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Predictor-corrector method for solving fractional order differential equation and its Application

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ABSTRACT

Nowadays there are many methods to solve fractional order differential equation, including finite difference method, finite element method, meshless method etc. In many methods of solution to ordinary differential equations, the most important one is predictor corrector. Based on the atmospheric diffusion theory, considering various influence factors of nuclear radiation substance diffusion which includes wind speed, wind direction, rainfall and other factors, this paper establishes a predictor corrector model, the result of which is solved by using MATLAB. Then the paper obtains favorable result which is in compliance with actual monitoring data. Finally, based on the model, the paper analyzes the diffusion from a global perspective and reaches the conclusions that nuclear dispersal has little effect on the southeast coast of China and the west coast of the United States.

KEYWORDS

Radioactive gas; Diffusion; Change of concentration; Predictor corrector; ADMS model.

INTRODUCTION

Fractional calculus is a differential and integral theory of arbitrary order. It is unified with integer order calculus and the promotion of the integer order calculus[1-8]. Integer order calculus as a mathematical tool of description of classical physics theory and other related subjects has been generally accepted. The mathematical model of many problems can be ultimately attributed to the problem of integer order differential equations. It is a relatively perfect theory both in theoretical analysis and numerical aspects[9-15]. But when people deal with the complex system and phenomenon, they may face the following problems in the description of the classical integer order differential equations, such as the need to construct nonlinear equation, the introduction of some artificial experience parameters and conditions inconsistent with the reality, the need to construct a new model as a result of small changes in material or external conditions and both the theoretical solution and numerical solution of these nonlinear model are very complicated. Based on the above reasons, people are eager to have an available mathematical tool and the basic principles, which can be the basis of the complex system modeling[16-20]. Fractional order calculus equation is very suitable for description materials and processes with memory and genetic properties. It has advantages for complex system such as simple modeling, parameter with physical meaning, accurate description, so it has become one of the important tools of mathematical modeling for complex mechanical and physical process[21-23].

For nearly three centuries, the research of fractional calculus theory is mainly confined to mathematicians in pure mathematics field. It seems only to have useful for mathematician[24-28]. However, in recent decades, fractional differential equations are increasingly used to describe the optical and thermal systems, rheology, the material and mechanical system, signal processing and system identification, control and robotics, and other problems of application areas. The theory of fractional calculus is widely concerned by more and more domestic and foreign scholars, especially the fractional order differential equation which is abstracted form the actual problem has become a research focus for many mathematicians. Due to the wider application of fractional differential equations in fields of science computing, it is imperative to study the theory of the fractional differential equations and numerical calculation. However, because the fractional order differential is Pseudo differential operator, though the memory of it (non local) on the realistic problems has beautiful characterizations, it caused great difficulties to the analysis and calculation at the same time[29-33].

In the aspect of theory research, almost all results are all assumed to satisfy the Lipschitz condition, and the method used in the proof is the same with the classical calculus equation. In other words, the work is basically just a extension of the classical calculus equation theory There are only a few systematic results for qualitative analysis of the fractional differential equations. Most of them just give some special equation, and the commonly used method has limitations[35-39].

In the numerical solution of fractional order differential equation, the numerical algorithm is not very mature, mainly for:

(1) In the process of numerical calculation, some challenging problem has not been completely solved, such as calculation of long time course and spatial domain calculation etc.;

(2) Mature numerical algorithm is relatively small, and now most study mainly focuses on the finite difference method and the finite element method;

(3) Fail to develop mature numerical calculation software and lag far behind the needs of application.

In view of this, the development of new numerical algorithms, especially to guarantee the accuracy of the calculation of reliability and premise, improving the computational efficiency, solving the problems of large quantities of computation and storage for the fractional order differential equations and the development of corresponding software calculating the mechanical application become common concern.

FRACTIONAL DIFFERENTIAL EQUATIONS

Decay or relaxation equation[14,17]

$$\begin{cases} \dfrac{d^a u(t)}{dt^a} = Bu(t), (a \in (0,1]) \\ u(0) = 10 \end{cases}$$

The equation was used to describe the abnormal decay or relaxation behavior in the field of machinery, semiconductor, electromagnetics and optics, the equations were solved numerically by the separation method of definition. According to the initial conditions, it can get the corresponding numerical results

When the data points are very close to the initial time, the error is relatively large. But the error is reducing rapidly with the time and soon became 0. So predictor corrector method has good stability in the process of numerical solution. Because the numerical method in the integral equation is the explicit multistep method, the stability of the prediction correction method is the condition stability. And due to the different orders of fractional order α , the maximum step size for keeping stability are also different.

