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# **Research on abrupt changes of Rainfull-Runoff Relationship**

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# ABSTRACT

It is importance to differentiate the effects on the change of regime in order to formulate robust water management strategies. The consistency has been destroyed in the influence of human activities and climate change. This work focuses on detecting abrupt changes in relationship of annual rainfall and runoff for the hydro meteorological stations in Wei River of the northwest China. Using double mass curve, this paper research the relationship of annual rainfall and runoff to test Wei River that have gone through decreases and abrupt changes in relationship rainfall and runoff for HX station from 1951 to 2010. In order to further verify the abrupt changes of the relationship, the Copula function was introduced into the paper. The study found that the distribution type of rainfall and runoff had changed in the variation before and after. The results show that the relationship of rainfall and runoff has undergone change in Wei Basin, and the runoff change in Wei River is significantly decreasing. Attribution analysis indicates that the primary reason of the change in relationship variation were human activities and climate change. The analysis of the variation changes of runoff would provide scientific understanding of Wei River basin and similar basins.

# KEYWORDS

Abrupt changes; Rainfall; Runoff; Wei river; Double mass curve; Copula function.



### **INTRODUCTION**

There are significantly changes in hydrology as runoff, flood and drought. Steamflow in a certain river basin is influenced by many factors, like climatic variables, human activities, subsurface drainage patterns, and various other climatologic and hydrological variables. Space-time distribution scope and scale of rainfall and runoff are changed<sup>[11]</sup>under the climatic change and human activities. Especially in the last decades, there are significantly changes in climate, such as temperature, rainfall, and runoff. The relationship of rainfall and runoff has been increasingly emphasized<sup>[2]</sup>. To date, most of the well-studied cases of abrupt change are focused on rainfall or runoff. Richter et al.,<sup>[3]</sup> who combined river ecosystem management objectives, annual runoff series variations were studied, and pointing out that the main driving force affecting river ecosystem is runoff magnitude variations. After a lot of calculation and verification, Perreault<sup>[4]</sup>used a Bayesian approach to detect changes in annual energy inflows of eight hydropower systems in Québec. Using the Mann–Kendall non-parametric test to detect abrupt changes and trends in runoff, Donald<sup>[5]</sup>investigated 18 hydrological variables reflecting various components of hydrological cycles in a network of 248 Canadian catchments selected to reflect natural conditions. Moreover various non-linear analysis methods have been introduced recently, such as entropy<sup>[6]</sup> techniques.

There are some certain relationships of rainfall and runoff in Wei River<sup>[7]</sup>that can be reflected in the double mass curve, and the slope of the curve changes to some extent reflects variations in rainfall-runoff relationship. Using Mann-Kendall test, double mass curve and bivariate Copula function, we analyzed variations of rainfall and runoff in Wei River, and the joint distribution function of rainfall and runoff, by comparing the differences in different periods of runoff, calculated contribution of climate changes and human activities on runoff changes.

## METHODOLOGY

#### Mann-kendall test

The Mann-Kendall test used to detect abrupt changes in runoff which is a statistical test widely used for the analysis of trend in climatologic and in hydrologic time series<sup>[8]</sup>.

Let the time series  $X = \{x_1, x_2, ..., x_n\}$ , the Mann-Kendall test is computed as follows:

$$D_{\tau} = \sum_{i=1}^{\tau} R_i \ (\tau = 1, 2, \dots n)$$
(1)

where  $R_i = \begin{cases} +1, & x_i > x_j; \\ 0, & x_i \le x_j \end{cases}$   $(j = 1, 2, \dots i)$ 

$$UF_{\tau} = \frac{\left|D_{\tau} - E(D_{\tau})\right|}{\sqrt{V(D_{\tau})}}$$
(2)

Where  $E(D_{\tau}) V(D_{\tau})$  are the mean and variance of  $D_{\tau}$ . In reverse sequence  $\{x_n, x_{n-1}, ..., x_1\}$ , repeat the above process, make  $UB_{\tau} = -UF_{\tau} (\tau = n, n-1, ..., 1)$ , draw graphs  $UF_{\tau}$  and  $UB_{\tau}$ , If  $UF_{\tau}$  or  $UB_{\tau}$  value is greater than 0, then the sequence on the rise, and vice versa decline; the intersection of the two curves is the abrupt change point.

### The double mass curve

Rainfall and runoff data were collected to examine the magnitude, distribution, and frequency of runoff and to determine selected factors affecting runoff from agricultural fields. The double mass curve is used to check the consistency of many kinds of hydrometeorological data by plotting the graph. The graph of the cumulative data of two variables is a straight line so long as the relation between the

variables is a fixed ratio. Breaks in the double-mass curve of such variables are caused by changes in the relation between the variables<sup>[9]</sup>. In this method proposed nearly a century, scholars have made great contributions to double cumulative curve<sup>[10]</sup>. Double mass curve is a simple method, computed as follows.

Let the observed variable values  $X_i$  and  $Y_j$ , the cumulative amount calculated:

$$X'_{i} = \sum_{j=1}^{i} X_{j}$$

$$(3)$$

$$Y_i' = \sum_{j=1}^{N} Y_j \tag{4}$$

Where,  $X_i$  represents rainfall, and  $Y_j$  is the runoff. The slope of the curve changes are abrupt changes points.

