Product and process innovations in a monopoly market with network externalities

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ABSTRACT

This study analyzes the R&D investment problem when a monopoly performs both process innovation and product innovation in a market presents network externalities. It considers both consumer fulfilled expectation and myopic expectation and finds that: (i) product and process innovations are complementary and the network externalities can not change this complementary relationship in both types of expectation; (ii) the complementary degree is not influenced by the network externalities in the myopic expectation case, but it increases with the intensity of network externalities in the fulfilled expectation case; (iii) the complementary degree in the fulfilled expectation case is higher than that in the myopic expectation case; (iv) the optimal product (or process) innovation efforts when the market exhibits network externalities are higher than that when it does not exhibit network externalities in both expectation cases.

KEYWORDS

Process innovation; Product innovation; Network externality; Complementarity; R&D; Monopoly.
INTRODUCTION

The network externality in an industry is that the benefit that consumers derive from buying one or several of its products depends on the number of other consumers that use the same or compatible products (Katz and Shapiro, 1992)[1]. The industries presenting network externalities in general are high-tech industries (e.g. the information goods and personal computers, software and telecommunication industries, and so on), in which products (e.g. CD players, personal computers, digital cameras and software products) are characterized by a constant improvement in the quality associated to a drop in price (i.e. producers constantly invest in both product innovation and process innovation) (Mantovani, 2006)[2].

There exists a vast literature on the economic aspects of R&D innovation, a wide spectrum of which studies process innovation and product innovation separately. In recent years, the theoretical work has devoted a closer attention to the role of complementarity of R&D activities. Lambertini and Orsini (2000)[3] analyze the incentive towards process and product innovations under social planner and producer; Bandyopadhyay and Rajat (2004)[4] investigate the complementarity between process innovation, which lowers the marginal cost of quality, and product innovation; Mantovani (2006)[2] considers a monopoly case when a firm invests in both product and process innovations. Some scholars think that process innovation and product innovation may be substitutes (Battaggion and Tedeschi, 2006; Bacchiega, Lambertini and Mantovani, 2011)[5,6]. However, all of them do not study how network externalities affect the relationship between process and product innovations. Only Xing, Wang and Zhang (2009)[7] examine product innovation and process innovation in a vertically differentiated duopoly market with network externalities. However, the above paper supposes that consumer expectations are fulfilled and does not analyze the monopoly case. There are two basic methods to deal with consumer expectations: the myopic-expectation approach and the fulfilled-expectation approach (Katz and Shapiro, 1992; Regibeau and Rockett, 1996)[1,8]. Considering both types of expectation, this paper investigates the relationship between process and product innovations in a monopoly market with network externalities.

The rest of this study is organized as follows. The basic model is present in section 2. The fulfilled expectation case is analyzed in section 3. The myopic expectation case is investigated in section 4. The paper is concluded in the final part.

THE BASIC MODEL

There is only one firm in a market with positive network externalities. The market is noted by the unit interval [0,1]. The firm produces only one type of product and locates in the left point of the market (i.e. ‘0’ point). Consumers are indexed by their preferences for the product, which is measured by the parameter \( x \). Assume that consumers are uniformly distributed over the interval [0,1]. The utility function for consumer at \( x \) is given by

\[
    u = a + u(q^e) - p - tx .
\]  

(1)

In (1), \( a \) is the intrinsic utility generated by the product itself; \( u(q^e) \) is the utility consumer obtaining from the network externalities; \( q^e \) is the output (or network scale) of firm that consumers expect; \( p \) is the product price; \( tx \) denotes the utility loss when a consumer purchases a product that differs from his/her most preferred product; \( t \) measures the degree of differentiation.

Since there always exist some consumers not purchasing any product in a market (i.e. the market is partly covered), this study supposes the market is not fully covered. Moreover, consumers are assumed to have unit demand (i.e. each consumer buys only one unit product or does not buy any product).
To increase the market demand, the firm engages in product innovation. When the firm derives the product R&D effort $\lambda$, it bears the investment cost $\gamma \lambda^2 / 2$. After product innovation, the intrinsic utility is given by

$$a = a_o + \lambda.$$  \hfill (2)

In (2), $a_o$ denotes the value of intrinsic utility before product innovation; $\gamma$ is a positive parameter characterizing product R&D cost (it measures the product innovation efficiency).

