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On the unification of macroscopic masses and charges and microscopic masses and charges

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Abstract

As I have already discussed In[1], only some simple theoretical ideas are needed wanting to unify the notions "mass", "charge", and "Metric". As I have already discussed In[1], this includes Einstein's field equations of gravitation and the equation of geodetic lines just like basic limiting cases such as Poisson's equations for masses and charges and Newton's equation of Motion for masses and charges. As I have already discussed In[1], the forces and fields of masses and charges just like the dynamics and statics of masses and charges then can be treated comprehensively under a generalized point of view defined by these simple ideas, in particular, defined by the ideas "charge-corrected mass", "mass charge–exchange relation", "Vacuum tension" or "generation tension", and "mass–charge metric". Certainly, these ideas are simple in comparison to other attempts to develop a unified theory of masses and charges based upon the notion "metric". Certainly, the physical interpretations established to gain a graphic access to these ideas require much more inspiration and transpiration in comparison to other attempts to develop a unified theory of masses and charges based upon the notion "metric". Certainly, some notional barriers must be overcome wanting to gain access to the congruity of these ideas. However, realizing what the observables are, these ideas are consistent with basic laws such as the energy–mass formula and the Lorentz transformation, also fulfilling the requirements of quantum systems ("wave–particle systems"), in comparison to the macroscopic masses and charges that have been assumed in[1], characterized by microscopic masses and charges. Let me work out this in more detail in this publication departing from the notions introduced in[1], in this context, a further time speaking of generalized Einstein field theory (GEFT).

Keywords

Generalized Einstein field theory (GEFT); Generalized Einstein field equations (GEFE); Wave–particle theory; Wave–particle system; Wave–particle energy momentum tensor; Wave–particle metric; Differential spin operators; Differential self-interaction operator.

INTRODUCTION

It is a common belief that Einstein's metric fields and Schrödinger's wave functions ("Hilbert state vectors") have nothing to do with one another. Indeed, Einstein's metric fields are exclusively dealing with masses, while Schrödinger's wave functions are mainly dealing with charges. Indeed, Einstein's metric fields can be expressed by real functions, i.e. complex functions are a formal trick utilizing the benefits of complex functions, while Schrödinger's wave functions must be expressed by complex functions, only in some special cases reducing to real functions, i.e. complex functions in no way are a formal trick utilizing the benefits of complex functions, but a physical necessity, and this also is true considering the alternatives to Schrödinger's formalism such as Heisenberg's formalism or Feynman's formalism, and this is true even more so as further attempts to create a completely real formalism failed. However, isn't it an astonishing issue that the physicists of nowadays have to reduce the description of quantum behavior to mathematical terms completed by classical physical terms such as eigenvalue equation and eigenvalue? Moreover, isn't it an astonishing issue that the physicists of nowadays certainly are able to unravel the physical meaning of the mathematical details considering macroscopic systems, but are not able to unravel the physical meaning of the mathematical details considering microscopic systems, in the latter case, unavoidably resorting to mathematical terms such as Hilbert state vector, eigenvalue equation, and eigenvalue? What if these circumstances are the formal expression of an physical background that still has to be discovered– a physical background that is not completely deducible resorting to traditional notions as given by classical notions of energy and momentum, mass and charge, wave and particle? Inspired by this idea, by way of trial, let us continue the lines of[1] and let us think unusual, in this manner, developing a network of ideas revealing the barriers quoted above in a different light, eventually suggesting a physical scheme supplying us with a unified image of macroscopic masses and charges and microscopic masses and charges under the patronage of a generalized type of Einstein field equations and a generalized type of equation of geodetic lines, in[1] termed "generalized Einstein field equations" and "generalized equation of geodetic lines", noted by the way, also including strong interaction aspects and weak interaction aspects, and of course, not contradicting Heisenberg's uncertainty principle interpreting the generalized equation of geodetic lines suitably.

Are potential equations occurring as consequence of Einstein's field equations of gravitation and showing a form known from potential equations covering charges only formal similarities? Are force terms occurring as consequence of the equation of geodetic lines and showing a form known from force terms covering charges only formal similarities? Are energy terms occurring as consequence of Einstein's field equations of gravitation and showing a form known from energy terms covering quantum systems ("wave–particle systems") only formal similarities? Certainly, following the lines of interpretation of physical circumstances of nowadays, this is true. However, slightly deviating from the lines of interpretation of physical circumstances of nowadays, this is not true anymore, nevertheless meeting calculations, computations, and experiments. Naturally, as already quoted above, we have to overcome traditional notions as given by classical notions of energy and momentum, mass and charge, wave and particle, thus developing an advanced interpretation of physical issues.

Dealing with macroscopic masses and charges, following[1], we have to establish the notion of a pure mass $\mathfrak{m}_0$ which is reduced to the observable mass $m_0 = \mathfrak{m}_0 + \lambda_C q$ with $\lambda_C = -2/Kc^2\varepsilon_0 U$ in the presence of a charge q, where U is a tension (voltage) showing the same sign as the charge q, and we have to establish the notion of a charge-related energy qU meeting the mass-related energy m_0c^2 as $m_0c^2 = qU$, in this manner, arriving at an extended notion of masses and charges, related mass/charge energies and related mass/charge potentials. Dealing with microscopic masses and charges, however, we have to supplement these extended notion of masses and charges, related mass/charge energies, and related mass/charge potentials as follows now.

THE WAVE–PARTICLE FEATURE OF QUANTUM SYSTEMS

Why are microscopic systems denoted as quantum systems?

Because microscopic systems exhibit wave properties and particle properties in a highly concatenated manner, eventually leading to quantization properties enforced by the intrinsic wave properties!

So to speak, quantum systems are "wave–particle systems".

Taking this vocable seriously, we postulate the existence of energies and momenta that structurally reflect the inseparability of wave properties and particle properties, then figuratively speaking of "wave–particle energies" and "wave–particle momenta".

But how could such wave–particle energies and wave–particle momenta look like?

Certainly, a wealth of structures in principle is possible. However, let us not think complicated here. Let us think simple here.

Let us think of product structures consisting of particle-related terms and wave-related terms, bound together by wave quantities directly replacing particle quantities occurring within the particle-related terms, eventually realizing that the product structures provided by quantum mechanics such as $E\psi$ showing particle-related terms such as $E = p^2/2m_0$ and wave-related terms such as ψ, bound together by wave quantities such as $p = \hbar k$ directly replacing particle quantities such as $p = m_0 v$, are such wave–particle energies, with E being the energy, p the momentum, k the wave vector, and ψ the wave function. Consequently, let us interpret further non-operative product structures such as $V\psi$ and further operative product structures such as $\hat{\mathbf{p}}^2\psi/2m_0$ as wave–particle energies, summing up these, defining related wave–particle energy balances, with V being a potential and $\hat{\mathbf{p}}$ the momentum operator. The extension to wave–particle momenta is straightforward.

This is a basic point, when we want to realize the actual problems of nowadays! Quantum systems are wave–particle systems, eventually calling for wave–particle energies and wave–particle momenta, in each case, characterized by a combination of a wave part

and a particle part, in total, setting up wave–particle energy momentum balances!

Certainly, from a mathematical point of view, these are terms of eigenvalue equations.

However, from a physical point of view, these are terms of related wave–particle energy momentum balances.

Along the same lines, we may establish "wave–particle energy momentum tensors", in each case, characterized by matrix elements defined by a combination of a wave part and a particle part.

This is also a basic point, when we want to realize the actual problems of nowadays! Quantum systems are wave–particle systems, eventually calling for wave–particle energy momentum tensors, in each case, characterized by matrix elements defined by a combination of a wave part and a particle part, in each case, specifying the special type of quantum system (wave–particle system)!

Mind you, all this simply reflects the entity of quantum systems (wave–particle systems)!

Going over from the traditonal notions to these advanced notions, we then may proceed as follows.

THE GENERALIZED EINSTEIN FIELD EQUATIONS (GEFE) AND THE WAVE–PARTICLE FEATURE OF QUANTUM SYSTEMS

In what follows, we observe Examples 3.1–3.7, collecting the basic formulae in LATEX style which I am preferring despite some differences face to face with the WORD style predetermined by the text template.

As I have already quoted in[1], not only allowing mass-related energy momentum tensors, but also allowing charge-related energy momentum tensors, I would like to rename the Einstein field equations of gravitation as generalized Einstein field equations (GEFE). As I have already quoted in[1], this terminology also takes into account further extensions needed dealing with quantum systems (wave–particle systems). In particular, consistent with general foundations of the Einstein field equations of gravitation, certainly requiring homogeneity and isotropy of space and time and transformational invariance of the form, but not exclusively requiring mass scenarios, we allow any constant $\mathcal{K}$ generalizing the gravitational constant K, and we allow any energy momentum tensor $\mathcal{T}_{\mu\nu}$ generalizing the mass–charge-related energy momentum tensors $T_{\mu\nu}$, and this includes "exotic tensors" such as wave–particle energy momentum tensors containing the wave function ψ as inherent part, eventually reflecting the entity of quantum systems (wave–particle systems). Therefore, we may express the generalized Einstein field equations (GEFE) in the form (A) or the form (B), where $R_{\mu\nu}$ is Riemann's tensor of curvature and R is Riemann's scalar of curvature, in the latter case, occurring in combination with the metric tensor $g_{\mu\nu}$.

$$-R_{\mu\nu} + R\,g_{\mu\nu}/2 = \mathcal{K}\mathcal{T}_{\mu\nu}, \qquad \text{(A)}$$

$$G_{\mu\nu} = \mathcal{K}\mathcal{T}_{\mu\nu},\ G_{\mu\nu} = -R_{\mu\nu} + R\,g_{\mu\nu}/2. \qquad \text{(B)}$$

(A) forms the starting point of[1], due to the mass–charge orientation, setting $\mathcal{T}_{\mu\nu} = T_{\mu\nu}$.

Looking for further arguments encouraging us to consider this as physics, let us proceed as follows.

First of all, we note that the transition to mechanical surroundings and quantum-mechanical surroundings is implemented by the Ansatz $g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}$, in a logically consistent manner, recasting the general form of the GEFE into a specialized form of the GEFE showing structural characteristics needed wanting to deal with mechanical surroundings and quantum-mechanical surroundings. Working out the deductive path from the GEFE to the Poissonian equations for masses and charges, this is elucidated in[1] for mechanical surroundings by example. Working out the deductive path from the GEFE to linear operators applied in quantum mechanics and from the GEFE to nonlinear operators that can be reduced to linear operators applied in quantum mechanics observing specific constraints, this is elucidated in[2-4] for quantum-mechanical surroundings by example. Firstly, it there is explicitly shown that we are led from (A) or (B) to (C) applying the Ansatz $g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}$, where the $\mathcal{W}_{\mu\nu,i}$ are placeholders for energy momentum terms based upon differential position/time operators, the $\eta_{\mu\nu}$, and the $\gamma_{\mu\nu}$. Secondly, it there is explicitly shown that the energy contribution to (C) is provided by (D) taking the implicit structure into consideration and by (E) taking the explicit structure into consideration applying the Ansatz $g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}$ where Δ is the Laplacian operator.

$$\mathcal{W}_{\mu\nu} = \mathcal{K}\mathcal{T}_{\mu\nu} - R\eta_{\mu\nu}/2 - R\gamma_{\mu\nu}/2,_{\mu\nu} = \textstyle\sum_i \mathcal{W}_{\mu\nu,i}\,(\eta_{\mu\nu},\gamma_{\mu\nu}), \qquad \text{(C)}$$

$$\mathcal{W}_{00} = \mathcal{K}\mathcal{T}_{00} - R\eta_{00}/2 - R\gamma_{00}/2,_{00} = \textstyle\sum_i \mathcal{W}_{00,i}\,(\eta_{\mu\nu},\gamma_{\mu\nu}), \qquad \text{(D)}$$

$$-\Delta\gamma_{00}/2 + \ldots = \mathcal{K}\mathcal{T}_{00} - R\eta_{00}/2 - R\gamma_{00}/2. \qquad \text{(E)}$$

(E) completes the starting point of[1], due to the mass–charge orientation, setting $\mathcal{T}_{00} = T_{00}$.

