is also handy in dealing with nonlinear problems. Compared with the standard numerical methods, this method has some advantages. For example, it needn't to be discretized; no errors need to be corrected, and it does not require a lot of computer memory and operation ability. So for many nonlinear models, Adomian method can provides approximate precision analytical solutions and the convergence is very fast. As a kind of iterative method, the algorithm is very regular and it can be realized by using symbolic computation.

ANALYSIS OF ADOMIAN DECOMPOSITION METHOD AND ITS SOLUTION REPRESENTATION OF EQUATION

By using the Adomian decomposition method, the solution of arbitrary order linear fractional differential equations with constant coefficients is given. First, there are several lemmas and propositions[9-11].

Lemma 2.1:if $f \in C^m, m \in N, \mu > -1, \alpha > 0$,so

1) $D_t^{-\alpha} D_t^{-\beta} f(t) = D_t^{-(\alpha+\beta)} f(t) = D_t^{-\beta} D_t^{-\alpha} f(t)$

2) $D_t^{-\alpha} t^{\mu} = \frac{\Gamma(\mu+1)}{\Gamma(\alpha+\mu+1)} t^{\mu+\alpha}$

3) ${}^{c}D_t^{\alpha} t^{\mu} = \frac{\Gamma(\mu+1)}{\Gamma(\mu-\alpha+1)} t^{\mu-\alpha}$

Lemma 2.2:if $f \in C^{m}$, $m \in N$, $m-1 < \alpha < m$,so

1) ${}^{c}D_t^{\alpha} D_t^{-\alpha} f(t) = f(t)$

2) ${}^{c}D_t^{\alpha} D_t^{-\alpha} f(t) = f(t) - \sum_{k=0}^{m-1} \frac{f^{(k)}(0)}{k!} t^{k}$, $t > 0$

Proposition 2.3

$$\sum_{k_1=0}^{\infty} \cdots \sum_{k_n=0}^{\infty} a_{k_1,k_2,\cdots,k_{n-1},k_n} = \sum_{m=0}^{\infty} \sum_{\substack{k_1,\cdots,k_{n-1},k_n \ge 0 \\ k_1+\cdots+k_{n-1}+k_n=m}} a_{k_1,k_2,\cdots,k_{n-1},k_n}$$

Proposition 2.4

$$\sum_{m=0}^{\infty} \cdots \sum_{\substack{k_1,\cdots,k_{n-1},k_n \ge 0 \\ k_1+\cdots+k_{n-1}+k_n=m}} a_{k_1,k_2,\cdots,k_{n-1},k_n} = \sum_{m=0}^{\infty} \sum_{\substack{k_1,\cdots,k_{n-1},k_n \ge 0 \\ k_1+\cdots+k_{n-1}+k_n=s}} a_{k_1,k_2,\cdots,k_{n-1},k_n}$$

Now consider the initial value of a kind of arbitrary order linear differential equations with constant coefficient[12-14]:

$$\mathbf{a_n\, {}^{c}D_t^{\alpha_n} y(t) + a_{n-1}\, {}^{c}D_t^{\alpha_{n-1}} y(t) + \cdots + a_1\, {}^{c}D_t^{\alpha_1} y(t) + a_0\, {}^{c}D_t^{\alpha_0} y(t) = f(t)} \quad \mathbf{(1)}$$

$$\mathbf{y^{(k)}(0) = c_k,\ k = 0,1,\cdots,[\alpha_n]} \quad \mathbf{(2)}$$

In particular, if α_n is an integer, then $k = 0,1,\cdots,\alpha_n - 1$. Among them, ${}^{c}D_t^{\alpha_n}$ is the fractional derivative in the sense of Caputo, $0 < \alpha_0 < \alpha_1 < \cdots < \alpha_{n-1} < \alpha_n$, $a_i\,(i = 0,1,\cdots,n)$ is an arbitrary constant and $a_n \neq 0$.

