

2014

# BioTechnology

*An Indian Journal*

FULL PAPER

BTAIJ, 10(12), 2014 [6289-6298]

## Preventive maintenance optimization model based on the availability economical restriction

Li Guo\*, Zhi-Jie Song, Zhi-Xiu Yang, Yu-zhong Zhao  
School of Economic and Management, Yan Shan University, Qin Huangdao 066004,  
(CHINA)

### ABSTRACT

To reduce maintenance cost and improve the efficiency of the equipment for enterprise, the theory of modern equipment maintenance is adopted. By considering the dynamic nature of preventive maintenance cost and the age reduction factor, a preventive maintenance decision-making model is established. Unit time minimum cost is regarded as the optimization goal and the availability is regarded as the constraint in the preventive update cycle of the model. Numerical example is given, and the enumeration approach and software simulation is proposed to get preventive maintenance cycle and cost under the different times of the preventive maintenance. The result shows that when equipment preventive maintenance cycle narrows gradually, maintenance cost decreases at first and increases later, and when the cost reaches the minimum, the availability reaches the maximum.

### KEYWORDS

Maintenance optimization model; Availability; Economy; Dynamic maintenance cost; Age reduction factor.



## INTRODUCTION

Equipment maintenance issue has been an important factor affecting the development of the industry, agriculture, military, and economy and in other areas, without unreasonable equipment preventive maintenance planning, it may lead to the problem of excess maintenance along with substantial waste of device effective use time as well as manpower and material and other resources; on the other hand it may lead to inadequate maintenance of equipment along with an increase in post-maintenance cost and unnecessary malfunction losses. Therefore, the development of a preventive maintenance strategy is crucial for enterprises to save costs, improve production efficiency and make business operations more economical.

Currently, there are a large number of scholars carried out extensive research on the issue of preventive maintenance. K.Das et al (2007)<sup>[1]</sup> proposed an optimal preventive maintenance strategy for integrated manufacturing system based on the system reliability and availability of resources. P.Beaurepaire et al (2012)<sup>[2]</sup> approached reliability-based optimization model of maintenance scheduling of mechanical components under fatigue, and analyzed the parameter sensitivity. A preventive maintenance policy for a continuously monitored systems with correlated wear indicators was studied by Mercier et al (2012)<sup>[3]</sup>. Minh et al (2013)<sup>[4]</sup> proposed a preventive maintenance strategy to improve reliability of device system through the state monitoring and estimating the state of a degradation system. K.T. Huynh et al (2012)<sup>[5]</sup> obtained age-based maintenance strategies model with minimal repairs in the dynamic environment. A hybrid preventive maintenance model for equipment maintenance features was established by Chun-jie Yang et al (2008)<sup>[6]</sup> based on two maintenance strategies of equipment failure rate decay and age decrease. Bang-jun Han et al (2003)<sup>[7]</sup> build an optimal preventive maintenance policy model in finite time horizon of the equipment concerning with the concept of age reduction factor. Moreover, Zhao-yong Mao et al (2010)<sup>[8]</sup> discussed the recursion relationship of failure rate before and after preventive maintenance concerning with the concept of age reduction factor and the difference of preventive maintenance period.

As described above literatures, we consider the economy and reliability of equipment maintenance problems, but that's not enough to consider these two aspects because the maintenance state and cost of the system will vary as the time changes. Therefore, different from the existing literature, a maintenance optimization strategy model is established in which unit time maintenance cost as the objective function and availability as constraint by taking into account that preventive maintenance cost changes. Numerical example is given, and the enumeration approach and software simulation is proposed to get optimal preventive maintenance cycle and cost under the different times of the preventive maintenance. This paper considers the isochronic cycle preventive maintenance strategies, which facilitate the control and management in the actual production and equipment maintenance, the optimization results of this model coincide with the actual situation, so this paper has a real sense of operability.