To lower the marginal cost, the firm engages in process innovation. When the firm derives the process R&D effort $\mu$, it bears investment cost $\gamma \mu^2 / 2$. After process innovation, the marginal cost is given by

$$c = c_o - \mu.$$  \hfill (3)

In (3), $c_o$ denotes the marginal cost before process innovation; $\gamma$ is a positive parameter characterizing process R&D cost (it measures the process innovation efficiency).

The timing of R&D innovations and pricing is as follows. Stage 1, the firm performs process innovation. Stage 2, the firm executes product innovation. Stage 3, the firm decides product price.

There are two basic methods to handle consumer expectations: the myopic-expectations approach, which considers the firm’s present network size, and the fulfilled-expectations approach, which employs the firm’s future network size. This study analyzes the relationship between process innovation and product innovation in both cases of expectation.

**THE FULFILLED EXPECTATION CASE**

In this section, consumer expectations are fulfilled (i.e. the firm’s output that consumers expect is equal to its actual output: $q' = q$). For simplicity, a linear unity function of network externalities is considered, i.e. $u(q') = \alpha q'$, in which $\alpha$ ($\alpha \geq 0$) is the intensity of network externalities.

Let $\overline{x}$ denote the marginal consumer who is indifferent between purchasing and not purchasing. Since consumers are uniformly distributed, $q = \int_0^\overline{x} dx = \overline{x}$. According to (1), $\overline{x}$ meets

$$a_o + \lambda + \alpha \overline{x} - p - \overline{x} = 0.$$  \hfill (4)

Thus, the firm’s demand function is given by

$$q = \overline{x} = \frac{a_o + \lambda - p}{t - \alpha}.$$  \hfill (5)

The profit function is

$$\pi = \frac{1}{t - \alpha} [p - (c_o - \mu)][(a_o + \lambda) - p] - \frac{1}{2}\gamma \lambda^2 - \frac{1}{2}\gamma \mu^2.$$  \hfill (6)

The model is solved by backward induction. Thus, the price stage is analyzed firstly.
Stage 3: Price

The first order condition of profit function (6) with respect to \( p \) is

\[
\frac{\partial \pi}{\partial p} = \frac{a_0 + \lambda + c_0 - \mu - 2p}{t - \alpha} = 0. \tag{7}
\]

Solving (7), the optimal price is given by

\[
p^* = \frac{a_0 + \lambda + c_0 - \mu}{2}. \tag{8}
\]

The optimal solution must meet the second order condition which requires \( \partial^2 \pi / \partial p^2 = -2/(t - \alpha) < 0 \) (this inequality holds when \( t > \alpha \)).

Plugging (8) into (6), the profit function, defined exclusively on the effort variables of innovations, is given by

\[
\pi = \frac{(a_0 - c_0 + \lambda + \mu)^2}{4(t - \alpha)} - \frac{1}{2} \gamma^2\lambda^2 - \frac{1}{2} \gamma^2 \mu^2. \tag{9}
\]

Stage 2: Product Innovation

Taking the derivative of (9) with respect to \( \lambda \), and setting it equal to zero

\[
\frac{\partial \pi}{\partial \lambda} = \frac{a_0 - c_0 + \lambda + \mu}{2(t - \alpha)} - \gamma \lambda = 0. \tag{10}
\]

Solving (10), the optimal effort of product innovation is

\[
\lambda^* = \frac{a_0 - c_0 + \mu}{2\gamma(t - \alpha) - I}. \tag{11}
\]

The second order condition requires \( \partial^2 \pi / \partial \lambda^2 = \gamma^2/(2(t - \alpha)) - \gamma < 0 \), which is met when \( \gamma > 1/[2(t - \alpha)] \).

Setting \( \varepsilon = \partial \lambda^*/\partial \mu \), the optimal effort of product innovation increases with the effort of process innovation if \( \varepsilon > 0 \) (i.e. process and product innovations are complementarity from the perspective of R&D investment) and decreases with the effort of process innovation if \( \varepsilon < 0 \) (i.e. process and product innovations are substitutes from the perspective of R&D investment). \( |\varepsilon| \) is defined as the complementarity (substitute) degree for two types of innovation.