Both sides of the equation (1) do the $D_t^{-\alpha_n}$ operation, according to lemma 2.1, lemma 2.2 and initial conditions (2), the results is as follows:

$$y(t) - \sum_{k=0}^{[\alpha_n]} \frac{c_k}{k!} t^{k} + \frac{a_{n-1}}{a_n}\left(D_t^{-(\alpha_n-\alpha_{n-1})} y(t) - \sum_{k=0}^{[a_{n-1}]} \frac{c_k t^{k+\alpha_n-\alpha_{n-1}}}{\Gamma(k+1+\alpha_n-\alpha_{n-1})} \right)$$

$$+ \frac{a_1}{a_n}\left(D_t^{-(\alpha_n-\alpha_1)} y(t) - \sum_{k=0}^{[a_1]} \frac{c_k t^{k+\alpha_n-\alpha_1}}{\Gamma(k+1+\alpha_n-\alpha_1)} \right)$$

$$+ \frac{a_0}{a_n}\left(D_t^{-(\alpha_n-\alpha_0)} y(t) - \sum_{k=0}^{[a_0]} \frac{c_k t^{k+\alpha_n-\alpha_0}}{\Gamma(k+1+\alpha_n-\alpha_0)} \right)$$

$$=\frac{1}{a_n}D_t^{-\alpha_n}f(t) \tag{3}$$

$$y(t)=\sum_{k=0}^{[\alpha_n]}\frac{c_k}{k!}t^k+\frac{1}{a_n}D_t^{-a_n}f(t)+\sum_{i=0}^{n-1}\frac{a_i}{a_n}\sum_{k=0}^{[\alpha_i]}\frac{c_k t^{k+\alpha_n-\alpha_i}}{\Gamma(k+1+\alpha_n-\alpha_i)}-\sum_{i=0}^{n-1}\frac{a_i}{a_n}D_t^{-(\alpha_n-\alpha_i)}y(t) \tag{4}$$

According to the Adomian decomposition method, solution of equation (1) is given by the following

$$y(t)=\sum_{j=0}^{\infty}y_j(t) \tag{5}$$

among them, $y_j(t)\ (j\geq 0)$ can be gotten by a recursive method.
take equation (5)into equation (4), got:

$$\begin{aligned}\sum_{j=0}^{\infty}y_j(t)&=\sum_{k=0}^{[\alpha_n]}\frac{c_k}{k!}t^k+\frac{1}{a_n}D_t^{-a_n}f(t)\\&+\sum_{i=0}^{n-1}\frac{a_i}{a_n}\sum_{k=0}^{[\alpha_i]}\frac{c_k t^{k+\alpha_n-\alpha_i}}{\Gamma(k+1+\alpha_n-\alpha_i)}-\sum_{i=0}^{n-1}\frac{a_i}{a_n}D_t^{-(\alpha_n-\alpha_i)}y_j(t)\end{aligned} \tag{6}$$

According to the Adomian decomposition method[15], we introduce the following recursive relationship

$$y_0(t)=\sum_{k=0}^{[\alpha_n]}\frac{c_k}{k!}t^k+\frac{1}{a_n}D_t^{-a_n}f(t)+\sum_{i=0}^{n-1}\frac{a_i}{a_n}\sum_{k=0}^{[\alpha_i]}\frac{c_k t^{k+\alpha_n-\alpha_i}}{\Gamma(k+1+\alpha_n-\alpha_i)} \tag{7}$$

$$y_{j+1}(t)=-\sum_{i=0}^{n-1}\frac{a_i}{a_n}D_t^{-(\alpha_n-\alpha_i)}y_j(t),\ j\geq 0. \tag{8}$$

Form (2.7) and (2.8), it will get the following components:

$$y_0(t)=\sum_{k=0}^{[\alpha_n]}\frac{c_k}{k!}t^k+\frac{1}{a_n}D_t^{-a_n}f(t)+\sum_{i=0}^{n-1}\frac{a_i}{a_n}\sum_{k=0}^{[\alpha_i]}\frac{c_k t^{k+\alpha_n-\alpha_i}}{\Gamma(k+1+\alpha_n-\alpha_i)}$$

$$y_1(t)=-\sum_{i=0}^{n-1}\frac{a_i}{a_n}D_t^{-(\alpha_n-\alpha_i)}y_0(t)$$

$$y_2(t)=(-1)^2\Big(\sum_{i=0}^{n-1}\frac{a_i}{a_n}D_t^{-(\alpha_n-\alpha_i)}\Big)^2y_0(t)$$

$$\vdots$$

$$y_j(t)=(-1)^j\Big(\sum_{i=0}^{n-1}\frac{a_i}{a_n}D_t^{-(\alpha_n-\alpha_i)}\Big)^jy_0(t)$$

$$\vdots \tag{9}$$

So the solution of equation (1) can be given in the following series form

$$y(t)=\sum_{j=0}^{\infty} y_j(t)$$

$$=\sum_{j=0}^{\infty}(-1)^j\left(\sum_{i=0}^{n-1}\frac{a_i}{a_n}D_t^{-(\alpha_n-\alpha_i)}\right)^j y_0(t)$$

