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Physics, electronics and entropies for plasma confinement

Abstract

We consider the problem of plasma confinement from the point of view of physics and electronics. The universal role of fuzzy Jaynes entropy for such investigation is given together with the maximum entropy principle. We compare fuzzy Jaynes entropy with the Tsallis entropy.

Key Words

Plasma confinement; Fuzzy Jaynes entropy; Tsallis entropy; Maximum entropy principle.

INTRODUCTION

Reliable diagnostic information will be of the primary importance for the success of the ITER experiment. On the one hand, this is essential to allow the observation of the new physics inside the ITER plasma. On the other hand, plasma diagnosis is vital for the real-time control of various plasma parameters (burn control).

To obtain the highest possible accuracy of measurements, in the plasma environment of ITER, advanced diagnostic techniques need to be developed. For example, the characterization of the behaviour of plasma impurities is vital for control of impurity levels in ITER and reactor grade plasmas. Study of impurity transport is also of key importance to the exhaust of helium, which would otherwise suffocate the nuclear reactions. An emissivity profile is reconstructed by Abel inversion, while recently also Tikhonov and Maximum Entropy regularization have been tested.

PHYSICAL POINT OF VIEW

The mathematical models we introduce to study equilibrium, stability and transport of the plasma lead to complicated questions about nonlinear differential equations that are related to Kolmogorov Arnold Moser (KAM) theory.

Equilibrium and stability of a fusion plasma are best described by a variational principle based on magnetostatics. This leads in a natural way to a weak formulation of the partial differential equations toroidal equilibrium that circumvents results about nonexistence of solutions[1]. All this is only physical point of view and it is not a view via the electronics tools. In toroidal geometry without two dimensional symmetry the KAM theory shows that smooth solutions with nested flux surfaces $s = \text{const.}$, should not be expected to exist. Computations based on the variational principle of magnetohydrodynamics suggest, however, that weak solutions can be found with current sheets that model small magnetic islands. A satisfactory test for stability is to search for multiple, or bifurcated, solutions of the equilibrium problem over one or two field period of the device.

The confinement times converge to an average value that is in substantial agreement with predictions from scaling laws for experimental data. When we see the picture of solutions it is easiliy conclude that the description with weak solutions, bifurcated solutions and scaling laws are equivivalent In modern approach it can be obtained by using of the theory of fuzzy scaling for partial Vlasov-

Poisson-Fokker-Planck equations. Also, the methods of artificial intelligence become achieveable. Our eyes (sensors) represent dual picture for the design.

A basic aspect of the L-H transition concerns the respective role of electron and ion channels. These, however, can only be separated at low densities. The density dependence of the L-H transition power threshold (P_{thr}) is well-known to be non-monotonic and exhibits a minimum at the density labeled $n_{e,\min}$ which depends on the device. The conditions to separate the ion and electron heat channels are particularly good in the case of electron cyclotron resonance (ECR) heating.

For machines, in which the density region is accessible with neutral beam heating, an increase in P_{thr} at low density is also observed. The power hysteresis is an important question for ITER, as the H-mode will be entered at densities around the minimum in P_{thr} and the consecutive increase in density migh lead to plasmas sustained with a heating power well below P_{thr}, so that an ELM (edge-localized mode) migh led to an H-L back transition[2]. These rules provide us the possibilities of consideration as power law behavior as exponential behavior inside of the fuzzy logic theory, i.e. fuzzy entropy for example.

In classical mechanics, Hamiltonian equations hold if and only if Lagrangian equations hold, and thus both are equivalent to Newton's second law. There is the natural extension of Newton's second law to stochastic mechanic system under standard constraints. One can obtain a method to construct stochastic Hamiltonian systems: for example, by selecting the vibrating hanging point as reference point, stochastic effects are shifted on the particles (or the mass points) of system according to principle of relative motion, which results in a stochastic Hamiltonian system. We can apply the Lagrange multiplier technique to handle the concrete constraint. The optimal control problem turns out to be a quadratic loss minimization problem and we can solve it using the stochastic maximum principle.

ELECTRONIC POINT OF VIEW

Image representation is a topic of growing interest in computer vision and image processing and has numerous applications. The sensors are used to obtain these images can thus be of various type and can lead to images corrupted with different noise models: Gaussian, Poisson,.... It is therefore important to take into account the physical nature of the images in statistical techniques for image segmentation.