Fractal derivative relaxation vibration equation[24,27]

$$\frac{du(t)}{dt^a} + Bu(t) = f(t)$$

Finite difference discrete the equation, get:

$$\frac{u(t_1) - u(t_2)}{t_1^a - t_2^a} + Bu(t_1) = f(t_1)$$

the following parameters are given:

$$B = 1, u(0) = 1, f(t) = 0$$

Using the MATLAB[5] to get numerical results as shown in Figure 1:

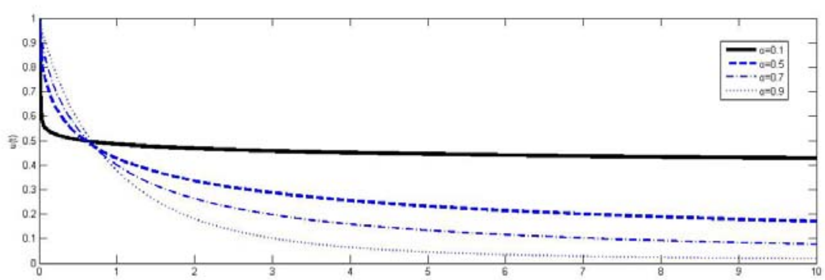

Figure 1 : The numerical solutions of Finite difference method

The predictor corrector method[7,9]

$$(-1)^{[a]-1} \frac{d^{|a|} f(t)}{dt^{|a|}} + Bu(t) = f(t) \ \ (B = w^a)$$

Here, Combined with fractional order derivative, the paper definse the form of positive fractional derivative definitions

$$\frac{d^{|a|}f(t)}{dt^{|a|}}=\begin{cases}\dfrac{-1}{aq(a)}\int_0^t\dfrac{f'(\tau)}{(t-\tau)^a}d\tau,(0<a\le 1)\\ \dfrac{1}{a(a-1)q(a)}\int_0^t\dfrac{f''(\tau)}{(t-\tau)^{a-1}}d\tau,(1<a<2)\end{cases}$$

We can get the relationship between the *Caputo* fractional derivative and the positive fractional derivative:

$$\phi(p)\frac{d^a u(t)}{dt^a}=\frac{d^{|a|}u(t)}{dt^a}$$

$$\phi(a)=\begin{cases}\dfrac{-\Gamma(1-a)}{aq(a)},(0<a\le 1)\\ \dfrac{\Gamma(2-a)}{a(a-1)q(a)},(1<a<2)\end{cases}$$

and

$$q(a)=\frac{\pi}{2\Gamma(a+1)\cos\left[(a+1)\dfrac{\pi}{2}\right]}$$

transposing the equation, we can get

$$\frac{d^{|a|}u(t)}{dt^{|a|}}=f(t)-Bu(t) \tag{1}$$

Using the equation (1), the equation is transformed into the fractional derivative relaxation - vibration equation

$$\frac{d^a u(t)}{dt^a}=\frac{f(t)}{\varphi(a)}-B\frac{u(t)}{\phi(a)}$$

Therefore, we can use the predictor corrector equation of fractional reciprocal relaxation - vibration equation to solve the equation. The following parameters are chosen:

$$B=1,u(0)=1,u'(0)=0,f(t)=0$$

The numerical results are shown as the following Figure 2:

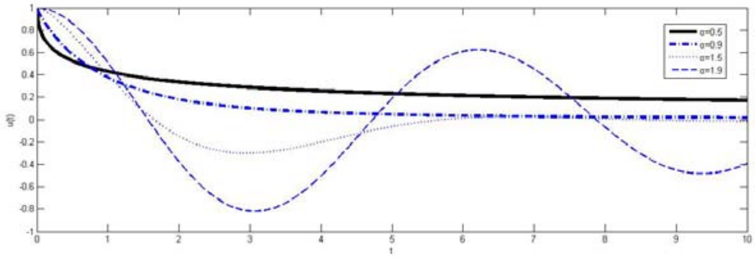

Figure 2 : The numerical solutions of predictor-corrector method

APPLICATION OF PREDICTOR CORRECTOR METHOD IN THE LIBERATION OF RADIOACTIVE GAS DIFFUSION[34,38]

Problems

A nuclear power melting down in natural disasters, radioactive gas with the concentration of p_0 is discharged with uniform velocity of $m\ kg/s$. In the case of no wind, the radioactive gas uniform spread around in the atmosphere, and the velocity is $s\ m/s$.

Problem 1: build a prediction model to describe radioactivity concentration in different periods from different distance around the region of nuclear power plants.

Problem 2: when the wind is $k\ m/s$, give the concentration changes situation of radioactive material in the surrounding of the nuclear power plant.

Problem 3, when the wind is $k\ m/s$, calculates the forecast model of the concentration of radioactive materials in the upwind and downwind L kilometers.

Problem 4, the model will be applied to the leakage of Fukushima nuclear power plant to calculate the impact of Fukushima nuclear power plant leaks on China's east coast and the west coast of the United States.