## PREVENTIVE MAINTENANCE MODEL

Assuming the device is a repairable system, and we implement  $N-1$  periodic preventive maintenance in infinite time horizon, the device does not return to new or initial state, however, its enlistment age will be reduced, as well as return to the state of the younger. During maintenance interval failure rate increases, then replacing equipment parts in  $N$ -th maintenance will be more economical. If a failure occurs then conduct the minimal maintenance. Periodic preventive maintenance activities can be shown as Figure 1.

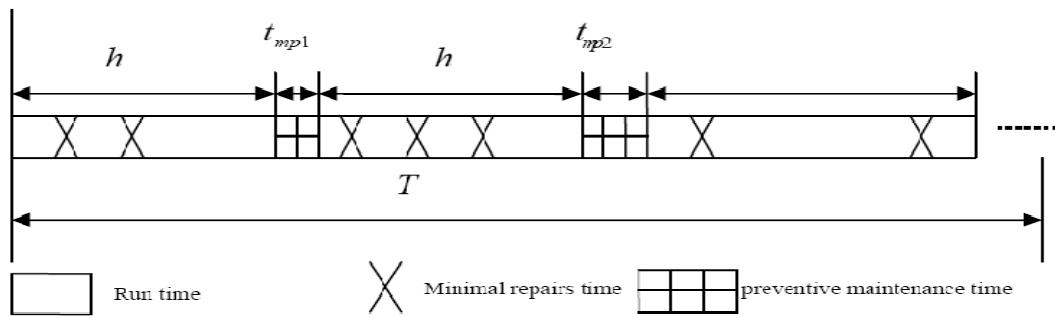


Figure 1 : The sequence chart of isochronic cycle preventive maintenance activities

Fundamental assumption:

- (1) Assuming the equipments implement preventive maintenance for an infinite time horizon ;
- (2) Implementing isochronic cycle preventive maintenance strategy in infinite time horizon, if the equipment failure occurred during the maintenance cycle, we adopt minimal maintenance measures, minimal maintenance does not change failure rate ;
- (3) The failure rate of devices increases with the passage of time, that is, the equipment system is degenerate;
- (4) We introduce age reduction factor to measure the available enlistment age after preventive maintenance, the minimal maintenance cost and the equipment replacement cost are fixed constants.

### AGE REDUCTION FACTOR AND FAILURE RATE

#### Age reduction factor

Let the preventive maintenance cost of every time be  $c_{pm_i}$ , then it is expressed as follow:

$$c_{pm_i} = c_f + ic_v \tag{1}$$

where  $i$  means the  $i$ -th preventive maintenance,  $c_f$  and  $c_v$  are the fixed cost and the variable cost, respectively. We can see from the above equation that preventive maintenance cost increases as maintenance frequency increases. After maintenance the equipment can not reach a new state, the more maintenance frequency, the faster equipment aging. Preventive maintenance optimization model can reduce the failure rate of the equipment and improve the life of equipment. In order to describe the effect of the performance of the equipment or system of the preventive maintenance optimization model, we introduce the age reduction factor to indicate the degree of equipment recovery :

$$\delta_i = \left( a \cdot \frac{c_{pm_i}}{c_{pr}} \right)^{b \cdot i}, i = 1, \dots, N - 1 \tag{2}$$

where  $N$  is preventive maintenance frequency,  $c_{pm_i}$  is the preventive maintenance cost each time,  $c_{pr}$  is the preventive replacement cost every time,  $c_{pm_i} \leq c_{pr}$ ,  $a$  is adjustment coefficient of maintenance cost, and  $1 \leq a \leq c_{pr} / c_{pm_i}$ ,  $b$  is adjustment parameter of preventive maintenance frequency,  $0 < b < 1$ . The equation also shows that  $\delta_i$  decreases as  $i$  increases.