As discussed in[1] and as illustrated by Example 3.1, specifying the energy matrix element $\mathcal{T}_{00} = T_{00}$ of (E) resorting to a mass density ρ_g, finally aiming at a solid-body-related energy momentum tensor, we directly arrive at a field equation where the field-related properties and the source-related properties occur separably, following[1], leading to the Poissonian equations for masses and charges additionally taking into account the connection of the generalized potential γ_{00} to the mass potential ϕ_g and the charge potential ϕ_C. As discussed in[2-4] and as illustrated by Example 3.2, however, specifying the energy matrix element $\mathcal{T}_{00}$ of (E) resorting to the product structure $\chi_{00}\gamma_{00}$, finally aiming at a wave–particle-related energy momentum tensor, we directly arrive at a field equation where the field-related properties and the source-related properties occur inseparably, following[2-4], leading to an equation showing the structure of the time-independent Schrödinger equation completed by linear and nonlinear, time-independent and time-dependent terms provided we additionally apply $\chi_{00} = Em_0/\hbar^2$ and $\gamma_{00} = \psi$. Wanting to deal with quantum systems (wave–particle systems), we thus consider the $\mathbb{W}$ form ("Wechselwirkungsform") of the GEFE defined by the Ansatz $g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}$. Certainly, without this Ansatz, relativistic specifications of the GEFE meeting quantum systems (wave–particle systems) could be entertained. However, relativistic specifications of the GEFE meeting quantum systems (wave–particle systems) go far beyond the scope of this publication. In time-dependent cases, the energy E may become time-dependent. In time-dependent cases, completions of $\chi_{00} = Em_0/\hbar^2$ may be necessary. Then E is no eigenvalue.

$\chi_{00} = Em_0/\hbar^2$ and $\gamma_{00} = \psi$ are also quoted in Example 3.2.

$\chi_{00} = Em_0/\hbar^2$ is suggested by the physical dimensions, consistent with the product structure $\chi_{00}\gamma_{00}$ reflecting awave–particle-related energy momentum tensor, finally suggesting to set $\gamma_{00} = \psi$.

Following[1], (3.2) is telling us that γ_{00} is the macroscopic reflection of the gravitational potential ϕ_g, due to our advanced notions, related to the charge-corrected mass $m_0 = \mathfrak{m}_0 + \lambda_C q$ with $\lambda_C = -2/Kc^2\varepsilon_0 U$.
Following[2], (3.4) is telling us that γ_{00} is the microscopic reflection of the gravitational potential ϕ_g, due to our advanced notions, related to the charge-corrected mass $m_0 = \mathfrak{m}_0 + \lambda_C q$ with $\lambda_C = -2/Kc^2\varepsilon_0 U$.
Continuing the above lines, (3.4) is a sum of a wave–particle energies such as the kinetic wave–particle energy, in total, establishing a wave–particle energy balance, with m_0 implementing the charge-corrected mass m_0 meeting the observable mass m_0. Continuing the above lines, (3.4) reflects the wave–particle dualism, and this wave–particle dualism is implemented by the product structure $\chi_{00}\gamma_{00}$ defining the total wave–particle energy. This terminology implies to speak of "wave–particle energies" considering the parameters or not considering the parameters implemented by $\chi_{00} = Em_0/\hbar^2$ and implemented by other types of free quantities.

What do we now know, collecting arguments encouraging us to consider the above statements as physics?

We do now know that the energy contribution to the $\mathbb{W}$ form ("Wechselwirkungsform") of the GEFE exhibits energy operators/terms known from Schrödinger's formalism as well as extensions of energy operators/terms known from Schrödinger's formalism, implementing quantum-mechanical variables/parameters such as the energy E and the Planck constant of action $\hbar$ choosing functions/parameters of the energy contribution to the $\mathbb{W}$ form ("Wechselwirkungsform") of the GEFE suitably. Mind you, resorting to our advanced notions, all this is consistent with the notions "mass" and "charge". Mind you, resorting to our advanced notions, all this is consistent with the usage of Einstein's field equations of gravitation in cosmology. Mind you, the gateway from a gravitational potential implemented by $\gamma_{00} = -2\phi_g/c^2$ to a wave function implemented by $\gamma_{00} = \psi$ simply is opened by the wave–particle dualism implemented by a wave–particle energy momentum tensor reflecting the entity of quantum systems (wave–particle systems) deviating from the entity of cosmic systems.

Going beyond formal similarities, it is reasonable to assign the energy operators/terms terms supplied by the $\mathbb{W}$ form of the GEFE to Schrödinger's formalism and extensions of Schrödinger's formalism. The examples that follow illustrate this.

Departing from the kinetic wave–particle energy already quoted in Example 3.2, Examples 3.3–3.7 collect the basic elements of a scheme enabling the interpretative access to the energy operators/terms terms supplied by the $\mathbb{W}$ form of the GEFE assuming the wave–particle specification implementing quantum parameters such as $\hbar$ deviating from the cosmic specification not implementing quantum parameters such as $\hbar$.
We firstly consider Examples 3.3–3.5.
Examples 3.3–3.5 compare mechanical expressions in a parameter-free version (bottom of arrow schemes) with energy operators supplied by the $\mathbb{W}$ form of the GEFE (top of arrow schemes), in the latter case, achieving a form typical for energy operators of Schrödinger's formalism applying $\chi_{00} = Em_0/\hbar^2$ (bottom of examples), and the two rows of arrows in close replacement rules, structurally recasting the mechanical expressions in a parameter-free version into the energy operators supplied by the $\mathbb{W}$ form of the GEFE, in each case, using a tilde in order to indicate parameter-free versions of energies, momenta, and related operators.
Considering Example 3.3, we realize that the parameter-free version $\tilde{T}$ of the kinetic energy T leads to the parameter-free version of the kinetic energy operator $\hat{\tilde{T}}$ resorting to the replacement rule $\tilde{\boldsymbol{p}} \rightarrow \pm \mathbf{i}\boldsymbol{\nabla}$, where $\tilde{\boldsymbol{p}}$ is formally adjusted to meet the physical dimension "wave vector" established by thenabla operators $\boldsymbol{\nabla}$ fixing the parameter-free version of the kinetic energy operator $\hat{\tilde{T}}$.
Considering Example 3.3, werealize that there are structurally similar forms of the kinetic energy and the kinetic energy operator of the rotational motion, where $\tilde{\boldsymbol{\omega}}_{\mathrm{I}}$ is the rotation frequency vector formally adjusted to meet the physical dimension "wave vector" established by thenabla

Example 3.1 (Separability of solid body properties and field properties).

Energy matrix element $\mathcal{T}_{00}$ of special solid body energy momentum tensor:

$$\mathcal{K}\mathcal{T}_{00} - \frac{R}{2}\eta_{00} - \frac{R}{2}\gamma_{00} := -\frac{K}{2}\rho_{\mathrm{g}}(\boldsymbol{x})c^2 \tag{3.1}$$

(solid-body-related quantity $\rho_{\mathrm{g}}(\boldsymbol{x})$).

Energy balance equation in $\mathbb{W}$ form of GEFE, stationary limiting case, relatively small space areas characterized by Cartesian coordinates, with (3.1):

$$\underbrace{-\frac{1}{2}\triangle\gamma_{00}(\boldsymbol{x}) + \cdots}_{\substack{\text{field-related properties} \\ \text{(“fields”)}}} \quad := \quad \underbrace{-\frac{K}{2}\rho_{\mathrm{g}}(\boldsymbol{x})c^2}_{\substack{\text{solid-body-related properties} \\ \text{(“sources”)}}} \tag{3.2}$$

$\Leftrightarrow$
separable!

$\gamma_{00} \leftrightarrow$ macroscopic reflection of the mass potential ϕ_{g}
which is considered as caused by charge-corrected masses $m_0 = \mathfrak{m}_0 + \lambda_C q$!

Example 3.2 (Inseparability of solid body properties and field properties).

Energy matrix element $\mathcal{T}_{00}$ of special wave–particle energy momentum tensor:

$$\mathcal{K}\mathcal{T}_{00} - \frac{R}{2}\eta_{00} - \frac{R}{2}\gamma_{00} := \chi_{00}(\boldsymbol{k})\gamma_{00}(\boldsymbol{x},\boldsymbol{k}) \ , \ \gamma_{00} = \psi \ , \ \chi_{00}(\boldsymbol{k}) = E\frac{m_0}{\hbar^2} \tag{3.3}$$

(wave-related part $\gamma_{00}(\boldsymbol{x},\boldsymbol{k})$, particle-related part $\chi_{00}(\boldsymbol{k})$,
wave-related wave vector $\boldsymbol{k}$ makes the inseparable unity perfect).

Energy balance equation in $\mathbb{W}$ form of GEFE, stationary limiting case, relatively small space areas characterized by Cartesian coordinates, with (3.3):

Example 3.3 (Kinetic energy $(\to T)$ including orbital motion $(\to T_{\rm l})$).

$$
\begin{array}{ccccc}
\hat{\tilde{T}} = -\frac{1}{2}(\nabla)^{\rm T}\nabla\,, & \hat{\tilde{T}} = -\frac{1}{2}(\nabla)^{\rm T}\nabla & & \hat{\tilde{T}} = -\frac{1}{2}(\nabla)^{\rm T}\nabla\,, \\
\qquad\quad\uparrow\quad\uparrow & \qquad\quad\uparrow\quad\uparrow & & \qquad\quad\uparrow\quad\uparrow \\
\uparrow \qquad \pm{\rm i}\nabla\ \pm{\rm i}\nabla & \uparrow \qquad \pm{\rm i}\nabla\ \pm{\rm i}\nabla & \Leftrightarrow & \uparrow \qquad \pm{\rm i}\nabla\ \pm{\rm i}\nabla \\
\qquad\quad\uparrow\quad\uparrow & \qquad\quad\uparrow\quad\uparrow & & \qquad\quad\uparrow\quad\uparrow \\
\tilde{T} = +\frac{1}{2}(\tilde{\boldsymbol{p}})^{\rm T}\tilde{\boldsymbol{p}}\,, & \tilde{T}_{\rm l} = +\frac{1}{2}(\tilde{\boldsymbol{p}})^{\rm T}\tilde{\boldsymbol{p}} & & \tilde{T}_{\rm l} = +\frac{1}{2}(\tilde{\boldsymbol{\omega}}_{\rm l})^{\rm T}\tilde{\boldsymbol{\omega}}_{\rm l}\,,
\end{array}
\tag{3.5}
$$

$$\chi_{00} := E\frac{m_0}{\hbar^2} \quad \to \quad \hat{T} = -\frac{\hbar^2}{2m_0}(\nabla)^{\rm T}\nabla\,, \quad \tilde{\boldsymbol{p}} = \boldsymbol{p}/\hbar\,, \quad \tilde{\boldsymbol{\omega}}_{\rm l} = \boldsymbol{\omega}_{\rm l}/c\,.$$