Problem solving

Now, a ADMS model which is more general will be built (The model includes PDF model, small convective scale model and Loft model):

(1) The PDF model[32,37]: under unstable conditions, using Weil PDF model to calculate the concentration of ground forlow buoyant nuclear pollutant, i.e.:

$$C = \frac{C_y}{\sqrt{2\pi\sigma_Y}} \exp\{-\frac{1}{2}[\frac{Y-Y_F}{\sigma_y}]^2\}$$

σ_Y in the equation is determined by:

$$\sigma_Y = \begin{cases} (\sigma_z x/u)/[1+0.5x/(uT_{xy})^{1/2}(F_m < 0.1)] \\ 1.6F_m^{1/3}X_m^{2/3}Z_i(F_m > 0.1, u/w_m \geq 2) \\ 0.8F_m^{1/3}X_m^{2/3}Z_i \end{cases}$$

C_y in the equation is determined by:

$$\frac{C_y uh}{Q} = \frac{2F_1}{\sqrt{2\pi\sigma_{x_1}}} \exp[-\frac{h_1^2}{2\sigma_{x_1}} + \frac{2F_2}{\sqrt{2\pi\sigma_{x_1}}}] \exp[-\frac{h_2^2}{2\sigma_{x_2}} + \frac{2F_1}{\sqrt{2\pi\sigma_{x_2}}}]$$

(2) Small convective scale model: under unstable conditions, using small convective scale model to calculate the concentration of the high buoyancy nuclear contaminants, i.e.
when $x < 10f / w*3$

$$C = 0.021Qw^3 * x^{1/3}(F^{4/3}Z_i)\exp[-\frac{1}{2}(\frac{Y-Y_p}{\sigma_y})^2]$$

$$\sigma_y = 1.6F^{1/3}X^{2/3}Z_i$$

when $x > 10f / w*3$

$$C = [Q/(wxh)\exp[-(\frac{7F}{zw^3})^{3/2}]\exp[-\frac{1}{2}(\frac{Y-Y_p}{\sigma_y})^2]]$$

$$\sigma_y = 0.6XZ_i$$

(3) Loft mode: using Weil Loft model for high buoyancy nuclear leakage in the near neutral conditions, i.e.:

$$C = \frac{Q}{\sqrt{2\pi}Z_1\sigma^y u}[1-erf(\Phi)]\exp[-\frac{1}{2}(\frac{Y-Y_p}{\sigma_y})^2]$$

$$\sigma_y = \begin{cases} 1.6F^{1/3}X^{2/3u-1}(L > 0 \ or \ L < O_3 \text{ and u/w} \geq 2) \\ \quad 0.8F^{1/3}X^{2/3u-1}(L > 0 \ and \text{ u/w<2}) \end{cases}$$

(4) Because the human body have a certain ability to resist the nuclear radiation. Only when the concentration of radioactive material to the surface of the earth is more than 50 mSv, it has obvious effects on the human body. In order to calculate nuclear radiation concentration of surface, we use ADTL model to calculate the pollutant concentration from surface sources which is based on the general Gauss model system. The application of the model depends on the specific circumstances. We divide them into surface source which is box arrangement. It is assumed that the source of strong spatial distribution and pollution diffusion follow certain rules. The calculation of ground concentration point is as follows:

$$C = \frac{Q}{\sqrt{2\pi}Z_1\sigma^y u}[1-erf(\Phi)]\exp[-\frac{1}{2}(\frac{Y-Y_p}{\sigma_y})^2]$$

$$K_A = [\frac{2}{\pi}]^{y_x}\frac{1}{u}\{Q_0\int_0^{L/2}\frac{1}{bx^q}\exp[-\frac{h^2}{2b^2x^2q}dx] + \sum_{i=1}^{N}Q_f\int_{(i-\frac{1}{2})L}^{(i+\frac{1}{2})L}\frac{1}{bx^q}\exp[-\frac{h^2}{2b^2x^2q}dx]\}$$

The results are shown in TABLE 1:

TABLE 1 : Calculated values of nuclear radiation substance concentration in different models and the measured average values /(mSv/m^3)

Monitoring point	Calculated values	Error (/%)	Measured values	sample number
Northeast	7.12	8.9	6.54	10
North China	17.31	7.8	16.06	10
Southeast	9.19	-11.3	10.36	10

CONCLUSIONS

The ADMS model in this paper is established to get the data which is consistent with the actual monitoring data. Unfortunately, due to the complexity of data processing, the numerical solution is not enough. This is what must be improved. Through the theory analysis of fractional differential equations, we mainly introduce two aspects, one of which is the nature of fractional order differential equations and the other is the method of solving differential equations of fractional order. The study of fractional differential equations is not mature enough, so the theoretical analysis made is still in the exploratory stage. The results are mostly simple extension of the classical theory of differential equations, differential equations of fractional order can only cover part of the special form. Many existing work is trying to seek for a new theoretical method, to break the existing limitations, and striving to construct a set of perfect theory of fractional order differential equations.

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CONFLICT OF INTERESTS

Authors declare that there are no conflicting interests regarding publication of this article.

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