### Available enlistment age and failure times

Let the equipment maintenance cycle be  $h$ , due to the influence of  $\delta_i$ , then available age before and after the first maintenance are given as follow respectively :

$$T_1^- = h, T_1^+ = h - \delta_1 h \quad (3)$$

that is, the remain age of the equipment increase by  $\delta_1 h$  after maintenance. The available age before and after the second maintenance are given as follow respectively :

$$T_2^- = T_1^+ + h = (1 - \delta_1)h + h = (2 - \delta_1)h \quad (4)$$

$$T_2^+ = T_2^- - \delta_2 h = (2 - \delta_1)h - \delta_2 h = (2 - \delta_1 - \delta_2)h \quad (5)$$

In the same way, the available age before  $i$ -th maintenance is:

$$T_i^- = T_{i-1}^+ + h = (i - \delta_1 - \delta_2 - \dots - \delta_{i-1})h = (i - \sum_{j=1}^{i-1} \delta_j)h \quad (6)$$

And the available age after  $i$ -th maintenance is :

$$T_i^+ = T_i^- - \delta_i h = (i - \sum_{j=1}^{i-1} \delta_j)h - \delta_i h = (i - \delta_1 - \dots - \delta_{i-1} - \delta_i)h = (i - \sum_{j=1}^i \delta_j)h \quad (7)$$

In age-based maintenance model, the age of equipment of  $i$ -th preventive maintenance cycle can decrease to  $(1 - \delta_i)h$ , that means equipment failure rate will be reduced to a certain extent after each preventive maintenance, then grow at a constant rate. Let the failure rate function is  $\lambda(t)$ , thus the failure rate function can be depicted as :

$$\left. \begin{aligned} \lambda_1(t) &= \lambda(t) \\ \lambda_2(t) &= \lambda(t + (1 - \delta_1)h) \\ \lambda_3(t) &= \lambda(t + (1 - \delta_1)h + (1 - \delta_2)h) \\ &\vdots \\ \lambda_i(t) &= \lambda\left(t + \left(i - 1 - \sum_{j=1}^{i-1} \delta_j\right)h\right) \end{aligned} \right\} \quad (8)$$

Weibull distribution is widely used in the description of mechanical products or equipment malfunction law. The failure rate expression of Weibull distribution is  $\lambda(t) = \frac{\beta}{\alpha} (t)^{\beta-1}$ , where  $\beta$  is form parameter,  $\alpha$  is scale parameter.  $\alpha, \beta$  usually due to the analysis of historical data of equipment malfunction utilizing mathematical statistics<sup>[9]</sup>.

Suppose that equipment failure density function follows two-parameter Weibull distribution, then failure rate function in  $i$ -th maintenance cycle is :

$$\lambda_i(t) = \frac{\beta}{\alpha} \left( t + \left( i - 1 - \sum_{j=1}^{i-1} \delta_j \right) h \right)^{\beta-1} \quad (9)$$

Hence in  $i$ -th maintenance cycle, the failure times of the system is :

$$n_i = \int_{T_{i-1}^+}^{T_i^-} \lambda_i(t) dt \tag{10}$$

### AVAILABILITY-BASED OPTIMIZATION MODEL

#### Maintenance cost

In this model, unit time minimum cost is regarded as the optimization goal and the availability is regarded as the constraint. The total maintenance cost consists of minimal repairs cost, periodic preventive maintenance cost, production loss cost and replacement cost.

#### Minimal repairs cost and the time required

Let the minimal repairs cost be  $c_{mr}$ , and suppose that each minimal repairs time is  $t_{mr}$ , both are constant, then total minimum maintenance cost and total minimum maintenance time during  $N$  preventive maintenance cycles are :

$$TC_{mr} = \sum_{i=1}^N c_{mr} n_i = c_{mr} (\int_0^{T_1^-} \lambda_1(t) dt + \int_{T_1^+}^{T_2^-} \lambda_2(t) dt + \dots + \int_{T_{N-1}^+}^{T_N^-} \lambda_N(t) dt) \tag{11}$$

$$T_{mr} = \sum_{i=1}^N t_{mr} n_i = t_{mr} (\int_0^{T_1^-} \lambda_1(t) dt + \int_{T_1^+}^{T_2^-} \lambda_2(t) dt + \dots + \int_{T_{N-1}^+}^{T_N^-} \lambda_N(t) dt) \tag{12}$$

