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Relevant study on general uniqueness of solutions to several nonlinear problems

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ABSTRACT

Study process of general uniqueness of solutions to nonlinearproblems is sophisticated. This process involves a wide scope of mathematics. The research process is conducted with reference to some supporting documents and papers. Some theorems and reasoning process are deducted. This does not only furnish good theoretical basis for this research, but also provide comprehensiveness for general uniqueness of solutions to nonlinearproblems. For the research idea, this paper contains some introduction to theorem of general uniqueness, and then clearly defines its application sphere. Based on theorem's features, its reasoning process is clearly demonstrated to make its application more successful. This also enlarges its application scope; then we try to research and analyze several nonlinearproblems based on corresponding theorems and their applications, which makes research more detailed and clear; at last, we discuss the unique fixed point of most non-expansive mappings, and this is a supplement to other classification that involved in general uniqueness of solutions to nonlinearproblems. All these make this research scientific and integral.

KEYWORDS

Solutions to nonlinearproblem; General uniqueness; Application; Research.

INTRODUCTION

From the perspectives of theorem and definition, uniqueness and general uniqueness are fundamentally different. Uniqueness serves that the statement is true; however, the general uniqueness applies to the most, not showing they are true. Their corresponding statements are given in the first part of this research; the following part is about the proving of theorems and deduced theorems, which makes deduction effective and makes it become supplement to the theorems of uniqueness and general uniqueness. Thus, nonlinearproblems can be solved from a wider scope; the last part is about theorem application. The practice shall verify theorem. This is the principal we are following in research process. Also the last part expounds general uniqueness of solutions to nonlinearproblems and makes this paper more scientific. We hope this could lay good theory and practice foundations to the further research.

THEOREMS OF UNIQUENESS AND GENERAL UNIQUENESS

From relevant bibliographies, we can see that theorems of uniqueness and general uniqueness are not that complex. But they play a pivotal role. Therefore, for the sake of integrity, this research contains the following statement:

We assume that, M and X are two topological spaces on Hausdorff, and S is one of the set-valued mappings between M→2^X; $u \in M$. Under the conditions of (C) , we can effectively research the uniqueness.

(C) has some certain relation with any open neighborhood of any two open sets. $G_1, G_2 \subset X_3$, and $G_1 \cap G_2 = \theta$. This makes $S(u) \cap G_1$ and θ not equal. However, there is $u' \in o \setminus \{u\}$ in open neighborhood O that makes $S(U) \cap G_1$ and θ equal[1].

We can easily see from the above statement that, $S(u)$ always works as a single point set. It is generally sufficient under condition (C), thus we can see its necessity. Even sometimes, its necessity & sufficiency can exist simultaneously. From the above statement, we can get corresponding theorem as follows:

Theorem 1: Assume S is one set-valued mapping of M→2X between u, and it is semi-continuous. As a single point set, u would be effective under condition （C）.

Proof: Assume G_1 and $G_2 \subset X$ are any two open set, and they meet two conditions that $G_1 \cap G_2 = \theta$ and $S(u)$ ∩G1 is not equal to θ . Then we assume, O is any open neighborhood of u. As mentioned in the above conditions, $S(u)$ is a single point set, this makes $S(u) \subset G_1$. However, S is semi-continuous on u. Then there exists any point $u' \in o \setminus \{u\}$ on any open neighborhood O_1 of u, and this ensures $S(u) \subset G_1$. Then, we get $G_1 \cap G_2 = \theta$, and we can also get the following relation:

$$S(u') \cap G_2 = \theta \quad \textbf{(1)}$$

This proof process can effectively demonstrate that (C) is effective in u.

Theorem 2: We assume that S is a set-valued mapping between M→2^X, and it is lower semi-continuous on u. From the last proving process, we can see that (C) is effective in u. Then $S(u)$ naturally is single point set.

Proof: From the above stated theorem we can see that, if S is lower semi-continuous on u, then there is $\bar{x} \in S(u)$ to make any open neighborhood U_1 is within u's open neighborhood O The two conditions can are available: for any open neighborhood, $u' \in O$, and $S(u') \cap U \neq \theta$ [2]. However, through proof by contradiction, we can also effectively verify. We suppose, S（u）isn't single point set, and $x' \in S(u)$ will exist and $x' \neq \bar{x}$. From the separability of Hausdorff's space, we can know two open sets are generated: G_1 and $G_2 \subset X_3$. If we make $x' \in G_1$, and $G_1 \cap G_2 = \theta$, then we will get two conditions:

$$S(u) \cap G_1 \neq \theta \text{ and } S(u) \cap G_2 \neq \theta \quad \textbf{(2)}$$

On one side, as G_2 is one open neighborhood in $\bar{x}$, and the above proof process sheds light that u's open neighborhood is $O(u)$, if we make one of them $u' \in o$ then we can get the following condition:

$$S(u') \cap G_2 \neq \theta \quad \textbf{(3)}$$

However, on the other side, as $G_1 \cap G_2 = \theta$, meanwhile, $S(u') \cap U \neq \theta$ and condition （C） are effective. Previously stated open neighborhood $O(u)$ exists, thus finally we can get:

$S(u')\cap G_2 \neq \theta$ **(4)**

This contradicts with previously proved opinion. But we can see $S(u)$ is a single point set. Therefore, the proof process finishes.