$$=\sum_{j=0}^{\infty}(-1)^j\left(\sum_{i=0}^{n-1}\frac{a_i}{a_n}D_t^{-(\alpha_n-\alpha_i)}\right)^j\left[\sum_{k=0}^{[\alpha_n]}\frac{c_k}{k!}t^k+\frac{1}{a_n}D_t^{-a_n}f(t)+\sum_{i=0}^{n-1}\frac{a_i}{a_n}\sum_{k=0}^{[\alpha_i]}\frac{c_k t^{k+\alpha_n-\alpha_i}}{\Gamma(k+1+\alpha_n-\alpha_i)}\right]$$

$$=\sum_{j=0}^{\infty}(-1)^j\sum_{\substack{m_0,m_1,\cdots,m_{n-1}\geq 0\\ m_0+m_1+\cdots+m_{n-1}=j}}\frac{j!}{m_0!m_1!\cdots m_{n-1}!}\left(\frac{a_{n-1}}{a_n}\right)^{m_{n-1}}\left(\frac{a_{n-1}}{a_n}\right)^{m_{n-2}}\cdots\left(\frac{a_{n-1}}{a_n}\right)^{m_0}$$

$$\left[\frac{1}{a_n}D^{-\{m_{n-1}(\alpha_n-\alpha_{n-1})+\ldots+m_0(\alpha_n-\alpha_0)+\alpha_n\}}f(t)+\sum_{k=0}^{\{\alpha_n\}}\frac{c_k t^{k+\alpha_n+m_{n-1}+(\alpha_n-\alpha_{n-1})+\ldots+m_0(\alpha_n-\alpha_{n-1})}}{\Gamma(m_{n-1}(\alpha_n-\alpha_{n-1})+\ldots+m(\alpha_n-\alpha_{n-1})+k+1}\right]$$

$$+\sum_{i=0}^{n-1}\frac{a_i}{a_n}\sum_{k=0}^{\{\alpha_n\}}\frac{c_k t^{k+\alpha_n+m_{n-1}+(\alpha_n-\alpha_{n-1})+\ldots+m_0(\alpha_n-\alpha_{n-1})}}{\Gamma(k+\alpha_n-\alpha_i+1+m_{n-1}(\alpha_n-\alpha_{n-1})+\ldots+m_0(\alpha_n-\alpha_0))} \quad \textbf{(10)}$$

In addition, the approximate analytic solution can be gotten by the truncated series

$$y_N(t)=\sum_{j=0}^{N-1} y_j(t) \quad \textbf{(11)}$$

Using the Adomian decomposition method, the exact solution of initial value can be obtained in a closed area in most cases. In the actual situation, the approximate solutions of the desired accuracy can also be obtained by truncating the series and the convergence speed of Adomian decomposition method is very quick.

APPLICATION OF ADOMIAN DECOMPOSITION IN THE CONVECTION DIFFUSION EQUATION

Establish the equation:
The normal convection diffusion equation[16-17]:

$$\frac{\partial c}{\partial t}+\nu\frac{\partial c}{\partial x}=M^*\frac{\partial^2 c}{\partial x^2},\ M^*=\frac{M}{\alpha} \quad \textbf{(12)}$$

c is the concentration. ν is the average velocity. M is coefficient of efficiency for the convection characterization. And α is turbulent coefficient.

According to the Fick law of diffusion, i.e. the relationship between flow and concentration gradient is:

$$\mathbf{J}(\mathbf{x},\mathbf{t}) = -\mathbf{M}^* \frac{\partial \mathbf{c}}{\partial \mathbf{x}} + \nu \mathbf{c} \tag{13}$$

J is the diffusion flow, take the above equation into the following formula which is the continuity equation or the mass conservation equation

$$\frac{\partial \mathbf{J}}{\partial \mathbf{x}} = -\frac{\partial \mathbf{c}}{\partial \mathbf{t}} \tag{14}$$

The above standard convection diffusion equation can be obtained.

H.Pascalland J.StePhenson had studied the nonlinear diffusion equation in fractal media. The foundation of nonlinear relationship of molecular diffusion in fractal media is nonlinear relationship between flow and concentration gradient. Based on this, the constitutive equations of nonlinear diffusion in fractal medium is established:

$$\mathbf{J} = -\mathbf{M}(\mathbf{x})\mathbf{c}^{\mathbf{m}} \left|\frac{\partial \mathbf{c}}{\partial \mathbf{x}}\right|^{\mathbf{n-1}} \frac{\partial \mathbf{c}}{\partial \mathbf{x}} \tag{15}$$

$M(x) = Mx^{-\theta}$ is the anomalous diffusion coefficient in fractal medium. M is constant. m, n are parameters. Because $c(x,t)$ is the monotone decreasing function of radial distance x, $\left|\frac{\partial c}{\partial x}\right| = -\frac{\partial c}{\partial x}$.