We can have online learning in discrete hidden Markov models. The learning process consists in presenting a series to the hidden Markov model which adapts its parameters in order to produce the sequences that mimic it. Depending of how data is presented, it can range from offline, when the whole data is given and parameters are calculated all at once, to online, when the data is given only by parts and a partial calculations of the parameters are made. We have parameters of the distributions. As each factor is a distribution over probabilities, the natural choise are the Dirichlet distributions. From MaxEnt and extremization of the Lagrangian we get the Dirichlet with normalization.

Most of the developed statistical segmentation techniques are based on a Bayesian formulation of the statistical model which generally leads to the optimization of a criterion that has at least one parameter to be tuned by the user. It is the case of Markov Random Fields (MRF) approaches.

The number of cameras, both visible and infrared, operational on JET has increased significantly in the last years[3]. In the last years, several image analysis methods have been explored to increase the efficiency of the data analysis. The interpretation of the videos of Tokamak cameras are complicated by various aspects, ranging from the variety of time scales involved to the strong changes in the background illumination. In case of big ELMs or disruptions, the supports of the cameras can vibrate affecting the images. Typically it is not possible to automatically reset the image. One idea being tested at the moment consists of caluculating a new type of nonextensive entropy, also called Tsallis entropy, to assess whether it can discriminate vibrations of the entire frames from the case of objects moving with frames. The general definition of this entropy is $S_{q0} = K\frac{1-\Sigma p_i^q}{q-1}$ where p_i are the probability of the i pixel value. Tsallis entropy is more sensitive to long range correlations than the usual definition of entropy.

The nonextensive entropy with q=0.1 is much more sensitive to vibrations and seems to be able to discriminate whether camera movements occur. With regard to image processing, these more sophisticate real time processing are required.

A distributed control systems is used to signal alarm conditions and send stop commands to the most important machinery of the future power station. This system ensures a constant monitoring of the equipment from which it receives signals, automatically managing their prediction. The data it contains can be used not only for control and protection, but also for trending the changing conditions of the machine under its control.

Many researchers have already successfully applied forecasting methods for predicting facility conditions. In the industrial sector analyzed, two models were developed on the basis of the methods proposed for predicting facility conditions : recurrent neural network and neurofuzzy systems.

The main problems of machine learning are sometimes specified as:

Classification : The inputs are (possibly noisy) multidimensional vectors and the outputs are discrete class labels.

Regression : The inputs are (possibly noisy) multidimensional vectors and the output is a continuous variable (also potentially noisy).

Detection : Detects statistical abnormalities in the input data.

The term sensor can be interpreted broadly as a provider of data for diagnostic. In general, data gathered from sensors could be both slowly-varying and computed by noise. Suppose the noisy signal z(t) is given by z(t) = x(t) + w(t) where x(t) is the signal and w(t) is the noise. A fundamental objective of signal processing in general is to remove as far as possible the noise and preserve the signal, or at least that component of the signal that carrying the desired information, i.e. a means of features extraction or dimension reduction- passing a time signal through a digital filter.

CONTROLLED MARKOV CHAINS

Entropy maximization as the classical variational principle of the statistical physics is an effective tool for modeling and solving a lot of applied problems. There are many definitions of entropy functions.

In this paper we suppose that the state space X of the controlled Markov chain can be written as the union of different ergodic classes X_i, for $i \in I$, where I is a countable (finite or infinite set), and a transient part X_*. We consider singularly perturbed countable Markov control processes. We shall suppose that for each $x \in X$ there exists a positive nondecreasing function ψ_x^G with $\lim_{t\to 0}\psi_x^G = 0$ such that for all actions $a \in A(x)$ and $(y, a') \in K$ it is true that

$$\| G(\bullet,(x,a)) - G(\bullet,(y,a')) \|_w \leq \psi_x^G(d(x,a),(y,a')) \quad (1)$$

where $\|\bullet\|_w$ is the w-norm

(a) a Borel space X, represents the state space

(b) a Borel space A, represents the control or action set

(c) a family {A(x)%x $\in X$} of nonempty measurable subsets A(x) of A, where A(x) denotes the set of feasible controls or actions when the system is in state $x \in X$, and with the property that $K = \{x,a) \mid x \in X, a \in A(x)\}$ is a measurable subset of X × A.