Example 3.4 (Orbital motion versus spin without magnetic field $(\to T_{\rm s})$, $T_{\rm s} = T_{\rm s}(\boldsymbol{\omega}_{\rm l})$).

$$
\begin{array}{cc}
\hat{\tilde{T}} = -\frac{1}{2}(\nabla)^{\rm T}\nabla\,, & \hat{\tilde{T}}_{\rm s} = -\frac{1}{2}(\theta_{\rm s}^{\ni}\nabla)^{\rm T}\nabla\,, \\
\qquad\quad\uparrow\quad\uparrow & \qquad\quad\uparrow\qquad\uparrow \\
\uparrow \qquad \pm{\rm i}\nabla\ \pm{\rm i}\nabla & \uparrow \qquad \pm{\rm i}\theta_{\rm s}^{\ni}\nabla\ \ \pm{\rm i}\nabla \\
\qquad\quad\uparrow\quad\uparrow & \qquad\quad\uparrow\qquad\uparrow \\
\tilde{T}_{\rm l} = +\frac{1}{2}(\tilde{\boldsymbol{\omega}}_{\rm l})^{\rm T}\tilde{\boldsymbol{\omega}}_{\rm l}\,, & \tilde{T}_{\rm s} = +\frac{1}{2}({\rm t}_{\rm s}\tilde{\boldsymbol{\omega}}_{\rm l})^{\rm T}\tilde{\boldsymbol{\omega}}_{\rm l}\,,
\end{array}
\tag{3.6}
$$

$$\tilde{\boldsymbol{\omega}}_{\rm l} = \boldsymbol{\omega}_{\rm l}/c\,, \tag{3.7}$$

$$\chi_{00} := E\frac{m_0}{\hbar^2} \quad \to \quad \hat{T} = -\frac{\hbar^2}{2m_0}(\nabla)^{\rm T}\nabla\,, \quad \hat{T}_{\rm s} = -\frac{\hbar^2}{2m_0}(\theta_{\rm s}^{\ni}\nabla)^{\rm T}\nabla\,. \tag{3.8}$$

Example 3.5 (Orbital motion versus spin without magnetic field $(\to T_{\rm s})$, $T_{\rm s} = T_{\rm s}(\boldsymbol{p})$).

$$
\begin{array}{cc}
\hat{\tilde{T}} = -\frac{1}{2}(\nabla)^{\rm T}\nabla\,, & \hat{\tilde{T}}_{\rm s} = -\frac{1}{2}(\theta_{\rm s}^{\ni}\nabla)^{\rm T}\nabla\,, \\
\qquad\quad\uparrow\quad\uparrow & \qquad\quad\uparrow\qquad\uparrow \\
\uparrow \qquad \pm{\rm i}\nabla\ \pm{\rm i}\nabla & \uparrow \qquad \pm{\rm i}\theta_{\rm s}^{\ni}\nabla\ \ \pm{\rm i}\nabla \\
\qquad\quad\uparrow\quad\uparrow & \qquad\quad\uparrow\qquad\uparrow \\
\tilde{T}_{\rm l} = +\frac{1}{2}(\tilde{\boldsymbol{p}})^{\rm T}\tilde{\boldsymbol{p}}\,, & \tilde{T}_{\rm s} = +\frac{1}{2}({\rm t}_{\rm s}\tilde{\boldsymbol{p}})^{\rm T}\tilde{\boldsymbol{p}}\,,
\end{array}
\tag{3.9}
$$

$$\tilde{\boldsymbol{p}} = \boldsymbol{p}/\hbar\,, \tag{3.10}$$

$$\chi_{00} := E\frac{m_0}{\hbar^2} \quad \to \quad \hat{T} = -\frac{\hbar^2}{2m_0}(\nabla)^{\rm T}\nabla\,, \quad \hat{T}_{\rm s} = -\frac{\hbar^2}{2m_0}(\theta_{\rm s}^{\ni}\nabla)^{\rm T}\nabla\,. \tag{3.11}$$

operators $\boldsymbol{\nabla}$ fixing the parameter-free version of the kinetic energy operator $\hat{T}$, not only covering the translational motion, but also covering the rotational motion, in the latter case, immediately rewritten into a form showing the angular momentum operator $\hat{\boldsymbol{l}} = \boldsymbol{r} \times \hat{\boldsymbol{p}}$, however, which we do not want to consider here. Straightforwardly, we realize that the parameter-free version $\tilde{T}$ of the kinetic energy T of the rotational motion leads to the parameter-free version of the kinetic energy operator $\hat{T}$, not only covering the translational motion, but also covering the rotational motion, resorting to the replacement rule $\tilde{\boldsymbol{p}} \rightarrow \pm\mathbf{i}\boldsymbol{\nabla}$ or $\tilde{\boldsymbol{\omega}}_l \rightarrow \pm\mathbf{i}\boldsymbol{\nabla}$. Considering Examples 3.4 and 3.5, we realize that the kinetic energy T_s of a spin, which deviates from the kinetic energy T_l of a rotational motion by the "tensor of inertia" and which is fixed by the spin frequency vector $\boldsymbol{\omega}_s$ instead of the rotation frequency vector $\boldsymbol{\omega}_l$, is immediately expressed by the rotation frequency vector $\boldsymbol{\omega}_l$ defining a rotational motion superimposing the spin provided we introduce a suitable transformation matrix t_S and is immediately expressed by the momentum $\tilde{\boldsymbol{p}}$ defining a rotational motion superimposing the spin provided we introduce a suitable transformation matrix t_S. Straightforwardly, we realize that the above replacement rule $\tilde{\boldsymbol{p}} \rightarrow \pm\mathbf{i}\boldsymbol{\nabla}$ or $\tilde{\boldsymbol{\omega}}_l \rightarrow \pm\mathbf{i}\boldsymbol{\nabla}$ completed by $t_S\tilde{\boldsymbol{p}} \rightarrow \pm\mathbf{i}\theta_S\boldsymbol{\nabla}$ or $t_S\tilde{\boldsymbol{\omega}}_l \rightarrow \pm\mathbf{i}\theta_S\boldsymbol{\nabla}$, with θ_s defining a suitable matrix obviously reflecting that spin scenarios require a suitable matrix on each level of consideration, leads to an extension of the parameter-free version of the kinetic energy operator $\hat{T}$ likewise defining an energy operator supplied by the $\mathbb{W}$ form of the GEFE, with $\chi_{00} = Em_0/\hbar^2$, passing into $\hat{T}_S$, covering spin scenarios.

Realizing that the kinetic energy T, the kinetic energy operator $\hat{\mathbf{T}}$, and Jordan's rule $\mathbf{p} \rightarrow \hat{\mathbf{p}} = -\mathbf{i}\hbar\boldsymbol{\nabla}$ thus are natural parts of GEFT occurring embedded in the comprehensive, superior stage that is provided by the $\mathbb{W}$ form of the GEFE, it is reasonable to assume that $\hat{\mathbf{T}}_S$ is an extension of Schrödinger's formalism not known in quantum mechanics, logically consistent, to be classified as kinetic spin energy operator.

We secondly consider Examples 3.6 and 3.7.

Example 3.6 introduces the structural elements of a formal scheme recasting the differential operator of a first special wave–particle energy supplied by the $\mathbb{W}$ form of the GEFE into the differential operator $\boldsymbol{B}\hat{\boldsymbol{l}}$ covering the interaction of an orbital motion with an applied magnetic field B. We note that this formal scheme fulfils the law of conservation of physical dimensions. We also note that this formal scheme is completely consistent with the formal scheme applied in[1] to recast comprehensive, superior forces supplied by the $\mathbb{W}$ form of the related equation of geodetic lines into Newtonian forces for masses and charges. We also note that this formal scheme is completely consistent with Schrödinger's formalism applying $\chi_{00} = Em_0/\hbar^2$. In particular, we then are led to the momentum operator $\hat{\boldsymbol{p}}$, the angular momentum operator $\hat{\boldsymbol{l}} = \boldsymbol{r} \times \hat{\boldsymbol{p}}$, and the magnetic field B concatenated in the way required by Schrödinger's formalism, and this includes the mass m_0 and the charge q, continuing the formal scheme applied in[1], fulfilling $m_0 = \mathfrak{m}_0 + \lambda_C q$ with $\lambda_C = -2/Kc^2\varepsilon_0 U$ and $m_0c^2 = qU$, with m_0 and q defining the observables, thus covering tabulated values.

Example 3.7 introduces the structural elements of a formal scheme recasting the differential operator of a second special wave–particle energy supplied by the $\mathbb{W}$ form of the GEFE into a differential operator $\boldsymbol{B}\hat{\mathfrak{s}}$ defining an extension of the differential operator $\boldsymbol{B}\hat{\boldsymbol{l}}$ covering the interaction of an orbital motion with a magnetic field B, in comparison to the differential operator $\boldsymbol{B}\hat{\boldsymbol{l}}$ covering the interaction of an orbital motion with a magnetic field B, exhibiting the additional matrix θ_s.

Realizing that the kinetic energy operator $\hat{\mathbf{T}}$ just like the differential operator $\boldsymbol{B}\hat{\boldsymbol{l}}$ covering the interaction of an orbital motion with an applied magnetic field B thus are natural parts of GEFT occurring embedded in the comprehensive, superior stage that is provided by the $\mathbb{W}$ form of the GEFE, and then firstly observing that the $\mathbb{W}$ form of the GEFE also includes differential operators $\hat{\mathbf{T}}_S$ and $\boldsymbol{B}\hat{\mathfrak{s}}$ differing from $\hat{\mathbf{T}}$ and $\boldsymbol{B}\hat{\boldsymbol{l}}$ by the additional matrix $\boldsymbol{\theta}_s$, and then secondly observing that the additional matrix $\boldsymbol{\theta}_s$ implements spin properties following the above considerations, it is reasonable to assume that not only $\hat{\mathbf{T}}_S$, but also $\boldsymbol{B}\hat{\mathfrak{s}}$ covers spin scenarios, logically consistent, in the first case to be classified as (differential) kinetic spin energy operator, and in the second case, to be classified as (differential) interaction operator covering the interrelation of a spin and an applied magnetic field B, in both cases, defining extensions of Schrödinger's formalism not known in quantum mechanics.

We here note in passing that the $\mathbb{W}$ form of the GEFE originally exhibits a general matrix θ_3 collecting the γ_{ij} of the Ansatz $g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}$ ($\mu = 0,1,2,3$; $i = 1,2,3$) as well as a general matrix θ^3 collecting the γ_{ij} of the contravariant form $g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}$ of the Ansatz $g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}$ but we here want to restrict ourselves to the specialized matrix $\theta^3 = \theta_s^3 = \theta_s$ collecting the γ_{ij} of the contravariant form $g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}$ of the Ansatz $g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}$ related to spin scenarios, neglecting other effects. We here note in passing that $\hat{T}_S$ and $\boldsymbol{B}\hat{\mathfrak{s}}$ can be used to develop analytical/numerical models for spin scenarios, in this manner, further proving the validity of the assignment to Schrödinger's

Example 3.6 (Angular momentum precession aspect).

