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Parametric cost estimation model for aircraft based on fractal theory

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ABSTRACT

Fractal theory possesses a congenital application foundation in Parametric Cost Estimation(PCE). Based on the fractal theory, a core equation of fractal parameters has been established, and on this basis, a grey fractal parametric cost estimation model is established by taking the comprehensive efficiency variables as the fractal variables, which is applicable to the early cost evaluation of aircraft manufacture. Example analysis shows: the aircraft cost estimation by the fractal parametric cost estimation model achieves desirable accuracy, and is superior to the method of conventional multiple linear regression in generalization ability and stability. Meanwhile, it can be concluded that efficiency is effective as the fractal unit and the cost has the fractal characteristics as the information set, which provide new ideas and methods for cost estimation.

KEYWORDS

Fractal theory; Parametric cost estimation (PCE); Grey fractal; Fractal unit;efficiency.



INTRODUCTION

Fractal theory is a new nonlinear science theory studying on self similarity of a system. This theory has drawn many attentions in various fields since it came into being and has become an important supplement to the research methods and results of each subject. It provides new perspectives and ideas for us, namely nonlinear connection between the whole and the part -- information isomorphism.

Fractal theory has risen as a methodology and epistemology for its extensive application in engineering, technological, biological, social and economic fields. Besides, the meaning of fractal has become a generalized fractal from the original narrow geometry fractal. Reference^[1] mentions the fractal model based on the concept of the Fractal Company, extending the fractal application to organizational structure design, information integration, resource distribution and other aspects so as to gain higher efficiency and benefit. References^[2-3] studies inventory management method based on fractal theory defining function point of each segment in the supply chain as a self similar structure, which is regarded as fractal. Reference^[4] puts forward the fractal structure of flexible manufacturing system. Reference^[5] studies fractal manufacturing system, which also uses the main characteristics of the fractal for modeling. Network data communication can be well described by fractal^[6]. As can be seen, with the in-depth and development of scientific research, the extension of fractal will be broader. But overall, the current development of fractal study focuses more on the system structure and function, while the researches on system information fractal are fewer.

Parametric cost estimation method (PCE) is a relatively mature early cost estimation method. Its core idea is to extract high correlation factors as cost driving factors from many cost related factors, that is, screening factors including the key information of cost as parameters and establishing mathematical relationship so as to estimate the whole cost. The central idea is still studying the relationship between the cost as a whole and its components, seeking the bridge transiting from the partial to the whole. This is in common with the fractal theory, both for understanding the overall through the part containing key information relating to the whole, which provides the possibility for the introduction of fractal theory.

In recent years, scholars abroad have just followed closely the application of fractal theory in parametric cost estimation method and combined the fractal theory with the system engineering cost model (COSYSMO)^[7]. However, the research content is limited to demonstration and deduction of the fractal application feasibility, and only the conclusion that the actual cost of the system can be estimated by using fractal concept model is obtained, while the research does not involve the specific application and the implementation approaches. Therefore, this paper is to establish the fractal parametric cost estimation model of the aircraft and conduct a preliminary exploration on the application.

FRactal Parametric Cost Estimation Model

Fractal refers to the similarity between the part and the whole where part is the relative epitome of the whole and contains all information of the whole. This kind of form of self similarity is called fractal. The self similarity of the part and the whole in form, function, information, time and space and any other aspects of statistical meaning can be included in the category of fractal^[8,9]. In the above, the relatively independent part containing key information of the whole is called the generating element or fractal unit. A fractal unit can be geometric entities, as well as the mathematical model supported by function or information^[8].

Then we can put forward a hypothesis. Whether the cost as a complex system, has some variables, containing all the information of the cost to a certain extent, can be the reproduction and epitome of the cost overall information. In fact, this aims to looking for the parameter that can indicate the essence of the cost in the parameter cost estimating model. From this point of view, the fractal theory has the congenital basis in parametric cost estimation. The traditional parametric cost estimation method is to extract a new concentrated variable or some component variables with the best explanation to the cost system from the cost element variables, and assuming that there are some relationships between the cost and them; and the fractal theory is starting directly from the complex system itself, that is, from the researched object with no simplification or decomposition itself to find the nonlinear relationship between the whole and the part. For example, in the partial least squares regression method (PLS) commonly used in parametric cost estimation, the principal components extracted from the analysis of principle component variables of the key steps correspond to the fractal element in a sense. In spite that the relationship between the fractal element and the dependent variable in fractal parameters estimation is the nonlinear self similar fractal relationship and the relationship between the dependent variable and the principal components in PLS are linear, the core idea and application mechanism of the two are similar, both are the extraction process of key information of the whole.