**Proposition 1:** In the fulfilled expectation case, (i) \( \lambda^* \) increases with \( \mu \); (ii) \( \varepsilon \) increases (resp. decreases) with \( \alpha \) (resp. \( \gamma \) and \( t \)).

**Proof.** (i) because \( \frac{\partial \lambda^*}{\partial \mu} = \frac{1}{2\gamma(t - \alpha) - I} > 0 \); (ii) because \( \frac{\partial \varepsilon}{\partial \alpha} = \frac{2\gamma}{[2\gamma(t - \alpha) - I]^2} > 0 \), \( \frac{\partial \varepsilon}{\partial \gamma} = -\frac{2(t - \alpha)}{[2\gamma(t - \alpha) - I]^2} < 0 \) and \( \frac{\partial \varepsilon}{\partial t} = -\frac{2\gamma}{[2\gamma(t - \alpha) - I]^2} < 0 \).

The above proposition indicates that the effort of product innovation increases with that of process innovation (i.e. two types of innovation are complementarity from the view of R&D investment) in the fulfilled expectation case. Moreover, the degree of complementarity increases (resp. decreases) with the intensity of network externalities and the product innovation efficiency (resp. the degree of differentiation).

Substituting (11) into (9), the resulting profit is
\[\pi = \frac{\gamma (a_o - c_o + \mu)}{2[2\gamma(t - \alpha) - I]} - \frac{1}{2} \gamma \mu.\]  
\hspace{5cm} (12)

**Stage 1: Process Innovation**

Consider the process innovation stage in this part. The first order condition of (12) with respect to \(\mu\) is

\[\frac{\partial \pi}{\partial \mu} = \frac{\gamma (a_o - c_o + \mu)}{2\gamma(t - \alpha) - I} - \gamma \mu = 0.\]  
\hspace{5cm} (13)

The optimal effort of process innovation is given by

\[\mu^* = \frac{\gamma (a_o - c_o)}{[2\gamma(t - \alpha) - I]\gamma - \gamma}.\]  
\hspace{5cm} (14)

The optimal effort of process innovation meets the second order condition when \(\gamma > \gamma/[2\gamma(t - \alpha) - I].\)

The resulting profit is

\[\pi^* = \frac{\gamma^2 (a_o - c_o)^2}{2([2\gamma(t - \alpha) - I]\gamma - \gamma}).\]  
\hspace{5cm} (15)

**Proposition 2**: the optimal effort of product (or process) innovation and the optimal profit are higher when the market exhibits network externalities than when it does not exhibit network externalities in the fulfilled expectation case.

**Proof.** According to (11), (14) and (15), \(\lambda_{a=0}^* < \lambda_{a>0}^*\), \(\mu_{a=0}^* < \mu_{a>0}^*\) and \(\pi_{a=0}^* < \pi_{a>0}^*\) can be proven.

The above proposition shows that, in the case of fulfilled expectation, the firm performs (resp. obtains) more R&D investment (resp. profit) when the market presents network externalities than when it does not.

When the firm engages in product innovation and does not engage in process innovation, the optimal profit is given by \(\pi^d = \gamma (a_o - c_o) / [2\gamma^2(t - \alpha) - I] \). Setting \(\tau = \pi^d / \pi\), \(\tau = [2\gamma(t - \alpha) - I] / [2\gamma^2(t - \alpha) - I - \gamma^2] > 0\) can be proven.

**Proposition 3**: in the fulfilled expectation case, (i) the firm obtains more profit when it executes both product and process innovations than when it only executes product innovation; (ii) \(\tau\) increases (resp. decreases) with \(\alpha\) (resp. \(\gamma\), \(\gamma\) and \(\gamma\)).

**Proof.** (i) \(\pi^d < \pi^*\) because of \(\tau > 1\); (ii) because \(\frac{\partial \tau}{\partial \alpha} = \frac{2\gamma^2}{\gamma^2} > 0\), \(\frac{\partial \tau}{\partial \gamma} = -\frac{1}{\gamma^2} < 0\), \(\frac{\partial \tau}{\partial \gamma} = -\frac{1}{\gamma^2} < 0\) and \(\frac{\partial \tau}{\partial \gamma} = -\frac{2\gamma^2}{\gamma^4} < 0\).