We focus on a first special wave–particle energy
of the energy balance equation in $\mathbb{W}$ form of the GEFE:

$$\chi_{00}\psi = -\frac{1}{2}\big(\nabla \times \boldsymbol{A}_{\ni}\big)^{\mathrm{T}}\boldsymbol{A}^{\ni} \times \nabla\psi + \ldots \ . \tag{3.12}$$

The vector $\boldsymbol{A}_{\ni}$ collecting scalar functions $\gamma_{i0} = \gamma_{0i}$ showing lower indices evokes the vector $\boldsymbol{B}_3 = \nabla \times \boldsymbol{A}_{\ni}$ eventually picturing interaction fields causing changes in the wave–particle system, whereas the vector $\boldsymbol{A}^{\ni}$ collecting scalar functions $\gamma^{i0} = \gamma^{0i}$ showing upper indices evokes the operator $\boldsymbol{A}^{\ni} \times \nabla$ eventually calculating the changes in the wave–particle system. For example, following the scheme below, we obtain a wave–particle energy known from conventional quantum mechanics covering the magnetic-field–"orbital-particle-motion" interaction leading to the precession aspect of an angular momentum vector in a magnetic field. In this case, the vector $\boldsymbol{A} \propto \boldsymbol{A}_{\ni}$, usually called "vector potential", evokes the vector $\boldsymbol{B} = \nabla \times \boldsymbol{A}$ eventually picturing the magnetic field causing the precession aspect of an angular momentum vector, whereas the vector $\boldsymbol{A}^{\ni} \propto \boldsymbol{r}$ evokes the operator $\hat{\boldsymbol{l}} = \boldsymbol{r} \times \hat{\boldsymbol{p}}$ eventually calculating the precession aspect of an angular momentum vector.

$$\begin{aligned}
\boldsymbol{B}_3 &:= \frac{c}{U}\boldsymbol{B}\ , \ \ \mathrm{Dim}\,[\boldsymbol{B}_3] = \frac{1}{\mathrm{m}} = \mathrm{Dim}\left[\frac{c}{U}\boldsymbol{B}\right] = \frac{\mathrm{m}}{\mathrm{s}}\frac{\mathrm{T}}{\mathrm{V}} = \frac{\mathrm{m}}{\mathrm{s}}\frac{1}{\mathrm{V}}\frac{\mathrm{V\,s}}{\mathrm{m}^2}\ ,\\
\boldsymbol{A}_{\ni} &:= \frac{c}{U}\boldsymbol{A}\ , \ \ \mathrm{Dim}\,[\boldsymbol{A}_{\ni}] = 1 = \mathrm{Dim}\left[\frac{c}{U}\boldsymbol{A}\right] = \frac{\mathrm{m}}{\mathrm{s}}\frac{1}{\mathrm{V}}\frac{\mathrm{V\,s}}{\mathrm{m}}\ ,\\
\boldsymbol{B}_3 &= \frac{c}{U}\boldsymbol{B} = \nabla \times \boldsymbol{A}_{\ni} = \frac{c}{U}\big(\nabla \times \boldsymbol{A}\big)\ , \ \ \mathrm{Dim}[\boldsymbol{B}_3] = \mathrm{Dim}\left[\frac{c}{U}\boldsymbol{B}\right] = \frac{1}{\mathrm{m}} =\\
&= \mathrm{Dim}\,[\nabla \times \boldsymbol{A}_{\ni}] = \frac{1}{\mathrm{m}} = \mathrm{Dim}\left[\frac{c}{U}\big(\nabla \times \boldsymbol{A}\big)\right] = \frac{1}{\mathrm{m}}\frac{\mathrm{m}}{\mathrm{s}}\frac{1}{\mathrm{V}}\frac{\mathrm{V\,s}}{\mathrm{m}}\ ,
\end{aligned} \tag{3.13}$$

$$\chi_{00} := E\frac{m_0}{\hbar^2}\ , \ \ \mathrm{Dim}\,[\chi_{00}] = \frac{1}{\mathrm{m}^2} = \mathrm{Dim}\left[E\frac{m_0}{\hbar^2}\right] = \mathrm{J}\frac{\mathrm{kg}}{\mathrm{J}^2\mathrm{s}^2} = \frac{\mathrm{kg\,s}^2}{\mathrm{kg\,m}^2\,\mathrm{s}^2} \tag{3.14}$$

$$\Downarrow$$

$$E\psi = -\frac{\hbar^2}{2m_0}\frac{c}{U}\boldsymbol{B}\boldsymbol{A}^{\ni} \times \nabla\psi + \ldots\ , \tag{3.15}$$

$$\begin{aligned}
\boldsymbol{A}^{\ni} &:= -\mathrm{i}\frac{qU}{\hbar c}\boldsymbol{r}\ ,\\
\mathrm{Dim}\left[\boldsymbol{A}^{\ni}\right] = 1 = \mathrm{Dim}\left[\frac{qU}{\hbar c}\boldsymbol{r}\right] &= \frac{\mathrm{C\,V\,s}}{\mathrm{J\,s\,m}}\mathrm{m} = \frac{\mathrm{A\,s\,V}}{\mathrm{J}} = \frac{\mathrm{A\,s\,V}}{\mathrm{W\,s}} = \frac{\mathrm{A\,s\,V}}{\mathrm{V\,A\,s}}
\end{aligned} \tag{3.16}$$

$$\Downarrow$$

$$E\psi = -\frac{q}{2m_0}\boldsymbol{B}\hat{\boldsymbol{l}}\psi + \ldots\ , \ \ \hat{\boldsymbol{l}} = \boldsymbol{r} \times \hat{\boldsymbol{p}}\ , \ \ \hat{\boldsymbol{p}} = -\mathrm{i}\hbar\nabla\ . \tag{3.17}$$

Example 3.7 (Spin precession aspect).

We focus on a second special wave–particle energy
of the energy balance equation in $\mathbb{W}$ form of the GEFE:

$$\chi_{00}\psi = -\frac{1}{2}\big(\nabla \times \boldsymbol{A}_{\ni}\big)^{\mathrm{T}}\boldsymbol{A}^{\ni} \times \theta^{\ni}\nabla\psi + \dots \; . \tag{3.18}$$

The vector $\boldsymbol{A}_{\ni}$ collecting scalar functions $\gamma_{i0} = \gamma_{0i}$ showing lower indices evokes the vector $\boldsymbol{B}_3 = \nabla \times \boldsymbol{A}_{\ni}$ eventually picturing interaction fields causing changes in the wave–particle system, whereas the vector $\boldsymbol{A}^{\ni}$ collecting scalar functions $\gamma^{i0} = \gamma^{0i}$ showing upper indices evokes the operator $\boldsymbol{A}^{\ni} \times \theta^{\ni}\nabla$ eventually calculating the changes in the wave–particle system. For example, following the scheme below, we obtain a wave–particle energy that supersedes the wave–particle energy known from conventional quantum mechanics based on spin (Pauli, $\mathrm{SU_n}$) matrices covering the magnetic-field–spin interaction leading to the precession aspect of a spin vector in a magnetic field. In this case, the vector $\boldsymbol{A} \propto \boldsymbol{A}_{\ni}$ evokes the vector $\boldsymbol{B} = \nabla \times \boldsymbol{A}$ eventually picturing the magnetic field causing the precession aspect of a spin vector, whereas the vector $\boldsymbol{A}^{\ni} \propto \boldsymbol{r}$ evokes the operator $\hat{\mathfrak{s}}_{\mathrm{s}} = \boldsymbol{r} \times \hat{\mathfrak{j}}_{\mathrm{s}}$ eventually calculating the precession aspect of a spin vector.

$$\begin{aligned}
\boldsymbol{B}_3 &:= \frac{c}{U}\boldsymbol{B}\;,\;\; \mathrm{Dim}\,[\boldsymbol{B}_3] = \frac{1}{\mathrm{m}} = \mathrm{Dim}\left[\frac{c}{U}\boldsymbol{B}\right] = \frac{\mathrm{m}}{\mathrm{s}}\frac{\mathrm{T}}{\mathrm{V}} = \frac{\mathrm{m}}{\mathrm{s}}\frac{1}{\mathrm{V}}\frac{\mathrm{V\,s}}{\mathrm{m}^2}\;,\\
\boldsymbol{A}_{\ni} &:= \frac{c}{U}\boldsymbol{A}\;,\;\; \mathrm{Dim}\,[\boldsymbol{A}_{\ni}] = 1 = \mathrm{Dim}\left[\frac{c}{U}\boldsymbol{A}\right] = \frac{\mathrm{m}}{\mathrm{s}}\frac{1}{\mathrm{V}}\frac{\mathrm{V\,s}}{\mathrm{m}}\;,\\
\boldsymbol{B}_3 &= \frac{c}{U}\boldsymbol{B} = \nabla \times \boldsymbol{A}_{\ni} = \frac{c}{U}\big(\nabla \times \boldsymbol{A}\big)\;,\;\; \mathrm{Dim}[\boldsymbol{B}_3] = \mathrm{Dim}\left[\frac{c}{U}\boldsymbol{B}\right] = \frac{1}{\mathrm{m}} =\\
&= \mathrm{Dim}\,[\nabla \times \boldsymbol{A}_{\ni}] = \frac{1}{\mathrm{m}} = \mathrm{Dim}\left[\frac{c}{U}(\nabla \times \boldsymbol{A})\right] = \frac{1}{\mathrm{m}}\frac{\mathrm{m}}{\mathrm{s}}\frac{1}{\mathrm{V}}\frac{\mathrm{V\,s}}{\mathrm{m}}\;,
\end{aligned} \tag{3.19}$$

$$\chi_{00} := E\frac{m_0}{\hbar^2}\;,\;\; \mathrm{Dim}\,[\chi_{00}] = \frac{1}{\mathrm{m}^2} = \mathrm{Dim}\left[E\frac{m_0}{\hbar^2}\right] = \mathrm{J}\frac{\mathrm{kg}}{\mathrm{J}^2\mathrm{s}^2} = \frac{\mathrm{kg\,s}^2}{\mathrm{kg\,m}^2\,\mathrm{s}^2} \tag{3.20}$$

$$\Downarrow$$

$$E\psi = -\frac{\hbar^2}{2m_0}\frac{c}{U}\boldsymbol{B}\boldsymbol{A}^{\ni} \times \theta^{\ni}\nabla\psi + \dots\;, \tag{3.21}$$

$$\begin{aligned}
&\boldsymbol{A}^{\ni} := -\mathrm{i}\frac{qU}{\hbar c}\boldsymbol{r}\;,\;\; \theta^{\ni} := \theta^{\ni}_{\mathrm{s}}\;,\;\; \boldsymbol{A}^{\ni} \times \theta^{\ni}\nabla := -\mathrm{i}\frac{qU}{\hbar c}\boldsymbol{r} \times \theta^{\ni}_{\mathrm{s}}\nabla\;,\\
&\mathrm{Dim}\left[\boldsymbol{A}^{\ni} \times \theta^{\ni}\nabla\right] = \frac{1}{\mathrm{m}} = \mathrm{Dim}\left[\frac{qU}{\hbar c}\boldsymbol{r} \times \theta^{\ni}_{\mathrm{s}}\nabla\right] = \frac{\mathrm{C\,V\,s}}{\mathrm{J\,s\,m}}\mathrm{m}\frac{1}{\mathrm{m}} = \frac{\mathrm{A\,s\,V}}{\mathrm{V\,A\,s\,m}}\frac{1}{\mathrm{m}}
\end{aligned} \tag{3.22}$$

$$\Downarrow$$

$$E\psi = -\frac{q}{2m_0}\boldsymbol{B}\hat{\mathfrak{s}}_{\mathrm{s}}\psi + \dots\;,\;\; \hat{\mathfrak{s}}_{\mathrm{s}} = \boldsymbol{r} \times \hat{\mathfrak{j}}_{\mathrm{s}}\;,\;\; \hat{\mathfrak{j}}_{\mathrm{s}} = -\mathrm{i}\hbar\theta^{\ni}_{\mathrm{s}}\nabla\;. \tag{3.23}$$

formalism. However, since these analytical/numerical models are lengthy, I cannot present these here, but have to refer to thefuture publication[2].