Therefore, we introduce the fractal theory, and set up a basic assumption: the cost information is considered as a complex nonlinear system affected by many factors, where there must be some factors include full information of the cost, namely the fractal unit. This factor can be an objective core component of the cost, and also be a kind of integrated variables, such as manufacturing complexity etc. In this circumstance of limited information and uncertainty, according to tacit recognition principle of the grey theory, if there is no reason to believe that this assumption is unsupported, it is defaulted to be established. So, we deem the relationship between whole and part as a nonlinear, use self similar fractal character as assumptions, and establish fractal parametric cost estimating model.

In the fractal theory, the relationship between the whole and the part can be described by the fractal dimension quantitatively. It represents the complexity between the whole and the part, and can serve as the main basis for judging the

fractal feature. Usually, people call objects with fraction as the Hausdorff dimension to be fractal, and the Hausdorff dimension is called fractal dimension of the fractal^[9]. The definition of fractal dimension is based on the thought of "measurement using standard r ", that is, for the fractal A , if the same dimension r is used for measurement, the determined measurements value $N(r)$ can be obtained; or, if the set A have fractal features, there must exist certain constant D , making measurement value and measurement standard r meet the following^[10]:

$$N(r) = cr^{-D} \quad (1)$$

Among them, D is the fractal dimension. The fractal dimension is generally fraction, and under few cases is an integer.

Take a simple example, suppose there is a three-dimensional sphere. We use the sphere with radius r to measure its volume V . Then the following measurement results can be obtained:

$$N(r) = \frac{V}{\frac{4}{3}\pi r^3} = \left(\frac{3V}{4\pi}\right)r^{-3} = cr^{-D} \quad (D=3, c = \frac{3V}{4\pi})$$

As can be seen, D as object dimension, c is structure factor that does not change with r . Usually, formula (1) is used to calculate the fractal dimension. Now assume that A is cost, based on the fractal unit containing full information of the cost for measurement. It can be seen from the derivation process that A as the known objective constant, is included in the constant C , and forms a constant with other structure constants together. Therefore, C can be decomposed into two parts c' and A , therefore:

$$N(r) = cr^{-D} = A \cdot c'r^{-D} \quad (2)$$

According to equation (2), we can draw the relationship of cost A :

$$A = [N(r) \cdot c'^{-1}] \cdot r^D = Cr^D \quad (3)$$

The equation is simplified as:

$$\ln A = c + D \ln r \quad (4)$$

In the formula, c is a constant.

This equation is the core equation of fractal parametric cost estimating. According to the fractal unit selection, then can use a two order equation to adjust the core equation. For the abstract information fractal, if the original hypothesis is supported, and cost information has fractal feature, there must exist a fractal unit containing the full information to make the formula (3) set up. In other words, when the relationship between the extracted fractal unit and the cost can be composed as a straight line, as formula (4) shows, the power-law relationship is significantly. The selection of fractal unit is reasonable. This cost estimation problem is transformed into the process of the key information extraction, and search cost fractal unit to make the power-law relationship be established. When the fractal unit containing the key cost information is "discovered", the establishment of a power law relation, a power law relation is established and silent acknowledge transforms into acknowledge.

GREY FRACTAL PARAMETRIC COST ESTIMATING MODEL OF AIRCRAFT

Fractal is divided into regular fractal and random fractal. The absolute regular fractal is the minority, and the vast majority of fractal is random fractal in the statistical sense. That is to say, the self similarity between the whole and the part does not strictly exist, while a certain degree of approximation or uncertainty exists between them. The self similarity of this uncertainty or approximation is widespread in reality. Researchers define the fractal of self similarity of uncertainty or approximation as grey fractal^[11]. Aircrafts are of complicate structure, and many factors affect the cost, and also the current situation is that the Chinese aircraft historical cost lacks data. Obviously researches on information structure of aircraft cost should be included in the research of grey fractal category.