Deduction 3: We assume S is lower semi-continuous on U, and （C） is effective on U, then we can prove $S(u)$ is single point set.

We can draw the following theorems based on theorem 1 and theorem 2

Theorem 4: We assume that S is both upper semi-continuous and lower semi-continuous, thus $S(u)$ is single point set and it makes condition （C） effective on U.

Deduction 5: We assume S is continuous on U, then $S(u)$ is single point set and it makes condition （C） effective on u.

From the above theorem and deduction, we can get:

Theorem 6: Assume M as Baire space, X as metric space. Then S is a USCO map between M→2^X. If (C) is effective under every condition in Baire space, and then every dense residual set can exist within Baire space. However, if we make any $\check{C}ech$, and if $S(u)$ is single point set, then, USCO mapping exists in Q set as single value form.

Deduction 7: This deduction is valuable, and it is widely used in theorem application. By process of above mentioned deduction and proof, we assume M is complete space of $\check{C}ech$. X is still a metric space, and then S is a USCO mapping between M→2^X. However, if every condition (C) is effective in $\check{C}ech$, thus every dense residual set can exist within $\check{C}ech$ space. if we make any $u \in Q$, then we get $S(u)$ is single point set, meanwhile, USCO map exists in Q set as single value form[3].

Theorem 8: Assume M as $\check{C}ech$'s complete space, and X belongs to φ. Then S is one USCO mapping between M→2^X. If every condition (C) is effective in $\check{C}ech$, thus every dense residual set can exist within $\check{C}ech$ space. If we make any $u \in Q$, then we get $S(u)$ is single point set, meanwhile, USCO map exists in Q set as single value form.

The following contents are supporting statements to the application of above theorems; meanwhile they are scientific practice to uniqueness of solutions to nonlinearproblem. During the assumption, M is problem space where every $u \in M_1$. However, here U is a nonlinear problem, and X is solution space. The nonlinear problem U can be shown in X space. S is a set-valued map between M→2^X. For every $u \in M_1$, here, $S(u)$ is set to be a set of all solution to nonlinear problem u, and we make value rage is not 0. However, $S(u)$ is single point set and when we use $\{x\}$ to represent, x is the only solution to nonlinear problem. When this solution is cast into U, and it is upper semi-continuous or lower upper semi-continuous, we just need to make every condition (C) effective in this point set U. By this proof, we can get that there is just one solution to nonlinear problem u. It is unique. However, for the uniqueness of solution to nonlinear problems, it does not always exist. Thus, we can research via general uniqueness. Relevant research processes involved in theorem 6, deduction 1 and theorem 8 that concern general uniqueness of solution to nonlinear problems provide basic structure and model[4].

APPLICATION

In this process, we mainly aim at the above mentioned theorems, their reasoning process and their application. Also we conduct corresponding research about maximum value, minimum value, and vector optimization of nonlinear problems. These all serve the discussion process of general uniqueness of solution to nonlinear problems.

There is unique solution to maximin problem of most semi-continuous function

We assume that X and Y are two nonempty sets. And f is one of the functions between $X \times Y \to R$. In this rage, the maximin problem can be solved via $(x^*, y^*) \in X \times Y$. We make the below equations effective:

$$\max_{x\in X}\min_{y\in Y} f(x,y) = \min_{y\in Y} f(x^*,y) = f(x^*,y^*) \quad \textbf{(5)}$$

In this process, if the condition (x^*, y^*) is available, it can be called a solution to max-min. Meanwhile, it is corresponding to the problem. Then we can solve $(x^*, y^*) \in X \times Y$ to make the following equation effective:

$$\min_{y\in Y}\max_{x\in X} f(x,y) = \max_{x\in X} f(x,y^*) = f(x^*,y^*) \quad \textbf{(6)}$$

Similar to this, usually we call (x^*, y^*) as one solution to min-max value.

In achieving the solution to min-max problem, we need to make $(x^*, y^*) \in X \times Y$ as one saddle point of function f. But the condition (x^*, y^*) must be not only a solution to min-max value but also to max-min problem. Form the above theorems, we can see that, the solution of max-min problem cannot be the solution of min-max problem, thus we can get that it is not a saddle point of function f[5]. Even though we have discussed the uniqueness in previous research process, this does not mean the research on max-min problem is complete.

We assume X and Y as non-empty compact subsets of E and F in topological space. And M_1 in max-min problem can be defined as:

$$M_1 = f : X \times Y \to R \quad \textbf{(7)}$$

Here we define distance of any point $f_1, f_2 \in M_1$

$$\rho_1(f_1, f_2) = \sup_{(x,y)\in X\times Y} \left| f_1(x,y) - f_2(x,y) \right| \quad \textbf{(8)}$$

Strictly-quasi monotonic vector optimization problems have unique solution

In this research and discussion, we mainly use unified methods to state interrelated theory, and make sufficient research about weakly efficient solution.