However, let me already here put on record that these analytical/numerical models directly lead to an advanced spin formalism[2] showing spin (Pauli, SUn) matrices as kernel including eigenvalue equations and commutation relations, as outlined in Example 3.7, superseding the conventional spin formalism[5,6] based upon spin (Pauli, SUn) matrices, noted by the way, also supplying us with wave structures related to the spin complementing wave structures related to the orbital motion, eventually supplying us with a deeper access to the measurement uncertainties characterizing spin scenarios[2].

It is important to have understood that the $\mathbb{W}$ form of the GEFE is related to a comprehensive, superior level of description of different classes of physical circumstances, reflecting the transformational invariance of the form of Einstein's field equations of gravitation establishing such a comprehensive, superior level of description of different classes of physical circumstances. Figures 1 and 2 illustrate the "covariant (superior) structure" of the GEFE considering orbital motion and spin, incorporating basic operators supplied by quantum mechanics as well as GEFT and basic operators supplied by GEFT, focusing on the principle geometrical situation without taking quantization aspects and coexistence aspects into consideration. Anticipating the future publication[2], I point out here that Figures 1 and 2 illustrate the operative basis reflecting classical geometrical circumstances. Anticipating the future publication[2], I point out here that typical quantum-mechanical scenarios such as the orbitals of the hydrogen atom exhibit geometrical circumstances differing from the operative basis reflecting classical geometrical circumstances. However, these advanced geometrical circumstances can be set up departing from these simple geometrical circumstances, explaining eigenvalues and "non-eigenvalues"[2].

Figure 1 shows a vectorial picture illustrating the geometrical interrelation of the momentum operator $\hat{\boldsymbol{p}}$ and the angular momentum operator $\hat{\boldsymbol{l}} = \boldsymbol{r} \times \hat{\boldsymbol{p}}$, directly related to eigenvalues $\boldsymbol{p}$ and $\boldsymbol{l}$, where $\boldsymbol{Q}$ indicates suitable sets of quantum parameters/numbers, and where θ is the corresponding scalar of inertia enabling the introduction of the angular frequency operator $\hat{\boldsymbol{\omega}}_l$. Figure 2 shows a vectorial picture illustrating the geometrical interrelation of the momentum operator $\hat{\boldsymbol{p}}$ completed by the advanced spin operator $\hat{\mathbf{i}}_s$ that emerges adding the matrix θ_s, the angular momentum operator $\hat{\boldsymbol{l}} = \boldsymbol{r} \times \hat{\boldsymbol{p}}$ completed by the advanced spin operator $\hat{\mathfrak{s}}$ that emerges adding the matrix θ_s, and the advanced spin operator $\hat{\boldsymbol{j}}_s$, directly related to eigenvalues $\boldsymbol{p}$, $\boldsymbol{l}$, $\mathbf{i}_s$, $\mathfrak{s}$, and $\boldsymbol{j}_s$, where θ is the corresponding tensor of inertia enabling the introduction of the spin frequency operator $\hat{\boldsymbol{\omega}}_s$. We firstly note that $\hat{\boldsymbol{j}}_s$ is readily established considering the kinetic spin energy in the main set up by $\hat{\boldsymbol{p}}$ and $\hat{\mathbf{i}}_s$ choosing θ_s suitably[2]. We secondly note that $\hat{\mathfrak{s}}$ is congruent to $\hat{\boldsymbol{j}}_s$ choosing θ_s suitably[2]. We thirdly note that $\hat{\mathfrak{s}}$ can be expressed using spin (Pauli, SUn) matrices choosing θ_s such that θ_s reflects the existence of an applied magnetic field[2], eventually recovering the spin operator of the conventional spin formalism[5,6].

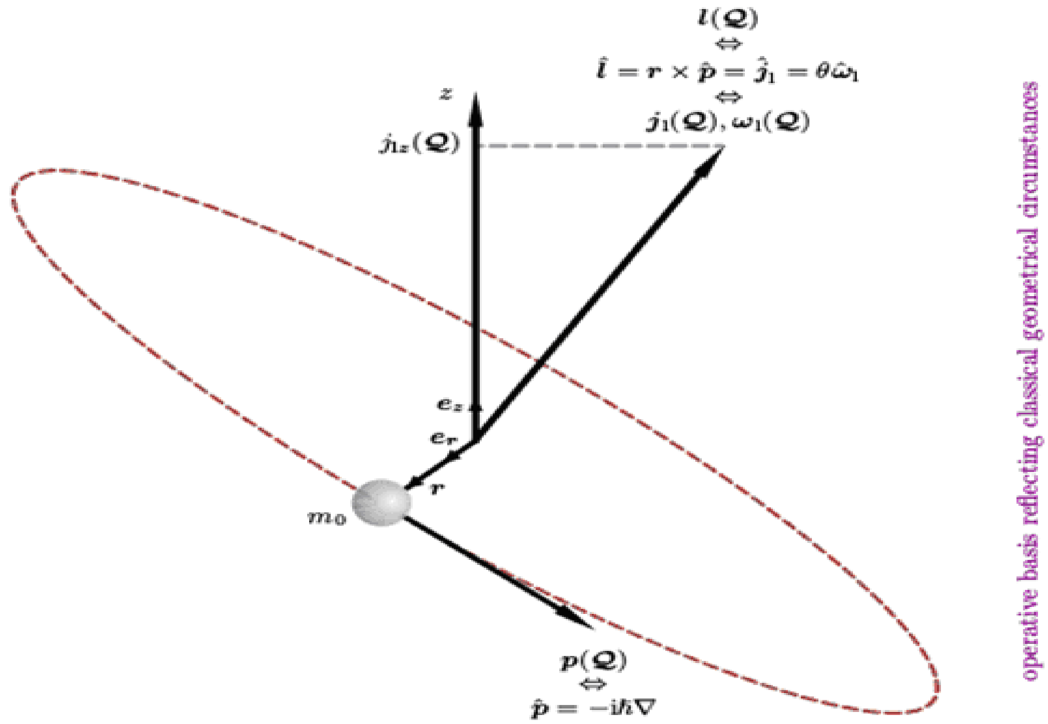
What do we now know, collecting arguments encouraging us to consider the above statements as physics?

We do now know that the energy contribution to the $\mathbb{W}$ form ("Wechselwirkungsform") of the GEFE exhibits energy operators/terms known from Schrödinger's formalism as well as extensions of energy operators/terms known from Schrödinger's formalism. We do now also know that these energy operators/terms occur in a form reflecting the "covariant (superior) structure" of the $\mathbb{W}$ form ("Wechselwirkungsform") of the GEFE, only after further steps of evaluation, adopting trusted shapes. We do now also know that these energy operators/terms can be proven further, developing analytical/numerical models. However, since these analytical/numerical models are lengthy, I cannot present these here, but have to refer to the future publication[2], for example, showing that the differential spin operators cover the "spin as it is" and the "spin in a magnetic field", eventually superseding spin operators based upon spin (Pauli, SUn) matrices.

Realizing that the product structure $\chi_{00}\gamma_{00}$ specified by $\chi_{00} = Em_0/\hbar^2$ and $\gamma_{00} = \psi$, in total, defining the energy element of awave–particle-related energy momentum tensor, thus recasts the energy contribution to the $\mathbb{W}$ form ("Wechselwirkungsform") of the GEFE into a wave–particle energy balance equation showing the structure of the time-independent Schrödinger equation completed by linear and nonlinear, time-independent and time-dependent terms, and realizing that these terms can be proven further developing analytical/numerical models, going beyond formal similarities, it is reasonable to assume that this wave–particle energy balance equation extends the time-independent Schrödinger equation into nonlinear, time-independent domains and nonlinear, time-dependent domains. In time-dependent cases, the energy E may become time-dependent. In time-dependent cases, completions of $\chi_{00} = Em_0/\hbar^2$ may be necessary. Then E is no eigenvalue.

But how can we understand this, observing the va-

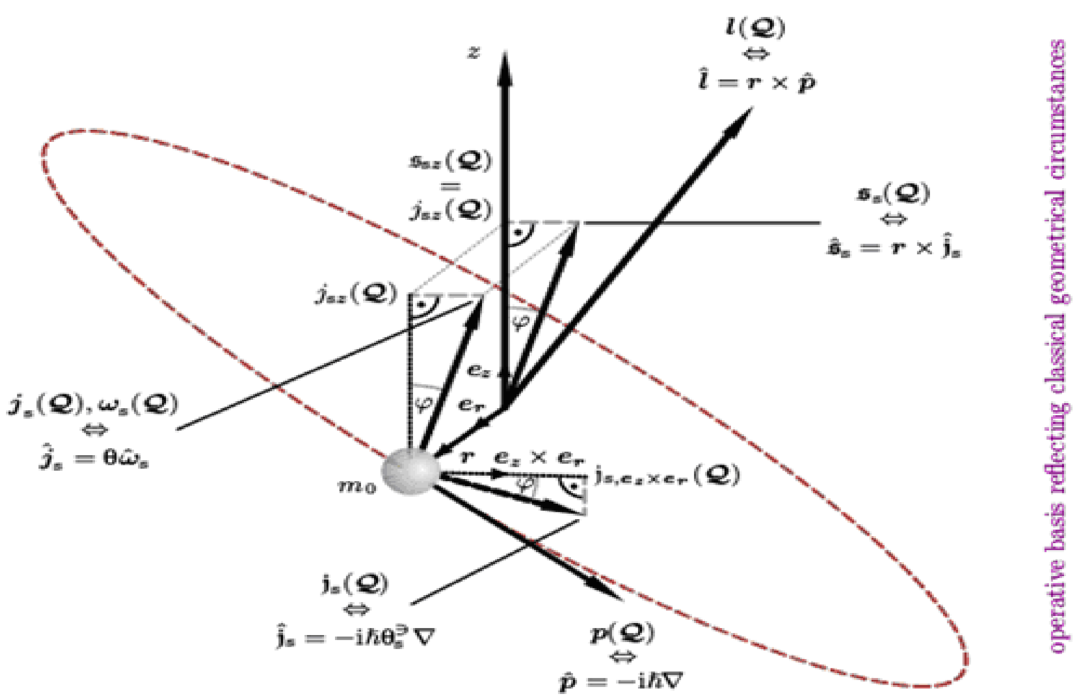
it is a well-known fact that the momentum operator $\hat{\boldsymbol{p}}$ fixes eigenvalues $\boldsymbol{l} = \boldsymbol{j}_1$ related to the orbital motion



we here consider a drawing that describes the principle geometrical situation without taking quantization aspects and coexistence aspects into consideration

Figure 1 : Momentum operator and angular momentum operator.

neither it is a miracle that the momentum operator $\hat{\boldsymbol{p}}$ fixes eigenvalues $\boldsymbol{l} = \boldsymbol{j}_1$ related to the orbital motion, nor it is a miracle that the momentum operator $\hat{\boldsymbol{p}}$ fixes eigenvalues $\boldsymbol{s}_s = \boldsymbol{j}_s$ related to the spin incorporating the matrix $\theta_s^{\ni}$ related to the spin, in fact it is the expression of the covariant (superior) structure of the generalized Einstein field equations



we here consider a drawing that describes the principle geometrical situation without taking quantization aspects and coexistence aspects into consideration

Figure 2 : Momentum operator and advanced spin operators.

lidity of the time-dependent Schrödinger equation?