#### Periodic preventive maintenance cost and time

Let per preventive maintenance cost be  $C_{pm_i}$ , then we can get the expression of the total cost of  $N-1$  preventive maintenances from formula (1), that is :

$$TC_{pm} = \sum_{i=1}^{N-1} c_{pm_i} = \sum_{i=1}^{N-1} (c_f + i c_v) = (N - 1)c_f + \frac{N(N - 1)c_v}{2} \tag{13}$$

Let the time required for per preventive maintenance be  $t_{pm_i}$ . With the increase of equipment age, the device is worn constantly, the time needed for per maintenance will be longer and longer, suppose that preventive maintenance cost is proportional to the preventive maintenance time<sup>[10]</sup>, that is :

$$c_{pm_i} = c_f + \eta \cdot t_{pm_i} \tag{14}$$

where  $\eta$  is the adjust parameter of preventive maintenance time. Therefore we can get total preventive maintenance from formula (1), (15), that is

$$T_{pm} = \sum_{i=1}^{N-1} t_{pm_i} = \sum_{i=1}^{N-1} \frac{i}{\eta} \cdot c_v = \frac{N(N - 1)c_v}{2\eta} \tag{15}$$

#### Production loss cost

Both minimum maintenance and periodic preventive maintenance would spend some time, which will cause some production loss. Let unit time production loss cost be  $c_s$ , according to formula (12) and (15), the total production loss cost of equipment during maintenance process is :

$$TC_s = c_s T_{mr} + c_s T_{pm} = \sum_{i=1}^N c_s t_{mr} n_i + c_s \frac{N(N-1)c_v}{2\eta} \quad (16)$$

### Replacement cost

After repeatedly minimal repairs and  $N-1$  periodic preventive maintenance, we carry out the preventive replacement of equipment in  $N$ -th time, so that the device can run properly and economically, assuming that the replacement cost of the equipment is  $C_{pr}$ , and ignoring equipment replacement time.

From formula (11)~(17), total repair costs in the periodic preventive maintenance optimization model can be expressed as follow:

$$\begin{aligned} TC &= TC_{mr} + TC_{pm} + TC_s + C_{pr} \\ &= c_{mr} \cdot \sum_{i=1}^N n_i + [(N-1)c_f + \frac{N(N-1)c_v}{2}] + \sum_{i=1}^N c_s t_{mr} n_i + c_s \frac{N(N-1)c_v}{2\eta} + C_{pr} \\ &= (c_{mr} + c_s \cdot t_{mr}) \left( \int_0^{T_1^-} \lambda_1(t) dt + \dots + \int_{T_{N-1}^+}^{T_N^-} \lambda_N(t) dt \right) + [(N-1)c_f + \frac{N(N-1)c_v}{2}] + c_s \frac{N(N-1)c_v}{2\eta} + C_{pr} \end{aligned} \quad (17)$$

The total run time  $T$  of equipment before replacing, including preventive maintenance time interval  $h$ , and preventive maintenance time required  $t_{pmi}$ , from the formula (17), we get the total run time expressed as follow:

$$T = Nh + T_{pm} = Nh + \frac{N(N-1)c_v}{2\eta} \quad (18)$$

### Availability

The average working hour of the long-run system in unit time is called availability of the system, that is  $A = \lim_{t \rightarrow \infty} \frac{A(t)}{t}$ , where  $A$  is availability,  $A(t)$  is total working time of the system's long-running,  $t$  is the life of system<sup>[11]</sup>.