For any $\varphi \in \Phi$, P^φ restricted to X_i is a w-ergodic kernel with unique invariant probability measure on X_i, denoted by $\Pi_i^\varphi, i \in I$[4].

On this way, we have got the result for estimation of different ergodic subclasses.

We are interested also in a dual problem of stochastic control: Given a dynamical system, a cost function, an action set, and a set of observation channels, does there exist an optimal observation channel? It means, in the context of Jaynes entropies, that we can find good constraints on the behaviour.

Let an observation channel Q be defined as a stochastic kernel from elements of controlled Markov process $\{X_t, t \in Z_+\}$ to a Borel set $Y \subset R^m$. Let a decision maker (DM) be located at the output of an observable channel Q, with inputs X_t and outputs Y_t. For a partially observed stochastic control problem, sometimes we have control over the observation channels by encoding (quantization). Given an uncertainity set for the observation channels, can one identify a worst element/ best element ? (robust control). It all is connected with the problem of design of experiments. Sometimes there exist best and worst channels in Q[5]. Under regularity conditions a sequence of channels $\{Q_n\}$ converges to a channel Q uniformly in total variation if

$$\lim_{n\to\infty} \sup_{x\in X} \| Q_n(\bullet,x) - Q(\bullet,x) \|_{TV} = 0 \quad (2)$$

Instead of arithmetic mean and standard deviation, in fuzzy logic we use words for classification of data.

Image parametrization is a technique for describing bitmapped images with numerical parameters- features and attributes. Traditionally, popular images features are the first and second order statistics, structural and spectral properties, and several others. Provided that images are described with informative numerical attributes, various machine learning algorithms can be used to generate a classification system (classfier) for diagnosis.

Dimensionality reduction is a mapping from a multidimensional space into a space of fewer dimensions. It is often the case that data analysis can be carried out in the space more accurately than in the original space. Principal componenent analysis (PCA) is a linear transformation that chooses a new coordinate system for a data such that the greatest variance by any projection of the dataset lies on the first axis (called first principal component), the second greatest variance on the second axis, and so on.. PCA can be used for reducing dimensionality in a dataset while retaining those characteristics of the dataset that contribute most to its variance by eliminating the lesser principal components (by a more or less heuristic decision). PCA is sometimes used to extract features directly from images in matrix form, where pixel intensity values are used as primary features.

POWER LAW BEHAVIOR

Fluctuation theorems have become a standard tool to characterize nonequilibrium states. The probability of positive and negative fluctuations of a given variable differ by an exponential weight proportional to the fluctuation's magnitude. The exponential weight does not scale linearly with the heat variable. We deal with anomalous (power-law distributed) fluctuations. The power-law distributions arise in Tsallis nonextensive statistical mechanics.

Normal distributions emerge naturally when formulating a central limit theorem, as solution of linear Fokker-Planck equations, or by maximizing Gibbs entropy under a second moment constraint. Similarly, q-Gaussian distributions are related to generalized central limit theorem, are solutions of a kind of nonlinear Fokker-Planck equations and maximize nonextensive Tsallis entropy under a generalized second moment constraint. We deal with superstatistics that consists of superpositions of different statistics for driven nonequilibrium systems with spatiotemporal inhomogeneities of an intensive parameter, for example, inverse temperature.

Instead of a standard exponential structure, we can assume[6]

$$p(\mu) \cong \frac{t^{\delta/2}}{C_q}\exp_q\left[-t^{\delta}C_q^{1-q}\varphi(\mu)\right] \quad (3)$$

The formalism of nonextensive statistical mechanics leads to a generalized Boltzmann factor in the form of a Tsallis distribution. This distribution os of high interest in many physical systems since it enables to model power-law phenomena. Tsallis' distribution (sometimes called Levy distributions) are derived by maximization of Tsallis entropy, under suitable constraints. A key for the apparition of Levy distributions and a probabilistic identification might be that it seems to appear in the case of modified, perturbed or displaced classical Boltzmann-Gibbs equilibrium. This means that the original MaxEnt formulation " find the closest distribution to a reference under a mean constraint " that displaces the equilibrium.