We can understand this assuming that the superposition principle is not a universal principle, but a restrictive constraint, restricting the area of validity of the time-dependent Schrödinger equation to quantum scenarios to be described by superpositions of "basis function" to be obtained as "eigenfunctions" of the time-independent Schrödinger equation!

Mind you, this is exactly the usage of the time-dependent Schrödinger equation in practice!
Mind you, we only have to understand that the superposition principle is not a universal principle.
We only have to understand that linearities supporting the superposition principle define a basic domain, while nonlinearities disturbing the superposition principle define an extended domain.
The virtue of the above assumption can be seen as follows.

What would young, unbiased students familiar with the basics of quantum mechanics say becoming confronted with self-interaction?

Probably they would say:

"$-\mathbf{i}\hbar\boldsymbol{\nabla}\boldsymbol{\psi}$ means motion of a particle. Therefore, $-\hbar^2 \boldsymbol{A}\boldsymbol{\nabla}\boldsymbol{\psi}\boldsymbol{\nabla}\psi$ should mean motion acts back onto motion. This should be self-interaction, with A defining the self-interaction strength."

Well, this energy operator/term is supplied by the $\mathbb{W}$ form ("Wechselwirkungsform") of the GEFE. In contrast to the energy linear operator/term applied in quantum mechanics, it is a nonlinear energy operator/term. In contrast to the energy linear operator/term applied in quantum mechanics, it does not lead to any singularities and does not need any renormalization procedures.

Moreover, if a particle consists of subparticles, this energy operator/term establishes the connection of the interaction fields of the subparticles to the self-interaction field, eventually supplying us with a comprehensive, superior access to interaction properties known as weak and strong.

Could this mean that self-interaction needs a nonlinear description? Could this mean that singularities and renormalization procedures are the expression of a nonlinear problem that is treated as a linear problem? Could this mean that particles, subparticles, self-interaction, weak interaction, strong interaction together with hierarchies of higher/lower order can be treated as self-consistent unity? Going beyond that, could this mean that the energy operator/terms that are supplied by the $\mathbb{W}$ form of the GEFE supply us with an advanced access to quantum scenarios, and this includes linear quantum scenarios and nonlinear quantum scenarios?
Exactly this is what I am assuming!
Wanting to prove this further, I meanwhile have developed a lot of applications collecting these in[2].
Anyway, the virtue of the above assumption already becomes obvious considering this example.
Furthermore, going far beyond the treatments of nowadays, a nonlinear domain attached to the linear domain enables us to lead nonlinearities of macroscopic physics back to nonlinearities of microscopic physics, and nonlinear concepts such as synergetic concepts can be extended to meet quantum scenarios.
Weaving these threads further, we realize that

The time-dependent Schrödinger equation is the energy contribution to the $\mathbb{W}$ form of the GEFE specified by the energy element of awave–particle-related energy momentum tensor exhibiting a time derivative only covering linear aspects of quantum systems, where as the wave–particle energy balance equation showing the structure of the time-independent Schrödinger equation completed by linear and nonlinear, time-independent and time-dependent terms is the energy contribution to the $\mathbb{W}$ form of the GEFE specified by an energy element of awave–particle-related energy momentum tensor, depending on the energy element of the wave–particle-related energy momentum tensor, covering linear aspects of quantum systems and/or covering nonlinear aspects of quantum systems, and in linear, stationary cases, showing a point of contact with the time-dependent Schrödinger equation.

Naturally, this implies that not all terms always must be considered at the same time.
This is a difference to the usage of Einstein's field equations of gravitation valid for cosmic systems.
The reason for this difference is that cosmic events are observed as superpositions of a lot of physical events evolving at the same time so that all terms always must be considered at the same time, while quantum events are observed as a few physical events allowing us to restrict ourselves to those terms that are needed. Moreover, dealing with cosmic events, terms of higher order usually do not define lower orders of magnitude, but dealing with quantum events, terms of higher order usually do define lower orders of magnitude allowing us to restrict ourselves to the leading terms. Therefore, GEFT does not establish a bottom-up approach, GEFT does establish a top-down approach, reversing the path of quantum mechanics. Going beyond that, GEFT not only does establish wave–particle energy balances, GEFT does establish wave–particle energy momentum balances, finally completing the paths of quantum mechanics.

Naturally, this is a metric field approach to quantum systems, in the following called "wave–particle theory"! Observing Figures 3-6, this metric field approach becomes graphically comprehensible.

THE GENERALIZED EINSTEIN FIELD EQUATIONS (GEFE), THE WAVE–PARTICLE FEATURE OF QUANTUM SYSTEMS, AND THE WAVE–PARTICLE METRIC

In what follows, we follow Figures 3-6, collecting the basic formulae in LATEX style which I am preferring despite some differences face to face with the WORD style predetermined by the text template.

What is the difference between a mechanical system and a quantum system (wave–particle system)?

Certainly, the masses are much smaller, the charges are much smaller, and the metric fields are much smaller dealing with a quantum system. Certainly, wave properties start to accompany particle properties approaching the domain of quantum system. However, going beyond that, let me here develop a graphic image explaining why wave

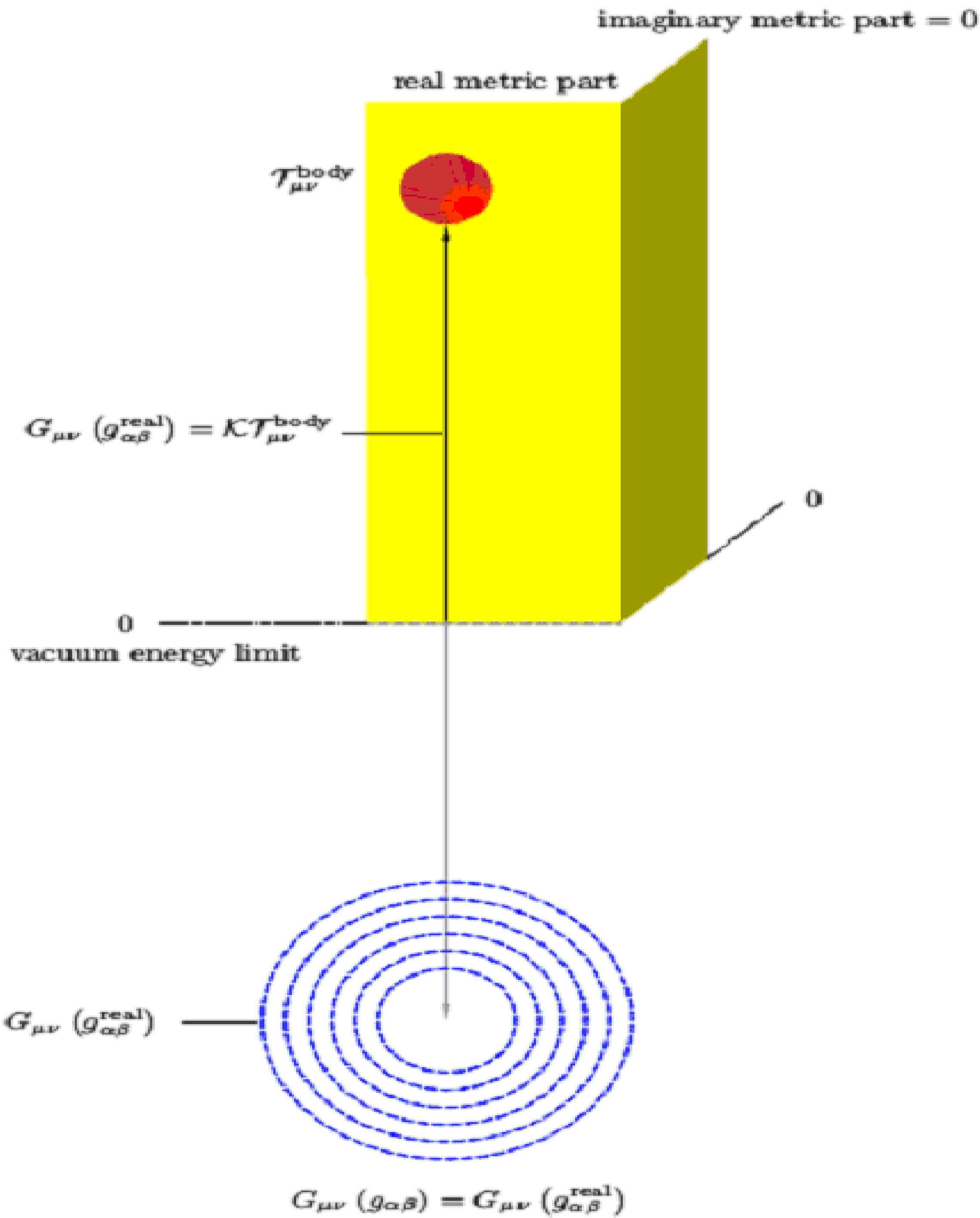


Figure 3 : Mechanical systems and metric fields.