We can get availability from above definitions :

$$A = \frac{Nh - T_{mr}}{Nh + T_{pm}} = \frac{Nh - \sum_{i=1}^N t_{mr} n_i}{Nh + \frac{N(N-1)c_v}{2\eta}} \quad (19)$$

The availability should be controlled on the minimum allowable value  $A_0$  in order to ensure equipment economy. In summary, we can get the following optimization model:

$$\begin{aligned} \min C(h, N) &= \frac{TC}{T} = \frac{TC_{mr} + TC_{pm} + TC_s + C_{pr}}{T} \\ &= \frac{(c_{mr} + c_s \cdot t_{mr}) \left( \int_0^{T_1^-} \lambda_1(t) dt + \dots + \int_{T_{N-1}^+}^{T_N^-} \lambda_N(t) dt \right) + [(N-1)c_f + \frac{N(N-1)c_v}{2}] + c_s \frac{N(N-1)c_v}{2\eta} + C_{pr}}{Nh + \frac{N(N-1)c_v}{2\eta}} \end{aligned} \quad (20)$$

$$s.t \quad A = \frac{Nh - T_{mr}}{Nh + TC_{pm}} = \frac{Nh - \sum_{i=1}^N t_{mr} n_i}{Nh + \frac{N(N-1)c_v}{2\eta}} \geq A_0 \tag{21}$$

$$h \geq 0$$

$$N \geq 0$$

### ANALYSIS OF EXAMPLES

To test the above described preventive maintenance optimization model, the following examples are given for analysis. Suppose that  $\beta = 3$ ,  $\alpha = 500$ ,  $c_s = 9000$  yuan,  $c_{mr} = 5000$  yuan,  $t_{mr} = 0.5$  days,  $c_f = 6000$  yuan,  $c_v = 50$  yuan,  $\eta = 0.05$ ,  $c_{pr} = 1000000$  yuan,  $a = 1$ ,  $b = 0.005$ ,  $A_0 = 0.9$ .

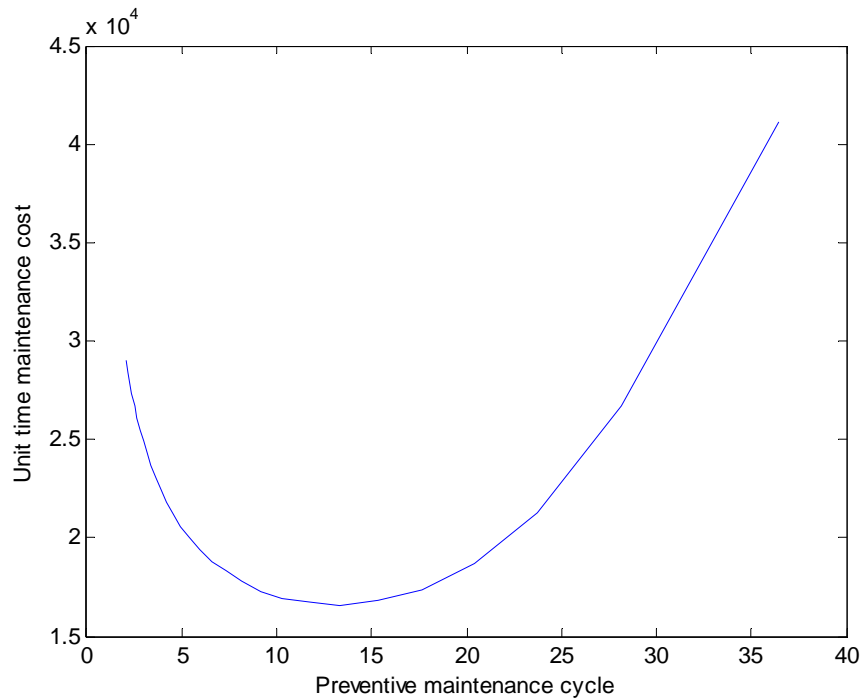
Based on the optimization model in this paper, we achieve the nonlinear inequality constrained optimization problems of examples using enumeration method and by programming. By calculation, we get the optimal preventive maintenance frequency and optimal preventive maintenance cycle under the fixed minimal availability, which are listed in TABLE 1.