Take the above constitutive equations which are radial symmetry into the continuity equation or mass conservation equation, and we can get nonlinear convection diffusion equation

$$\frac{\partial \mathbf{c}(\mathbf{x},\mathbf{t})}{\partial \mathbf{t}} + \nu \frac{\partial \mathbf{c}(\mathbf{x},\mathbf{t})}{\partial \mathbf{x}} = -\mathbf{M} \frac{\partial}{\partial \mathbf{x}}\left[\mathbf{x}^{-\theta}\mathbf{c}^{\mathbf{m}}(\mathbf{x},\mathbf{t})\left(-\frac{\partial \mathbf{c}(\mathbf{x},\mathbf{t})}{\partial \mathbf{x}}\right)^{\mathbf{n}}\right] \tag{16}$$

When $m = 0, n = 1, \theta = 0$, it is the standard convection diffusion equation.

Based on the above equation, using the fractional derivative of time instead of integer order derivatives will be more reasonable, thus established fractional order nonlinear convection diffusion equation which contains force and absorption time:

$$^{c}D_t^{\alpha} c(x,t) + \nu \frac{\partial c(x,t)}{\partial x} = -M \frac{\partial}{\partial x}\left[x^{-\theta}c^{m}(x,t)\left(-\frac{\partial c(x,t)}{\partial x}\right)^{n}\right]$$

$$-\frac{\partial}{\partial \mathbf{x}}(\mathbf{F}(\mathbf{x},\mathbf{t})\mathbf{c}(\mathbf{x},\mathbf{t})) + \mathbf{a}(\mathbf{t})\mathbf{c}(\mathbf{x},\mathbf{t}) \tag{17}$$

Here, $^{c}D_t^{\alpha}$ is the fractional differential operator in sense of CaPuto, $F(x,t)$ is the external force, $a(t)c(x,t)$ symbolizes the effect of adsorption process. When $m = 0, n = 1, F(x,t) = 0, \theta = 0, a(t) = 0$, it is the standard flow diffusion equation.

In the sense of Riemann-Liouvilie, people established nonlinear convection diffusion equation of fractional order with the external force and absorption, used the LaPlace transform, generalized finite Hnakel transformation, and the corresponding inverse transform to get analytical solution of which the main form is Mittag-Leffier. This paper e established nonlinear convection diffusion equation (17) of fractional order with the external force and absorption. In the sense of Caputo, and the series form solution of the equation can be obtained by the Adoman decomposition method.

$$\mathbf{c(x,0) = \varphi(x)} \quad \textbf{(18)}$$

When solving the nonlinear differential equations of fractional order, the Adomian decomposition method provides a series solution whose convergence is fast. The results can be written in both closed form and sum form. And the prime indicators of sum are very good approximations. S.Monmani gave an approximate analytical solution of fractional order heat and wave equations with variable coefficients, S.Monmani and K.Al-khaled used this method to give the approximate solution of fractional diffusion wave equation.

Now consider the initial conditions:

$$c(x,0) = \varphi(x)$$

The fractional nonlinear convection diffusion equation of time with force and absorption is as follows.

$${}^{c}D_t^{\alpha} c(x,t) + \nu \frac{\partial c(x,t)}{\partial x} = -M \frac{\partial}{\partial x}\left[x^{-\theta} c^m(x,t)\left(-\frac{\partial c(x,t)}{\partial x} \right)^n \right]$$

$$-\frac{\partial}{\partial x}(F(x,t)c(x,t) + a(t)c(x,t)$$

Among them, $0 < \alpha < 1$, $c(x,t)$ is concentration, ν is the average velocity, M is anomalous diffusion coefficient, m, n are parameters, and ${}^{c}D_t^{\alpha}$ is a α order derivative in the sense of Caputo.