The physical models emerge from dimensionality constraints on the exponents of a scaling function over the confinement energy W. As operation parameters entering the scaling function serve the electron density n, toroidal magnetic field B, absorbed power P and a (the radius)

$$W^{theor} \propto n^{\alpha_n} B^{\alpha_B} P^{\alpha_P} a^{\alpha_a} \quad (4)$$

The analysis of discrimination, feature and model selection- conduct to the discussion of the relationships between different classification formalisms based on machine learning methods, such as Support Vector Machine (SVM), Bayesian and Maximum Entropy (MaxEnt) inference[7].

APPROACH VIA ENTROPIES

Tsallis entropy in discrete and continuous system are respectively defined as follows

$$S_q^{(d)} = \left(\frac{1}{q-1}\right)\left(1-\sum_{i=1}^{\infty} p_i^q\right), \ (q \in R^+) \quad (5)$$

$$S_q^{(c)} = \left(\frac{1}{q-1}\right)\left(1-\int f(x)^q dx\right), \ (q \in R^+) \quad (6)$$

where $\{\langle p_i \rangle_{i=1}^{\infty}$ is a probability distribution and f is a probability density function. According to the L'Hospital rule, Tsallis entropy recovers Shannon entropy when $q \to 1$,

$$\lim_{q\to 1} S_q^{(d)} = -\sum_{i=1}^{\infty} p_i \ln p_i \quad (7)$$

$$\lim_{q\to 1} S_q^{(c)} = -\int f(x)\ln f(x) dx \quad (8)$$

The maximum entropy principle for Tsallis entropy $S_q^{(c)}$ under the constraints

$$\int f(x)dx = 1 \ , \ \left(\int f(y)^q dy\right)^{-1}\left(\int x^2 f(x)^q dx\right) = \sigma^2 \quad (9)$$

yields the so-called q-Gaussian probability density function[8].

The power form in q-Gaussian has been found to be fairly suited to many physical systems which cannot be systematicaly studied in the usual Boltzmann-Gibbs statistical mechanics. There exist interesting mathematics for such purpose. But, our aim is to prove the equivalent formulation of such phenomena inside of the theory of fuzzy Jaynes entropy[9].

It is well known that any binomial distribution converges to a Gaussian distribution when n goes to infinity. This is a typical example of the central limit theorem in the usual probability theory. It open a possibility of the existence central limit theorem in Tsallis statistics, in which any distributions converge to a q-Gaussian distribution as nonextensive generalization of a Gaussian distribution. The central limit theorem in Tsallis statistics mathematically explains the reason of ubiquitous existence of power-law behaviours in nature. For $q \approx 1$ the soft computing of these transitions could be expressed in linquistic form with fuzzy Jaynes entropy and the approach is successful in description of L-H and H-L transitions of plasma confinement at tokamaks and stellarators. It is similar to the situation when Poisson distribution replaces binomial distribution.

The diagnostic set-up for Wendelstein 7-X, a magnetic fusion device presently under construction, is currently in the design process to optimize the outcome under given technical constraints. The aim is to find the optimal design by maximizing and expected utility function that quantifies the goals of the experiment. The design of plasma diagnostic is typical task to be resolved along the preparation of fusion experiments. The design has to meet with requirements like highest accuracy of measurements, high resolution, robustness and extensibility, as well with constraints such as accessibility or economic restrictions.

Our problem is to maximize the fuzzy entropy functional

$$H\left[f^{\vartriangle}\right]=-\int_0^{\infty} f^{\vartriangle}(x)\ln f^{\vartriangle}(x)dx \quad (10)$$

Subject to some $\mu_j=\left\{E(X^{\alpha_j}\right\}_{j=0}^{M}$ where the based MaxEnt PDF is given as follows

$$f_M^{\vartriangle}(x)=\exp(-\sum_{j=0}^{M}\lambda_j x^{\alpha_j}) \quad (11)$$

where $\lambda_0,....\lambda_M$ are the Lagrange multipliers corresponding to the following M constraints $\mu_{\alpha_j}=E(X^{\alpha_j})=\int_0^{\infty}x^{\alpha_j}f_M^{\vartriangle}(x)dx$, j=0,1,2,...M where a_0 = 0. Then the entropy is represented by

$$H\left[f_M^{\vartriangle}\right]=-\int_0^{\infty}f_M^{\vartriangle}(x)\ln f_M^{\vartriangle}(x)dx-\sum_{j=0}^{M}\lambda_j\mu_{\alpha_j} \quad (12)$$

We can propose a new adaptive nonlinear activation function for deterministic case, and Bayesian method of learning and classification for stochastic case.