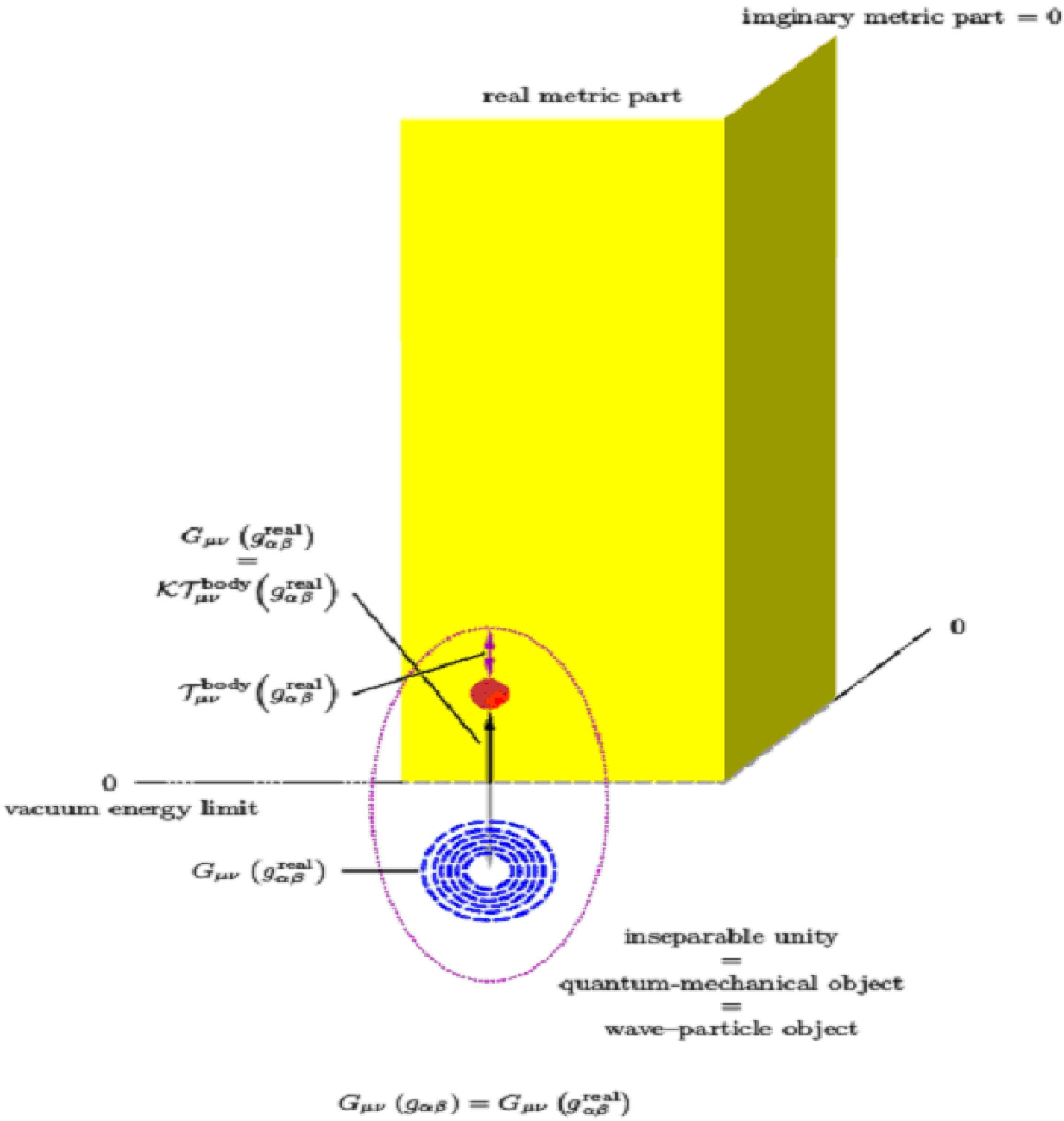


Figure 4 : Quantum systems (wave–particle systems) and metric fields.

properties start to accompany particle properties approaching the domain of quantum systems continuing the considerations of[1], assuming a metric field energy density/ metric field energy $\varrho_m / \oint V \varrho_m$ postulating energy densities/energies related to warpings or distortions of the "vacuum" as a consequence of mass–charge generation such that the energy is conserved during the mass–charge generation starting from the vacuum energy density/ vacuum energy $\varrho_v / \oint V \varrho_v = 0$ reflecting "unobservable states" not showing any kind of observable matter properties such as observable mass properties or observable charge properties. Dealing with macroscopic entities, the energetic distance between both energetic levels of consideration has to be assumed as relatively big so that no overlay of mass–charge properties and metric field properties is to be expected. Dealing with microscopic entities, however, the energetic distance between both energetic levels of consideration has to be assumed as relatively small so that an overlay of mass–charge properties and metric field properties is to be expected, eventually providing a graphic access to quantum systems ("wave–particle systems"), explaining the inseparability of wave properties (metric field properties) and particle properties (mass–charge properties).

Resorting to this graphic image, the difference between a mechanical system and a quantum system ("wave–particle system") thus is provided by the different energetic distances of metric field energy and mass–charge energy, in the latter case, then occurring mixed with wave properties/metric field properties.

In Figure 7 and Figure 8 presented in[1], these energetic circumstances are illustrated on the classical stage supplied by the generalized Poissonian equation. In Figure 3 and Figure 4, these energetic circumstances are illustrated on an advanced stage supplied by the GEFE. In Figure 4 and 6, the specific example of a spin in a magnetic field is considered, on the one hand, resorting to the Einstein picture directly established by the GEFE, and on the other hand, resorting to the Schrödinger picture directly established by the wave–particle energy balance equation showing the structure of the time-independent Schrödinger equation completed by linear and nonlinear, time-independent and time-dependent terms. Observing mechanical systems, the energetic distance of metric field energy implemented by G_{00} and mass–charge energy implemented by $\mathcal{T}_{00}$ is relatively big so that no overlay comes into being. This is illustrated by Figure 3. Observing mechanical quantum systems, the energetic distance of metric field energy implemented by G_{00} and mass–charge energy implemented by $\mathcal{T}_{00}$ is relatively small so that an overlay comes into being. This is illustrated by Figure 4. While the first case is covered by an energy momentum tensor $\mathcal{T}_{\mu\nu}$ not showing intrinsic dependencies from the metric tensor $g_{\mu\nu}$, the second case is covered by an an energy momentum tensor $\mathcal{T}_{\mu\nu}(g_{\mu\nu})$ showing intrinsic dependencies from the metric tensor $g_{\mu\nu}$, directly reflecting what we want to call a "wave–particle energy momentum tensor", here restricting ourselves to product structures $\chi_{\mu\nu}\gamma_{\mu\nu}$ related to the Ansatz $g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}$.

In the first case, in total, described by the energy momen-

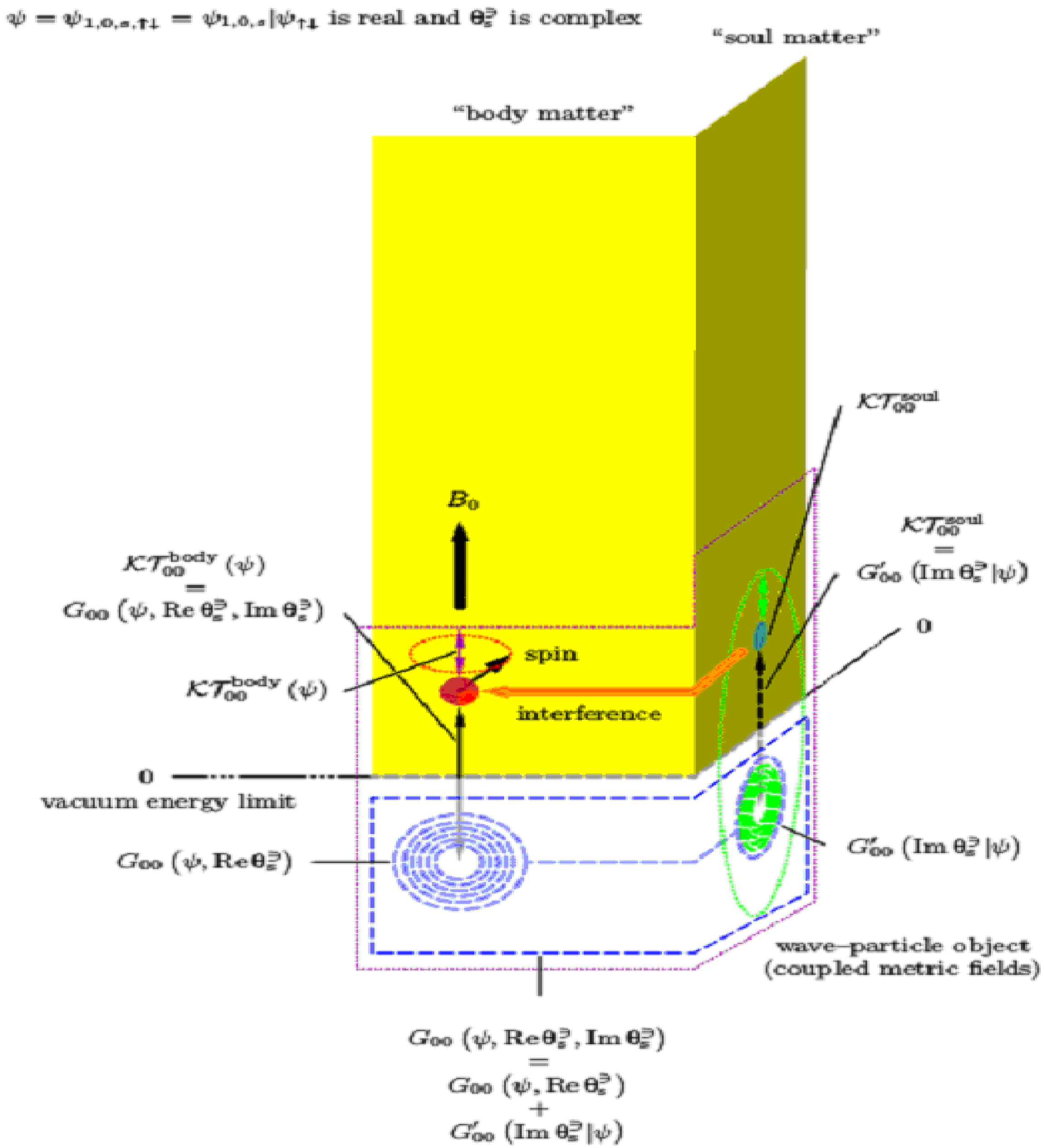


Figure 5 : Spin in a magnetic field. Body and soul, Einstein picture.

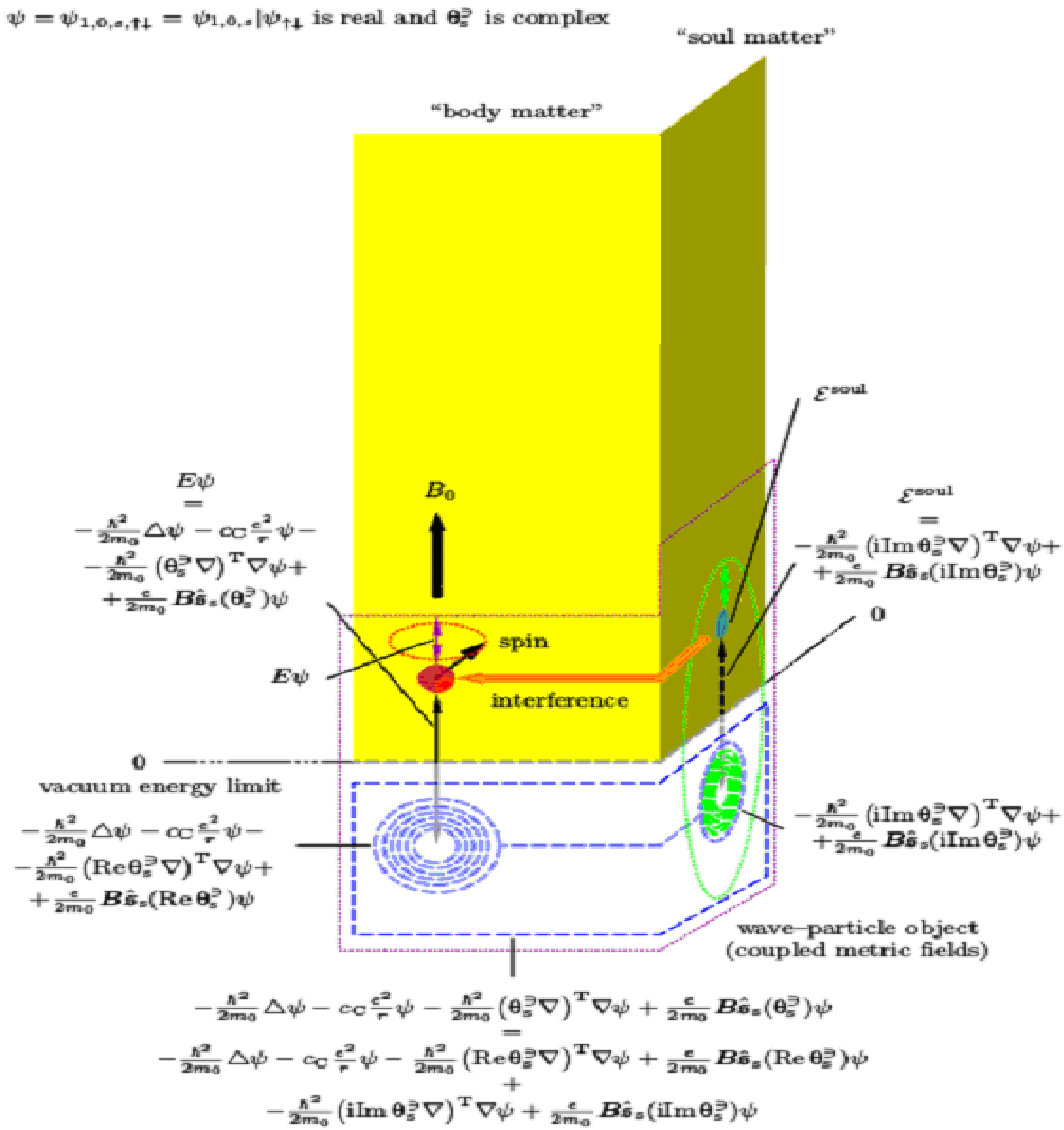


Figure 6 : Spin in a magnetic field. Body and soul, Schrödinger picture.