According to the original form of grey fractal:

$$N(r) \cong cr^{-D(\otimes)} \quad (5)$$

In the formula, \cong indicates the approximate relationship; $D(\otimes)$ is grey fractal dimension. The core equation for fractal parameter cost estimating should accordingly be defined as:

$$A \cong cr^{D(\otimes)} \tag{6}$$

Among them, $D(\otimes)$ is as the grey fractal dimension of cost A. The equation can be simplified as:

$$\ln A = c + D(\otimes) \ln r \tag{7}$$

Therefore, the estimation of cost is mainly focused on determining the grey fractal dimension $D(\otimes)$ and constant C . The simplest way for determination is the linear fitting. The central idea of line fitting is graying the relevant parameters, then conduct linear regression^[11]. Therefore, the relevant parameters of the grey fractal model (7) are grayed and transformed into the following form:

$$\ln A(\otimes) = c + D(\otimes) \ln r(\otimes) \tag{8}$$

Further, several groups of $r(\otimes)$ and $A(\otimes)$ with different measures can be fitted with the least squares fitting. Fitting results as follows:

$$D(\otimes) = \frac{\sum (\ln r(\otimes) - \overline{\ln r(\otimes)})(\ln A(\otimes) - \overline{\ln A(\otimes)})}{\sum (\ln r(\otimes) - \overline{\ln r(\otimes)})^2} \quad c = \overline{\ln A(\otimes)} - D(\otimes) \overline{\ln r(\otimes)}$$

Finally, a whitening value of $D(\otimes)$ is obtained by equal weight and mean whitening. Then the cost estimation of relationship between the fractal unit A and cost can be obtained.

When establishing of core equation for estimating fractal parameters of cost, the main problem is focused on the selection and extraction of fractal unit. In order to look for the cost information in the huge variable system, we can readily consider some comprehensive variables as fractal units. Compared with the objective of physical or technical variables, integrated variables are more likely to be relatively complete in representing the cost information in a complex system. Especially when aircraft structure is becoming more and more complex nowadays, the data collections become huge and difficult projects, involving decomposition, analysis, classification, and elimination of technical parameters in combination with all kinds of complicated parameters of the plane system.

Moreover, with the technology development, the key factors affecting the cost constantly changed, but the goal and the efficiency level as required can be easily identified and evaluated. Especially in the early stage of a new system, technology parameters are difficult to be specifically established and extracted. The intuitive logical synthesis variables provide benefits in rapidly and efficiently evaluating the cost. Therefore, this paper uses the comprehensive variable of efficiency as the fractal unit. In theory, the various factors affecting the efficiency affect the cost, and any efficiency of the plane is at the price of cost consumption. From this point of view, the efficiency and the cost have a similarity in information, which can be seen as a epitome of cost information.

Reference^[12] points out it is more objective and direct to use the efficiency parameter to replace each index of parameter cost evaluation model, and takes the aircraft as an example and selects nine efficiency parameter to replace the conventional physical and technical parameters, which is finally converted into three independent comprehensive efficiency factors, and uses multiple linear regression method for cost estimation and achieves favorable effects. This paper will refer to the efficiency variable data above, and analyze whether the satisfactory effect can be achieved when using fractal parameter cost evaluation model by taking the efficiency as the parameter compared.

Thus, this paper also chooses nine efficiency parameters: equivalent range, the equivalent bomb load, mobility, firepower, target detection capability, control effectiveness, survivability, voyage and ECM. The calculation of index value can refer to references^[13-15]. And also, the three efficiency comprehensive factors are defined on the basis of the nine performance parameters, respectively being:

Design level (REACH): representing the comprehensive performance level for the aircraft to achieve, that is, the level of the existing technology.

$$REACH = (ORIGIN \rightarrow NEW)$$

where ORIGIN is the origin, and NEW is a new system in research.