Let $(X,Y)=\{X_n,Y_n\}_{n\in N}$ be a joint process in which X is unobserved and Y is observed. We assume that X and Y are both discrete with $X_n \in \{1, 2,..., \}$ and $Y_n \in \{1, 2,...,\infty\}$ for all $n \in N$. In some applications it is relevant to compute the fuzzy entropy. Given an observation $\{y_n\}_{n=0}^{N}$ if we want to compute

$$H(x_{0,N};y_{0,N})=-\sum_{i=0}^{N}\sum_{j=0}^{\infty}p_{ij}^{\vartriangle}$$

$$\ln p_{ij}^{\vartriangle}=-\sum_{x_{0,N}}p^{\vartriangle}(x_{0,n};y_{0,n})\ln p^{\vartriangle}(x_{0,n};y_{0,n}) \quad (13)$$

It means that the problem of partial observability, can be transformed into the problem of fuzzy entropy. Fuzzy entropy is described by words, i.e. with some intervals of numbers. We work with power law behaviour. When process becomes completely observable fuzzy entropy is transformed into regulary Shannon entropy (Boltzmann-Gibbs entropy). In such case we talk about exponentially sstabilizable process. Fuzzy entropy is true approach for the problem of plasma confinement, because we can have interplay between relativistic and nonrelativistic effects.

For the fuzzy continuous case of MaxEnt, we have got maximize

$$H(p^{\vartriangle})=-\int p^{\vartriangle}(x)\ln p^{\vartriangle}(x)dx \quad (14)$$

with constraints $\int p^{\vartriangle}(x)\phi_k(x)dx=d_k$ k=1,2,..., K (15)

Again writing the expression of the Lagrangian

$$L=-\int p^{\vartriangle}(x)\ln p^{\vartriangle}(x)dx+\sum_{k=0}^{K}\lambda_k(\int p^{\vartriangle}(x)\phi_k(x)dx-d_k) \quad (16)$$

and finding its stationary point, we obtain

$$p^{\vartriangle}(x)=\frac{1}{Z(\lambda^*)}\exp\left[-\sum_{k=1}^{K}\lambda_k^*\phi_k(x)\right] \quad (17)$$

where $Z(\lambda)=\exp(\lambda_0)=\int\exp\left[-\sum_{k=1}^{K}\lambda_k\phi_k(\varpi_j)\right]dx$ (18)

This solution has the following properties $H=\lambda_0+\sum_k\lambda_k E\ \ \phi_k(x)$ and $H_{\max}=\lambda_0+\sum_k\lambda_k d_k$. Instead of the problems with K expected values, we can have the problems with N direct, indirect or noisy samples. After taking the discrete step of times and pseudopartition of unity or with the Fourier series for deterministic case, fuzzy continuous case will be transferred in discrete fuzzy vector space and we will be able make classification of data with previous methods.

CONCLUSIONS

So, we can conclude that for investigations of different plasma regimes at tokamaks and stellarators, we should use interplay between fractional entropy[10] for anomalous heat propagation, Boltzmann-Gibbs entropy[11] for visualization and identification of plasma confinement regimes, imprecise Shannon entropy[12] for exponential behaviour of plasma, Tsallis entropy[13] for the problems of detection. It all can be interpreted via unique fuzzy Jaynes entropy with several constraints[14-17].

The power laws approximate the power spectrum over limited frequency ranges, whereas the exponential is very close to the observed power spectrum over the entire frequency range. This distinction is of fundamental significance because exponential power spectra in the time signals of nonlinear dynamic models indicate the presence of chaotic behavior, which differs from Self-Organized Criticality (SOC), in that the former is deterministic process while the latter is stochastic. Uncertainities can lead toward precise (robots) or approximate (turbulence) reasoning.

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