tum balance equation (F), we may think of a planet surrounded by a gravitational field, where the gravitational field is described by a real metric tensor $g_{\mu\nu}$, and where the mass is described by a real energy momentum tensor $\mathcal{T}_{\mu\nu}$. In the second case, in total, described by the wave–particle energy momentum balance equation (G), we may think of the s orbital of the hydrogen atom, where the s orbital is described by a real metric tensor $g_{\mu\nu}$, after application of the Ansatz $g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}$, to be replaced by the real deviation tensor $\gamma_{\mu\nu}$ including the real wave function $\gamma_{00} = \psi$ of the s orbital, and where the mass–charge properties occur mixed with the wave properties/metric field properties, eventually calling for a real wave–particle energy momentum tensor $\mathcal{T}_{\mu\nu}(g_{\mu\nu})$ and thus $\mathcal{T}_{\mu\nu}(\gamma_{\mu\nu})$ exhibiting the $g_{\mu\nu}$ and thus the $\gamma_{\mu\nu}$ as intrinsic parts. As discussed in[1], (F) directly leads to the Poissonian equation for masses (H). As discussed in[2-4], (G) directly leads to the time-independent Schrödinger equation (I), following Example 3.2, focusing onto the energy contribution of (G) exclusively observing $\chi_{00}\gamma_{00}$ with $\chi_{00} = Em_0/\hbar^2$ and $\gamma_{00} = \psi$.

$$\boldsymbol{G}_{\mu\nu}(\boldsymbol{g}_{\mu\nu}) = \mathcal{K}\mathcal{T}_{\mu\nu}, \quad \text{(F)}$$

$$\boldsymbol{G}_{\mu\nu}(\boldsymbol{g}_{\mu\nu}) = \mathcal{K}\mathcal{T}_{\mu\nu}(\boldsymbol{g}_{\mu\nu}), \quad \text{(G)}$$

$$-\Delta\boldsymbol{\phi}_g/2 = 2\pi G\boldsymbol{\rho}_g, \quad \text{(H)}$$

$$\hat{\boldsymbol{p}}^2\psi/2\boldsymbol{m}_0 + V\boldsymbol{\psi} = \boldsymbol{E}\psi. \quad \text{(I)}$$

Establishing metric fields $g_{\mu\nu}$ showing a real part and an imaginary part meeting the Hilbert space and nonlinear generalizations $\mathbb{W}$ of the Hilbert space required by wave–particle theory, we here always consider two planes each, one hosting real parts of a metric field and related equations and one hosting imaginary parts of a metric field and related equations, in the latter case, including dependencies on real parts of a metric field. In each case, the vertical axis indicates energy levels measured relatively to the vacuum energy level which is set equal zero. In each case, the horizontal axis is not taken as dependent on any parameters. Figures 5 and 6 show the example of a spin in a magnetic field, again considering thes orbital of the hydrogen atom. While Figure 5 illustrates the situation resorting to the Einstein picture, Figure 6 illustrates the situation resorting to the Schrödinger picture. Consistent with Figure 4, we consider the real wave function $\gamma_{00} = \psi$ of the s orbital, but deviating from Figure 4, we additionally consider the spin-related matrix θ_s, in the special case of a spin in a magnetic field, showing a complex structure given by a real part and an imaginary part.

As it is indicated in Figures 5 and 6, the real parts as well as the imaginary parts of the spin-related matrix θ_s are needed within the wave–particle energy balance equation. As it is also indicated in Figures 5 and 6, this can be seen as a coupling of metric field contributions located in the real domain, as it is further explained in a following segment, termed "body matter", and metric field contributions located in the imaginary domain, as it is further explained in a following segment, termed "soul matter". As it is also indicated in Figures 5 and 6, we then may combine the imaginary parts of the spin-related matrix θ_s located in the imaginary domain with an energy contribution to an energy momentum tensor, on the Einstein stage, formally denoted by $\mathcal{T}_{00}$, and on the Schrödinger stage formally denoted by ε, in both cases, showing the upper decoration "soul", contrasting the corresponding terms of the real domain, showing the upper decoration "body".

Resorting to this graphic image, $\gamma_{00} = \psi$ and θ_s are contributions to a special type of metric, here denoted as "wave–particle metric", not only comprising real parts, but also imaginary parts, together with mass–charge properties, forming an inseparable unity, meeting the observable quality of microscopic entities.

The extension to further wave–particle aspects is straightforward.

We here firstly note that the complex structure of wave–particle metrics is a direct structural consequence of wave–particle-related energy momentum tensors implementing wave–particle characteristics. We here secondly note that body-related energy momentum tensors usually cannot be assumed as independent of soul-related energy momentum tensors, in Figures 5 and 6 indicated adding the complementary hint "interference". We here thirdly note that the structural transition from Poisson-type equations dealing with macroscopic entities such as the generalized Poissonian equation taken as the basis in Figure 7 of[1] to Poisson-type equations dealing with microscopic entities such as the generalized Poissonian equation taken as the basis in Figure 8 of[1] is initiated by combinations of mass–charge densities $\rho_{g,C}$ and mass–charge potentials $\phi_{g,C}$ directly reflecting the idea of wave–particle energy momentum tensors. We here fourthly note that the structural transition from such extended types of classical field equations to Schrödinger-type equations eventually is evoked implementing conditions leading from spacious masses to point masses, as it is also worked out by example in[2].

Certainly, it is comfortable to assume that the complex structure of wave–particle metrics is pure mathematics. However, since it seems not to be possible to replace the complex formalisms by real formalisms describing quantum systems, it seems indispensable to me to consider the complex structure of wave–particle metrics as actual physics. Consistently, I suppose that the notion of soul-related energy momentum tensors supplementing body-related energy momentum tensors is actual physics, in the first case, putting "body matter" pointing at observable matter such as masses and charges into concrete terms, and in the second case, putting "soul matter" pointing at not observable matter into concrete terms, noted by the way, meeting the philosophical idea of "body and soul", in last consequence, explaining the decorations "body and soul".

SUMMARY

The simple ideas "charge-corrected mass", "mass–charge exchange relation", and "tension (voltage)" *U*, completed by the notion "metric field energy", define basic elements of a proposal aiming at a unified treatment of masses and charges. Supplementing these ideas by the idea of a wave–particle energy momentum tensor putting the observable quality of microscopic entities into concrete terms, we are led to an extended proposal aiming at the unified treatment of macroscopic masses and charges and microscopic masses and charges based upon a generalized form of Einstein's field equations of gravitation, following[1-4], denoted as the generalized Einstein field equations. As it is here shown by example, in agreement with classical physics, cosmic physics, and quantum physics, the generalized Einstein field equations lead to energy operators/terms known from quantum physics and to energy operators/terms extending the energy operators/terms known from quantum physics. Certainly, in their basic version, these quantities reflect the comprehensive, superior stage of the generalized Einstein field equations. However, specifying these quantities further, these meet the structure known from quantum physics, and this includes parameters such as $\hbar$. Moreover, these quantities can be visualized geometrically, and this includes terms known from quantum physics and terms not known from quantum physics. Going beyond that, these quantities can be seen in the light of the notion "metric field", and this includes wave functions $\gamma_{00} = \psi$ and spin-related matrices θ_s in real specification, imaginary specification, and complex specification. Certainly, the ideas needed to achieve this level of unification are quite unusual. However, these do not contradict calculations, computations, and experiments. Only some changes within the scheme of interpretations of nowadays are needed.

However, can we really trust in such a metric field approach comprising linear and nonlinear issues including real, imaginary, and complex specifications, in all cases, related to physical circumstances?

Of course, wanting to establish trustable relations, further applications are needed!

A selection of first applications I have collected in[2] to be published in near future. The applications presented there reach from the hydrogen atom via superconductivity[7-9] to the synergetics[10,11] of electron–positron annihilation and also include further blueprints of technological applications reaching from energy production via photon drives to the generation of chemical elements. As already outlined in[1], the work[2] is still under construction. As already outlined in[1], a "developer version" of the work[2] can be requested from the author. However, already in advance, a summary of important consequences for energy production techniques, space technologies, and material technologies[12] is presented in a subsequent publication.

Certainly, due to the restricted space, I here could not prequote answers to all questions I am expecting. Therefore, let me here finish this work outlining that a wealth of questions I am expecting has already been answered in[2]. For instance, I there work out the relation to quantum geodetics defined by the equation of geodetic lines observing Heisenberg's uncertainty principle. For instance, I there work out the meaning, the applicability, and the limits of normalization procedures including the probability density normalization and the metric field normalization observing linear structures, nonlinear structures and experimental results.

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REFERENCES

[1] V.A.Weberruss; On the Idea of Superunified Fields, Journal of Space Exploration, **(2014/2015)**.

[2] V.A.Weberruss; Unified Field Theory and Wave–Particle Theory, to be published.

[3] V.A.Weberruss; Nichtlineare Quantenphysik, Shaker-Verlag, **(2008)**.

[4] V.A.Weberruss; The Metric Field Gateway to Quantum Physics, AVM, **(2012)**.

[5] G. Mahler, V.A.Weberruss; Quantum Networks, Springer, **(1998)**.

[6] M.Mehring, V.A.Weberruss; Object-Oriented Magnetic Resonance, Academic Press, **(2001)**.

[7] F. London, H.London; Z.Phys., **96**, 359-364 **(1935)**.

[8] V.L.Ginzburg, L.D.Landau; Zh.Eksp.Teor.Fiz., **20**, 1064 **(1950)**.

[9] J.Bardeen, L.N.Cooper, J.R.Schrieffer; Phys.Rev., **108(5)**, 1175-1205 **(1957)**.

[10] H.Haken; Advanced Synergetics, Springer, **(1983)**.

[11] J.Main, G.Wunner; Physical Review E, **57**, 6 **(1998)**.

[12] F.Aldinger, V.A.Weberruss; Advanced Ceramics and Future Materials, Wiley-VCH, **(2010)**.