**TABLE 1 : The preventive maintenance intervals and unit time maintenance cost under fixed minimal availability**

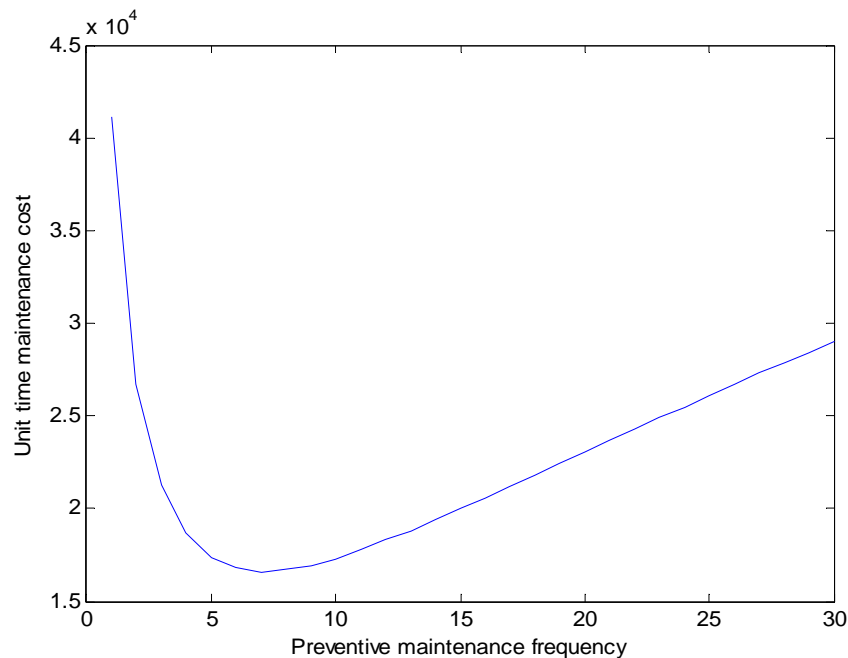
Preventive maintenance Frequency /time	Optimal maintenance cycle /month	Optimal unit time maintenance cost/yuan	Availability	Preventive maintenance Frequency /time	Optimal maintenance cycle /month	Optimal Unit time maintenance cost/yuan	Availability
1	36.4743	41100	0.9557	16	4.9756	20600	0.9764
2	28.1927	26700	0.9714	17	4.5781	21200	0.9754
3	23.703	21300	0.977	18	4.2119	21800	0.9747
4	20.3832	18700	0.9797	19	3.9118	22400	0.9736
5	17.623	17400	0.9812	20	3.6421	23100	0.9726
6	15.353	16800	0.9818	21	3.4003	23700	0.9716
7	13.3596	16600	0.982	22	3.1713	24300	0.9708
8	11.7351	16700	0.9818	23	2.9822	24900	0.9697
9	10.2931	16900	0.9816	24	2.8095	25500	0.9686
10	9.1465	17300	0.9809	25	2.6519	26100	0.9675
11	8.1576	17800	0.9803	26	2.5083	26700	0.9664
12	7.2912	18300	0.9797	27	2.3694	27300	0.9655
13	6.5849	18800	0.9788	28	2.2528	27900	0.9643
14	5.972	19400	0.978	29	2.1448	28400	0.9631
15	5.4264	20000	0.9773	30	2.0448	29000	0.9619

From the table we can find that, when  $N = 7$ , the optimal unit time preventive maintenance cost is 16600, and the homologous optimal preventive maintenance interval is  $h = 13.3596$ , that is, we should conduct preventive maintenance every 13.3596 months, then replace equipment parts in seventh after six preventive maintenance activities. In this case, the availability reaches 0.982, which is the maximum.

The unit time maintenance cost decreases first and increases later as the maintenance cycle increases, it reaches minimum when  $h = 13.3596$ , is shown as Figure 2. The preventive maintenance cost also decreases first and then increases as maintenance frequency increases, as shown in Figure 3. Then we have an optimal maintenance strategy combinations  $(h^*, N^*)$ .