Both sides of the equation (1) do the $D_t^{-\alpha_n}$ and use lemma (2.1) and (2.2)

$$c(x,t) = c(x,0) - D_t^{-\alpha}\left(\nu \frac{\partial c(x,t)}{\partial x} \right)$$

$$+ D_t^{-\alpha}\left[-M \frac{\partial}{\partial x}\left(x^{-\theta} c^m(x,t)\left(-\frac{\partial c(x,t)}{\partial x} \right)^n \right) \right]$$

$$-\mathbf{D_t^{-\alpha}}\left[\frac{\partial}{\partial \mathbf{x}}\mathbf{(F(x,t)c(x,t)) + a(t)c(x,t)} \right] \quad \textbf{(19)}$$

From the above formula, the following can be obtained

$$c(x,t) = \varphi(x) - \nu D_t^{-\alpha} c_x(x,t) - M D_t^{-\alpha}\left(x^{-\theta} c^m(x,t)(-c_x(x,t))^n \right)$$

$$-\mathbf{D_t^{-\alpha}}\left[\frac{\partial}{\partial \mathbf{x}}\mathbf{(F(x,t)c(x,t))_x + a(t)c(x,t)} \right] \quad \textbf{(20)}$$

$c_x(x,t)$ represents the derivative $c(x,t)$ for x.

According to the Adomian decomposition method, $c(x,t)$ is given by the following series form

$$\mathbf{c(x,t) = \sum_{j}^{\infty} c_j(x,t)} \quad \textbf{(21)}$$

the nonlinear term of equation (15) is given by the follows

$$\mathbf{x^{-\theta}c^{m}(x,t)(-c_{x}(x,t))^{n}=\sum_{j=0}^{\infty}A_{j}(c_{0},c_{1},...,c_{j})} \tag{22}$$

Among them, A_j is the polynomial determined by $c_0, c_1, ..., c_j$.
take equation (16), (17) into (15), got

$$\sum_{j=0}^{\infty} c_j(x,t) = \varphi(x) - \nu D_t^{-\alpha}\left(\sum_{j=0}^{\infty} c_j(x,t)\right)_x - MD_t^{-\alpha}\left(\sum_{j=0}^{\infty} A_j\right)$$

$$\mathbf{-D_t^{-\alpha}\left[\left(F(x,t)\sum_{j}^{\infty} c_j(x,t)\right)_x + a(t)\left(\sum_{j=0}^{\infty} c_j(x,t)\right)\right]} \tag{23}$$

By the Adomian decomposition method, type (23) iterative can be given by the following recursive relations

$$c_0(x,t) = \varphi(x) \quad c_1(x.t) = -\nu D_t^{-\alpha}(c_0(x,t))_x - MD_t^{-\alpha}A_0 - D_t^{-\alpha}\left[(F(x,t)c_0(x,t))_x + a(t)c_0(x,t)\right]$$

$$c_2(x.t) = -\nu D_t^{-\alpha}(c_1(x,t))_x - MD_t^{-\alpha}A_1 - D_t^{-\alpha}\left[(F(x,t)c_1(x,t))_x + a(t)c_1(x,t)\right]$$

$$\mathbf{c_{j+1}(x.t) = -\nu D_t^{-\alpha}(c_j(x,t))_x - MD_t^{-\alpha}A_j - D_t^{-\alpha}\left[(F(x,t)c_j(x,t))_x + a(t)c_j(x,t)\right]} \tag{24}$$

In the nonlinear term, the Adomian polynomial can be calculated according to the specific algorithm by Adomian. Introduce a parameter λ, suppose that

$$c_\lambda(x,t) = \sum_{j=0}^{\infty} c_j(x,t)\lambda^k, \ j \ge 0$$

Here $c_\lambda(x,t)$ is a representation of $c(x,t)$ with parameters λ. The general form of the Adomian polynomial is as follows:

$$\mathbf{A_j = \frac{1}{j!}\frac{d^j}{d\lambda^j}\left[x^{-\theta}c_\lambda^m(x,t)(-(c_\lambda(x,t))_x)\right]_{\lambda=0}} \tag{25}$$

So the whole series solution can be obtained by (24), (25) to

$$c(x,t) = \sum_{j=0}^{\infty} c_j(x,t)$$

In addition, we can get the approximate solution by truncating the series

$$c_N(x,t) = \sum_{j=0}^{N-1} c_j(x,t)$$

$$c(x,t) = \lim_{N \to \infty} c_N(x,t)$$

When $\alpha = 1$, the equation (13) is integer order nonlinear convection diffusion equation with force and absorption. And its approximate solution can be obtained by the above algorithm.

CONCLUSIONS

In this paper, Adomian decomposition method not only has the characteristics of good convergence but is easy to calculate and the solution is in series form. Compared with the standard numerical solution, it does not need to be discretized; no errors need to be corrected, and it does not require a lot of computer memory and operation ability. The algorithm can be better applied to the viscoelastic mechanics, fractional order models of neurons, data fitting, etc.

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CONFLICT OF INTERESTS

Authors declare that there are no conflicting interests regarding publication of this article.

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