In the fulfilled expectation case, the first part of proposition 3 shows that, product and process innovations are complementarity from the perspective of profit; the second part of proposition 3 indicates that the degree of complementarity increases (resp. decreases) with the intensity of network externalities and the innovation efficiency (resp. the degree of differentiation).

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**THE MYOPIC EXPECTATION CASE**
This section assumes that consumers can not accurately predict the firm's output and their utility deriving from network externalities depends on the firm's output in the past (i.e., consumer expectations are myopic).

Let $\bar{x}$ denote the marginal consumer who is indifferent between purchasing and not purchasing. According to (1), $\bar{x}$ meets

$$a_0 + \lambda + u(q') - p - \bar{x} = 0 . \quad (16)$$

Thus, the demand function is

$$q = \bar{x} = \frac{a_0 + \lambda + u(q') - p}{t} . \quad (17)$$

The profit function is

$$\pi = \frac{1}{t}\left[ (p - (c_0 - \mu))[(a_0 + \lambda) + u(q') - p] - \frac{1}{2} \gamma \lambda^2 - \frac{1}{2} \gamma \mu^2 \right] . \quad (18)$$

The model is also solved by backward induction. Firstly, the price stage is analyzed.

**Stage 3: Price**

Solving the first order condition of (18) with respect to $p$, the optimal price is given by

$$p^* = \frac{1}{2}\left[ a_0 + c_0 + \lambda - \mu + u(q') \right] . \quad (19)$$

The optimal solution must meet the second order condition which requires $\partial^2 \pi / \partial p^2 = -2/t < 0$ (this inequality holds when $t > 0$).

Plugging (19) into (18), the profit function, defined exclusively on the R&D effort variables, is given by

$$\pi = \frac{1}{4t}\left[ a_0 - c_0 + \lambda + \mu + u(q') \right]^2 - \frac{1}{2} \gamma \lambda^2 - \frac{1}{2} \gamma \mu^2 . \quad (20)$$

**Stage 2: Product Innovation**

Solving the first order condition of (20) with respect to $\lambda$, the optimal effort of product innovation is given by

$$\lambda^* = \frac{a_0 - c_0 + u(q') + \mu}{2\gamma - 1} . \quad (21)$$

The second order condition requires that $\gamma > 1/(2t)$.

Setting $\dot{e} = \partial \lambda^*/\partial \mu$, the following proposition is obtained.

**Proposition 4:** In the myopic expectation case, (i) $\lambda^*$ increases with $\mu$; (ii) $\dot{e}$ does not affect by $u(q')$; (iii) $\dot{e}$ decreases with $\gamma$ and $t$. 
Proof. (i) because \( \frac{\partial \lambda^*}{\partial \mu} = \frac{1}{2\gamma - 1} > 0 \); (ii) because \( \frac{\partial \lambda}{\partial u(q^*)} = 0 \); (iii) because \( \frac{\partial \lambda}{\partial \gamma} = -\frac{2t}{(2\gamma - 1)^2} < 0 \) and \( \frac{\partial \gamma}{\partial \gamma} = -\frac{2\gamma}{(2\gamma - 1)^2} < 0 \).

Proposition 4 indicates that, in the case of myopic expectation, the optimal effort of product innovation increases with the effort of process innovation (i.e. two types of innovation are complementarity in the view of investment). The degree of complementarity increases with the efficiency of product innovation and decreases with the degree of differentiation. However, the degree of complementarity does not depend on network externalities. This is different from the fulfilled expectation case.

Substituting (21) into (20), the profit is

\[
\pi = \frac{\gamma}{2(2\gamma - 1)}[a_o - c_o + u(q^*) + \mu]\left[1 - \gamma^*\left(\frac{2\gamma}{(2\gamma - 1)}\right)\right].
\]

Stage 1: Process Innovation

Solving the first-order condition of (22) with respect to \( \mu \), the optimal effort of process innovation is

\[
\mu^* = \frac{\gamma[a_o - c_o + u(q^*)]}{(2\gamma - 1)\gamma^* - \gamma}.
\]

The optimal effort of process innovation meets the second order condition if \( \gamma > \gamma/(2\gamma - 1) \).