**Figure 2 : The relationship between preventive maintenance cycle and cost**

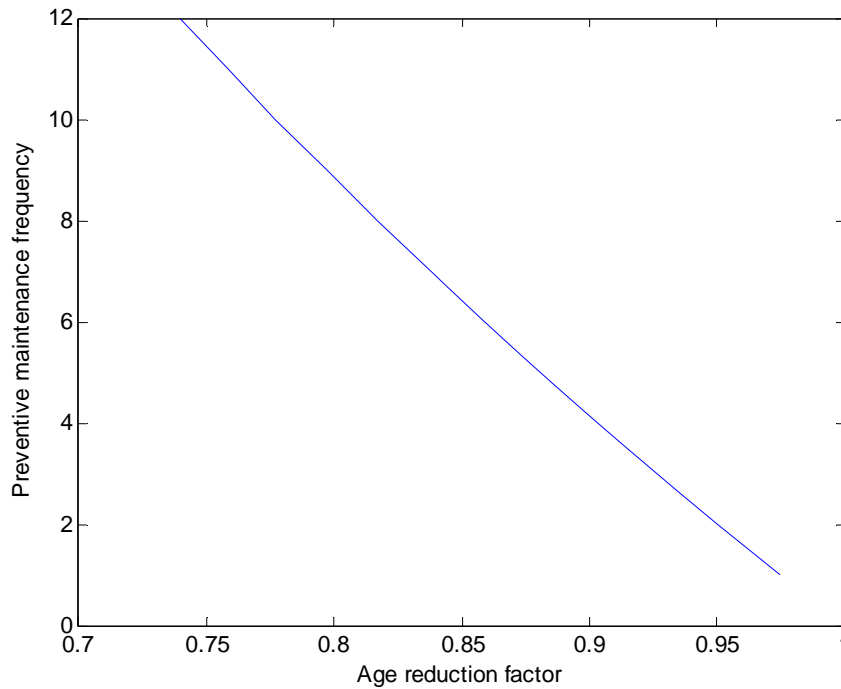


**Figure 3 : The relationship between preventive maintenance frequency and cost**

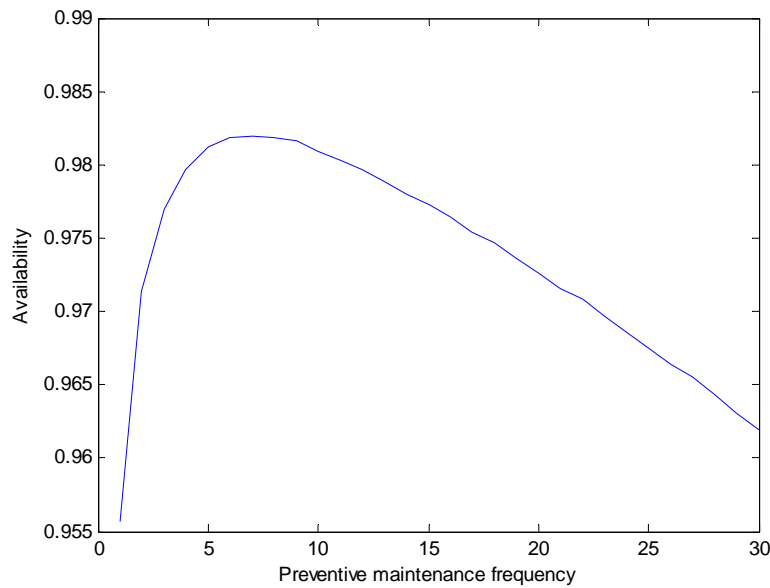
The age reduction factor curve is shown as Figure 4, it decreases as maintenance frequency increases. In the first preventive maintenance, age reduction factor is 0.9748, indicating that the device can be restored to almost new condition, while in the 12th preventive maintenance, only return to 86.19% of the initial state of the device.



This paper also shows the relationship between the preventive maintenance frequency and the equipment availability, see Figure 5, we can know from the figure that when the maintenance times reach at 7, equipment availability achieve the maximal degree. This indicates that the availability would increases in early preventive maintenance because of low repair difficulty; With the increase of preventive maintenance times and equipment aging, equipment maintenance will be more difficult while the availability will decrease. Meanwhile, from the figure, when the device reaches the maximal availability, the corresponding unit time maintenance cost reaches minimum. This is consistent with the actual production.