## **Copulas functions**

Copulas functions are useful tools for the construction of multivariate models<sup>[11]</sup>. The joint cumulative distribution function<sup>[12]</sup> H(x, y) H(x, y) of any pair (*X*, *Y*) of random variables may be written in the form.

$$H(x, y) = C[F(x), G(y)], \quad x, y \in R$$
(5)

F(x) and G(y) are marginal distributions, and  $C:[0,1]^2 \rightarrow [0,1]$ .

The Copula theory can be extended to multi-dimensional joint distribution. Previous studies of the joint distribution required making some assumptions on the random variables, but copula function can avoid the limitations of assumptions.

BivariateT-copula function is described equation (6).

$$C^{t}(u,v;\rho,k) = \int_{-\infty}^{t_{k}^{-1}(u)} \int_{-\infty}^{t_{k}^{-1}(v)} \frac{1}{2\pi\sqrt{1-\rho^{2}}} \left[1 + \frac{s^{2} - 2\rho st + t^{2}}{k(1-\rho^{2})}\right]^{-(k+2)/2} ds dt$$
(6)

 $\rho$  is the linear correlation coefficient, k is the degree of freedom,  $t^{-1}$  is the inverse function of the distribution of T.

Archimedean Copula function contains more types and the Frank-Copula relatively wide scope of application. Bivariate Frank-Copula function is described as equation (5).

$$C^{Fr}(u,v;\theta) = -\frac{1}{\theta} \ln\left[1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{(e^{-\theta} - 1)}\right], \theta \in (-\infty,\infty) \setminus \{0\}$$

$$\tag{7}$$

Let  $(x_i, y_i)(i = 1, 2, \dots, n)$  be samples taken from the general (X, Y).  $F_n(x)$  and  $G_n(y)$  are empirical distribution functions X and Y respectively.

$$\hat{C}_{n}(u,v) = \frac{1}{n} \sum_{i=1}^{n} I_{[F_{n}(x_{i}) \le u]} I_{[G_{n}(y_{i}) \le v]}, \ u, v \in [0,1]$$
(8)

 $I_{[\bullet]}$  is indicator function, when  $F_n(x_i) \le u$ ,  $I_{[F_n(x_i) \le u]} = 1$  else  $I_{[F_n(x_i) \le u]} = 0$ 

The Euclidean distance (ED) of E-Copula and bivariate copula is defined as equation (9)

$$d = \sqrt{\sum_{i=1}^{n} \left| C(u_i, v_i) - \hat{C}_n(u_i, v_i) \right|^2}$$

Euclidean distance reflects goodness of fit, the smaller the value, the model is more appropriate.

### **RESULTS AND DISSCUSS**

The Wei River originates in Weiyuan County, Gansu Province, northwest China. It passes through several major cities, including Tianshui, Xianyang and Weinan situated in the southern margins of the Loess Plateau before discharging into the Yellow River at Tongguan. The basin is in the continental monsoon climate zone, with cold, dry winters and hot, wet summers governed by the Mongolian and West Pacific subtropical high pressure systems, respectively.

# Trends and abrupt changes of rainfall and runoff

As can be seen from Figure 1, runoff significantly declined in the Wei River, rainfall tends to decrease. There were abrupt changes in rainfall at HX in 1972. There were abrupt changes in runoff at HX in 1973, 1982 and 1994.





Figure 1b : M-K values of runoff in HX

#### Figure 1 : M-K test values for rainfall and runoff at 95% confidence level (the horizontal line) at HX

In the Euclidean distance criterion, before the variation of rainfall-runoff relationship, T-Copula fits better than Frank-Copula (Frank-Copula has been better than T-Copula at XY), but Frank-Copula better after the variation. For most stations, the probability density function more obeys a certain copula function type before variation; however, even to obey another function after variation, as shown in TABLE 1.

The abrupt changes of rainfall-runoff relation



Figure 2 : Double mass curve of HX (the turning point is variation corresponding to the year)

(9)



Figure 3a : Bivariate T-Copula density function at HX in 1951~1972





Figure 3b : Bivariate T-Copula density function at HX in 1973~2010



HX in 1951~1972

Figure 3c : Bivariate frank-copula density function at Figure 3d : Bivariate frank-copula density function at HX in 1973~2010

#### Figure 3 : Joint probability density function of rainfall and runoff in HX

HX, for example, the copula function shown in Figure 3,

	Station	Time	Туре	Parameters	ED
НХ	1951-1973	1051 1072	Т	0.5693	0.1004
		1951-1975	Frank	5.0797	0.1363
	1974-2010	1074 2010	Т	0.2916	0.1211
		Frank	2.8874	0.0991	

**TABLE 1 : The parameters of copula function** 

# CONCLUSIONS

During the period of records, Wei River had been becoming drier. Meantime, the local human activities had become more and more extensive. The results obtained by applying the Mann-Kendall test, double mass curve, Copula function to the runoff series of Huaxian station show that variations occurred in 1972. Observed annual rainfall and stream flow were detected as decreasing trends. ENSO events, variations in West Pacific subtropical high pressure systems and various identified human activities appear to have contributed to the changes in runoff. However, sunspot activity patterns, other climatic factors and other human activities may also have contributed. Thus, further research is required to elucidate the causal factors comprehensively. The results could be useful for water resources planners

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and managers to understand the changing process of hydrological cycle and driving factors for runoff change and could also be a reference for water resources planning and management in the Wei River Basin.

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