The resulting profit is

\[
\pi^* = \frac{\gamma^*[a_o - c_o + u(q^*)]}{2(2\gamma - 1)\gamma^* - \gamma}.
\]

Proposition 5: The optimal effort of product (or process) innovation and the optimal profit are higher when the market exhibits network externalities than when it does not exhibit network externalities in the myopic expectation case.

Proof. When the market does not present network externalities, \( u(q^*) = 0 \). According to (21), (23) and (24), \( \lambda^*\neq_{u(q^*)=0}^{u(q^*)>0} \), \( \mu^*\neq_{u(q^*)=0}^{u(q^*)>0} \), and \( \pi^*\neq_{u(q^*)=0}^{u(q^*)>0} \) can be proven.

The above proposition demonstrates that, the firm performs more R&D investment and obtains greater profit when the market presents network externalities than when it does not in the myopic expectation case. This conclusion is similar to the fulfilled expectation case.

When the firm carries out product innovation and does not carry out process innovation in the myopic expectation case, the optimal profit is given by \( \pi^0 = \frac{\gamma[a_o + u(q^*) - c_o]}{2(2\gamma - 1)} \). Setting \( \tau = \frac{\gamma}{2\gamma - 1} \),

\[
\tau = \frac{(2\gamma - 1)/(2\gamma - 1)}{\gamma^*}(2\gamma - 1 - \gamma^*) > 0
\]

Proposition 6: in the myopic expectation case, (i) the firm obtains more profit when it performs both product and process innovations than when it only performs product innovation; (ii) \( \tau \) does not affect by \( u(q^*) \); (iii) \( \tau \) decreases with \( \gamma, \gamma^* \) and \( t \).
Proof. (i) $\pi^{\ast} < \pi^{0}$ because of $\tau > 1$; (ii) because $\frac{\partial \pi^{0}}{\partial u(q^0)} = 0$; (iii) because $\frac{\partial \pi^{0}}{\partial x} = - \frac{1}{\gamma(2t - 1 - \gamma)} < 0$,

$\frac{\partial \pi^{0}}{\partial q} = - \frac{\gamma(2t - 1)}{\gamma(2t - 1 - \gamma)} < 0$ and $\frac{\partial \pi^{0}}{\partial x} = - \frac{2}{\gamma(2t - 1 - \gamma)} < 0$. □

Proposition 6 indicates that, in the myopic expectation case, product and process innovations are complementarity in the view of profit. The degree of complementarity increases with the efficiency of innovation and decreases with the degree of differentiation. However, the degree of complementarity does not affect by network externalities. This is different from the fulfilled expectation case.

Proposition 7: (i) $\varepsilon > \varepsilon^{0}$; (ii) $\tau > \tau^{0}$.

Proof. (i) because $\frac{1}{2t - 1} < \frac{1}{2t - 1 - \gamma}$, $2\gamma(t - \alpha) - 1 > 0$ and $\alpha > 0$, $\varepsilon > \varepsilon^{0}$; (ii) because $\frac{2\gamma - 1}{2t - 1 - \gamma} < \frac{2\gamma(t - \alpha) - 1}{2\gamma(t - \alpha) - 1 - \gamma}$, $\tau > \tau^{0}$. □

The above proposition shows that, form the perspective of both R&D investment and profit, the degree of complementarity in the fulfilled expectation case is higher than that in the myopic expectation case.

Note that: (i) this study analyses the case that the firm executes process innovation firstly and then product innovation. When the firm executes product innovation firstly and then process innovation, the analysis methods and results are similar to this paper; (ii) this study requires the optimal solutions meet the second order conditions and supposes the model parameters satisfy these conditions.

CONCLUSIONS

Building a mathematical model, this paper analyzes the relationship between product and process innovations in a monopoly market presenting network externalities. According to consumer expectations on firm’s network size, both fulfilled expectation and myopic expectation are considered. The following results are found: (i) in both fulfilled and myopic expectation cases, product and process innovations are complementary and the network externalities can’t change this relationship in the view of both R&D investment and profit; (ii) the degree of complementarity increases with the intensity of network externalities in the fulfilled expectation case, but it is not affected by network externalities in the myopic expectation case; (iii) standing point of both R&D investment and profit, the degree of complementarity in the fulfilled expectation case is higher than that in the myopic expectation case; (iv) the optimal product (or process) innovation effort and profit are higher when the market exhibits network externalities than when it does not in both expectation cases.

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