Advancedlevel (ADVANCE): representing the progressiveness of the new system in research (NEW) compared with the existing technical prototype OLD.

$$ADVANCE=(ORIGIN \rightarrow NEW)-(ORIGIN \rightarrow OLD) =REACH(NEW)- REACH(OLD)$$

Redesign (REDESIGN): representing different technical distinctions as applied for achieving the same design requirement.

The three respectively represent three main factors determining the cost trend from the effectiveness perspective: the overall complexity of the new weapon system, the inventive and creative part of the technique and the reduced cost part brought by redesign. As can be seen, these three factors are not related to or are dependent on each other. The three independent comprehensive variables represent the information of original variables and further as grey fractal units. Then fractal unit R can be expressed as:

$$r(\otimes) =a*REACH+b*ADVANCE-c*REDESIGN \tag{9}$$

According to the principle of least information of gray theory, we take the whitening value with equal weight of the three variables as the fractal unit, i.e.:

$$r= REACH+ ADVANCE - REDESIGN \tag{10}$$

Accordingly, establish the gray fractal parametric cost estimation model using the efficiency as a fractal unit. The simultaneous equations as follows:

$$\begin{cases} A \cong cr^{D(\otimes)} \\ r= REACH+ ADVANCE - REDESIGN \end{cases}$$

Finally, the above can be simplified into type (11):

$$\ln A = C + D(\otimes) \ln(REACH+ ADVANCE - REDESIGN) \tag{11}$$

The power-law relationship is converted into the linear regression equation for straight line fitting, and for significance analysis. Use the grey fractal dimension for cost analysis and estimation.

EMPIRICAL STUDY

According to the the actual task requirements of combat aircraft, use the nine performance parameters equivalent range (A₁), the equivalent bomb load (A₂), mobility(A₃), firepower(A₄), target detection capability(A₅), control effectiveness(A₆), survivability(A₇), voyage(A₈) and ECM(A₉) to replace the technical parameters for calculation, and evaluate the nine efficiency parameters of the aircraft to be measured by combining with the expert scoring method. Each aircraft the system can uniquely determine a point in nine dimensional spaces based on these nine values. Finally, the specific evaluation values are converted to three comprehensive effectiveness factors. The calculation formula is as follows:

$$REACH(i) = [A_1^2(i) + A_2^2(i) + \dots + A_9^2(i)]^{1/2} \tag{12}$$

$$ADVANCE(i) = REACH(i) - REACH(k) \tag{13}$$

$$REDESIGN(i) = \left\{ \left[\frac{REACH(k)}{REACH(i)} A_1(i) - A_1(k) \right]^2 + \left[\frac{REACH(k)}{REACH(i)} A_2(i) - A_2(k) \right]^2 + \dots + \left[\frac{REACH(k)}{REACH(i)} A_9(i) - A_9(k) \right]^2 \right\}^{1/2} \tag{14}$$

In the formula, k is the prototype of i .

Select the reliable data of 12 types of aircrafts as the samples to estimate the cost model. Since the parameter calculation, expert evaluation and statistical process do not belong to the research range of this paper, to explain the problems here, the comprehensive efficiency factors calculated from formulas (12) (13)and(14) can directly refer to the follows^[12]:

TABLE 1 : The comprehensive efficacy factor of 12 kinds of aircrafts.

Serial number	REACH	ADVANCE	REDESIGN	Cost/million US dollar
1	22.38	1.21	0.425	54.0
2	24.21	4.32	2.310	72.4
3	18.03	0.91	1.640	40.0
4	25.38	1.17	0.923	58.5
5	25.75	1.54	1.500	61.5
6	28.68	5.32	0.681	92.6
7	25.44	3.04	1.380	69.4
8	23.36	2.35	1.890	59.3
9	20.47	3.58	0.520	65.2
10	17.76	3.61	2.030	57.3
11	29.90	2.45	1.890	75.8
12	21.30	1.88	0.623	58.2

Assuming that A is a cost, it can be obtained according to sample data of group 12 and by using multiple linear regression models that:

$$A = 1.90 + 2.07 \times REACH + 6.28 \times ADVANCE - 2.52 \times REDESIGN \tag{15}$$

Given that $\alpha=0.05$, F test and t test are both past. The regression relationship is remarkable.