In research process, we assume X as E's nonempty compact subset in Hausdorff topological space. And C is nonempty closed convex cone in Banach space. Also we make $\operatorname{int} C$ is not 0[6]. Here, $\operatorname{int} C$'s implication is in C's topological space. Thus we can get $\operatorname{int} C$'s convex cone by above theorems and deduction, also the condition $\operatorname{int} C + C \subset \operatorname{int} \mathrm{C}$ can be met. For the value range of ε, it shall be over 0, and marked as open ball $B^0(\varepsilon) := \left\{ z \in H : \|z\| < \varepsilon \right\}$ and closed ball $B(\varepsilon) := \left\{ z \in H : \|z\| \le \varepsilon \right\}$.

Theorem 9: We assume f as a vector-valued function between X→H. Its harmony point $x^* \in X$ is one effective weakly efficient solution. of vector optimization. If any point could meet the condition of $y \in X$, then there will be the following relation:

$$f(x^*) - f(y) \notin \operatorname{int} C \quad \textbf{(9)}$$

However, when in cases that $H = \mathbb{R}^n$ and $C = \mathbb{R}_+^n$, the above stated vector problems can change into problem of target optimization. Then its weakly efficient solution could meet the following condition:

$$f(x^*) - f(y) \notin \operatorname{int} \mathbb{R}_+^n, \forall y \in X \quad \textbf{(10)}$$

MOST NONEXPANSIVE MAPPINGS HAVE UNIQUE FIXED POINT

In this research process, we also use unified methods to conduct corresponding research about unique fixed point of nonexpansive mappings. This provides sufficient practice for application of nonlinear problems.

During the research, we firstly assume X as a nonempty compact convex set in Banach space. Meanwhile, we assume M_3 as mapping set of function f within X→Y. We shall ensure that: X→Y is non-expansive. Then we can get mutual relation:

$$\| f(x) - f(y) \| \le \| x - y \|, \forall x, y \in X \quad \textbf{(11)}$$

In this relation, we can effectively define distance for any $f_1, f_2 \in M_3$.

$$\rho_3(f_1, f_2) = \max_{x\in X} \| f_1(x) - f_2(x) \| \quad \textbf{(12)}$$

For $f_1, f_2 \in M_3$, we can obviously see f_1 and f_2 are continuous. This makes corresponding sense for ρ_3, and make it more simple to prove a complete metric space when (M_3, ρ_3).

However, for any $f \in M_3$, we can prove that fixed point function f exist in X via corresponding fixed point theorem. We can see from the following equation relation:

$$S_3(f) := \left\{ x \in X : f(x) = x \right\} \ne 0 \quad \textbf{(13)}$$

In this equation, $f \to S_3(f)$ effectively defines set-valued map, and it is $S_3 : M_3 \to 2^X$.

Proof: As X is nonempty compactset, and based on the above theorem, we just need to effectively prove S3's closed mapping, i.e. $Graph(S_3)$ is one closed set of M3×X. Then the following equation relation shall be met[7].

$$Graph(S_3) = \{(f, x) \in M_3 \times X : x \in S_3(f)\} \tag{14}$$

Before proof, we first assume that $f_n \in M_3, f_n \to f_0 \in M_3, x_n \in S_3(f_n), x_n \to x_0 \in X$ are effective. We also need to prove the condition that $x_0 \in S_3(f_0)$, by which we can get the below equation relation:

$$f_n(x_n) = x_n \tag{15}$$

As $f_n \to f_0, x_n \to x_0$ exists and is effective, naturally, the following equation relation can be achieved.

$$\begin{aligned} \| f_n(x_n) - f_0(x_0) \| &\le \| f_n(x_n) - f_0(x_n) \| + \| f_0(x_n) - f_0(x_0) \| \\ &\le \rho_3(f_n, f_0) + \| x_n - x_0 \| \to 0 \end{aligned} \tag{16}$$

In previous proof process, we make $n \to \infty$ as known conditions, thus we can effectively prove $f_0(x_0) = x_0$, and get $x_0 \in S_3(f_0)$. Till now, the proof process finishes.

CONCLUSION

The above contents are relevant research and proof process about general uniqueness of solutions to several nonlinear problems. The key points of this paper are the researches of uniqueness and general uniqueness theorems. In this paper, we try to give sufficient deduction and proof process to make this research more theoretical, meanwhile we practically apply this research to nonlinear problems. As sufficient discussion about general uniqueness of solution to nonlinear problems has not been proposed, this research and proof process have more practical. From the idea of research and proof, this can contribute to the extension of theorem and definition; meanwhile contribute to its applications. The research and proof process convey our expectation that scholars and relevant researchers could gain more solid theoretical base for their further researches.

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