**Figure 4 : The relationship between preventive maintenance frequency and age reduction factor**



**Figure 5 : The relationship between preventive maintenance frequency and availability**

## CONCLUSION

Based on the preventive maintenance cost changing with the change in maintenance frequency, this paper studies the optimization problem of repairable system. Some similar studies have aimed to minimize the average cost of the long-run equipment, and device availability as the bound for objective function. Furthermore, those papers assume that preventive maintenance cost is fixed, which ignores the limited availability and variable preventive maintenance cost. Considering the variable preventive maintenance cost, the age reduction factor is introduced to determine the device age and failure rate and then a trade-off model about total maintenance costs and availability is established. This is a comprehensive analysis of equipment preventive maintenance strategy which have important theoretical significance and practical guidance. Finally, the article introduces the numerical examples to achieve the validation of the model.

## ACKNOWLEDGMENTS

The author expresses his gratitude to the editor and the anonymous reviewers for their detailed comments and valuable suggestions to improve the exposition of this paper. This paper is supported by Hebei Province Department of Education Science Research Program, Hebei Province Key Project of Natural Sciences in Colleges (ZH2012021).

## REFERENCES

- [1] Das, Lashkari, Sengupta; Machine Reliability and Preventive Maintenance Planning for Cellular Manufacturing Systems, *European Journal of Operation Research J.*, **183**, 162-180 (2007).
- [2] P.Beaurepaire, M.A.Valdebenito, H.A.Jensen; Reliability-based Optimization of Maintenance Scheduling of Mechanical Components under Fatigue, *Computer Methods Applied Mechanics and Engineering J.*, **221(222)**, 24-40 (2012).
- [3] Mercier, M.Pha; A Preventive Maintenance Policy for a Continuously Monitored Systems with Correlated Wear Indicators, *European Journal of Operational Research J.*, **222**, 263-272 (2012).
- [4] D.Minh, Cher T.Ming; Optimal Maintenance Strategy of Deteriorating System under Imperfect Maintenance and Inspection Using Missed Inspection Scheduling, *Reliability Engineering and System Safety J.*, **113**, 21-29 (2013).
- [5] K.T.Huynh, I.T.Castro, A.Barros; Modeling Age-based Maintenance Strategies with Minimal Repairs for Systems Subject to Competing Failure Modes Due to Degradation and Shocks, *European Journal of Operational Research J.*, **218**, 140-151 (2012).
- [6] Chun-Jie Yang, Sheng Tong, Chang-Sheng Sun, Zhi-Huan Song; Hybrid Preventive Maintenance Model Based on Reliability Constraint, *Journal of Zhejiang University J.*, **42(8)**, 1376-1379 (2008).
- [7] Bang-Jun Han, Xiu-Min Fan, Deng-Zhe Ma; Optimal Policy Research of Preventive Maintenance in Finite Time Horizon, *Journal of Shanghai Jiaotong University J.*, **37(5)**, 679-682 (2003).
- [8] Zhao-Yong Mao, Bao-Wei Song, Guang Pan; Optimal System Preventive Maintenance Model of Different Preventive Maintenance Period, *Fire and Command Control J.*, **35(3)**, 58-60 (2010).
- [9] Malik; Reliable Preventive Maintenance Policy, *AIIE Transactions J.*, **11(3)**, 221-228 (1979).
- [10] Tian-Wen Sheng, Xiao-Hui Chen, Shu-Ping Yi; Preventive maintenance policy for life-type equipment, *CIMS J.*, **15(3)**, 598-603 (2009).
- [11] Lin Tan, Tong Cheng, Bo Guo; Optimal Maintenance Strategy for the Repairable System Consisting of One Component Based on Geometric Process, *Systems Engineering J.*, **26(6)**, 88-92 (2008).