When grey fractal cost is used to form estimation mode, the data require logarithmic transformation processing. The data in TABLE 1 are substituted into formulas (10) and (11) for the least squares fitting. The fitting results are:

$$\ln A = 0.75 + 1.06 \times \ln r \tag{16}$$

Given that $\alpha=0.05$, the significance test is passed. The test results are as follows: P-value=9.17E-05

As can be seen, P value is smaller than the critical value and the linear relationship is considered to be remarkable, proving that the efficiency as the fractal unit has a power-law relationship with cost information.

Carry out the model analysis according to the sample test results. Use these three indicators average absolute percent error, mean square error andStatistic Accuracy Percent for evaluating the accuracy of the estimation model, the generalization ability and stability:

The average absolute percentage error (MAPE)

$$MAPE = \frac{1}{n} \sum_{i=1}^n \left| \frac{\hat{y}_i - y_i}{y_i} \times 100 \right| \tag{17}$$

In the formula, \hat{y}_i and y_i respectively represent the estimated cost and actual cost. Generally, the smaller MAPE indicates higher accuracy of the model.

Mean Square Error (MSE)

$$MSE = \frac{1}{n} \sum_{i=1}^n (\hat{y}_i - y_i)^2 \tag{18}$$

Smaller mean square errors indicate better generalization abilities of the models.

Statistic Accuracy Percent(SAP)

The accuracy rate is the percentage of the samples with estimation cost and actual cost between error in [0.1, + of 0.1] accounting for the total samples. Higher accuracies indicate better stabilities of the models.

The analysis results refers to TABLE 2.

TABLE 2 : The comparison between fractal cost estimation model and the linear regression model.

Models	MAPE(%)	MSE	SAP(%)
Grey fractal Parametric Cost Estimation model	1.65	3.91E-04	0.83
Multiple linear Regression model	1.42	1.12	0.17

As can be seen, the estimation accuracy of fractal parametric cost estimating models has little difference from that of multiple linear regression method. However, the generalization ability and stability are obviously better. Multiple linear regression model estimation is of high volatility and unstable results. In addition, we can draw the following three conclusions by analysis of fractal cost estimation model relationship formula.

(1) The efficacy and the cost has a significant power-law relationship; and also according to the linear fitting results, 95% confidence interval of the grey fractal dimension $D(\otimes)$ in the fractal cost estimating model is (0.6985, 1.415531), and the weighted average values albino value $D = 1.06 > 1$ and the fractal dimension are fractions, which is consistent with the basic assumptions of the paper, and is a reflection of cost information fractal set hypothesis, and is also a basis of the silent acknowledge being converted into acknowledge, indicating costs as a set of multi information has a fractal feature.

(2) Although the fractal dimension is a fraction and is greater than 1, the dimension is very low of being closed to 1, indicating the efficacy as the fractal element cost is comparatively effective. According to the fractal theory, the self similarities of fractal have distinctions (that is, the times of using the generating element or the magnification) respect to the hierarchy or level, where the generator of the lowest level is the zero level generator and the generator of the highest level is the whole of the fractal. The elements of closer level have larger similarity, and the elements of farther levels have smaller similarity, and even have no similarity.^[8] As can be seen, the cost as a whole having information structure fractal features and containing the fractal element with the full information is certain to be the fractal element of higher level, indicating the effectiveness as the fractal element has many similarities with the whole.

(3) Extract efficacy comprehensive variables as the fractal elements of cost information represent the common ingredients of the original variable system, which effectively achieves dimension reduction. The final form of model is a unitary linear regression, and the amount of samples as required is greatly reduced.

CONCLUSION

In this paper, the fractal theory is introduced into the parametric cost estimation, thus establishing a new fractal parametric cost estimation model and achieving a better estimation result. In addition, a good model should be a model that can be extended and enlarged. This paper mainly provides a core equation for estimating fractal parameter cost, which can further be adjusted and improved according to the practical conditions in the future research and be comprised of increased equation sets and simultaneous equations. Later, better variables may serve as the fractal elements in replacement of the efficacy. This paper aims to provide a new thinking and a theoretical basis for parametric cost estimation and to explore a new method of cost estimation from a new perspective.

CONFLICT OF INTEREST

This article content has no conflict of interest.

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