ISSN : 0974 - 7435 Volume 10 Issue 24

2014

BioTechnology

An Indian Journal

FULL PAPER

BTAIJ, 10(24), 2014 [16360-16371]

Nonlinear robust control of a wastewater treatment

Qiao Xinghong[1,2*], Luo Fei[1], Xu Yuge[1], Lin Jinmei[1]

[1]College of Automation Science and engineering south China university of technology, Guangzhou (CHINA)

[2]Guangzhou Institute of Energy Testing, Guangzhou (CHINA)

Email : qiaoxinghong@126.com

ABSTRACT

In this paper, a robust feedback control improving the performance of a wastewater treatment is presented. In operational terms, the main control objective is to keep the concentration of the biomass proportional to the influent flow rate with the various disturbance compensation, noisy measurements and varying process model parameters. The H-inf control design is based on a simplified Activated Sludge Model No. 1 (ASM1) with the description of the nonlinearities by bounded structured uncertainty. It is found that the proposed controller yields excellent responses in the face of parameter uncertainties, load disturbances and set-point changes.

KEYWORDS

Activated sludge model; Robust control; Structured uncertainty; Linear fractional transformation (LFT).

INTRODUCTION

Water pollution is the top issue facing the world, the pollution control subjects have been listed on the World Environmental Protection Organization's work schedules. In order to improve the performance of the wastewater treatment system, robust control design for uncertain nonlinear systems is becoming inevitable especially. The most popular wastewater treatment, however, is biological[1]. In particular, the technology of activated sludge Process (ASP) can offer a very good result for phosphorus, organic load and nitrogen removal in wastewater. The actual wastewater treatment process has complex dynamical processes. Often activated sludge Process may have various parameters or the structure of the model which are not perfectly known due to measurement inaccuracy, process noise, structural uncertainty and mode changes[2,3].

To tackle these different uncertainty reasons, several control system algorithms have been developed which can provide an acceptable performances and system stability: PI/PID robust tuning algorithms[4,5], the linear quadratic optimal control[6], adaptive approach[7-9] and knowledge-based expert system (KBES)[10-11]. A lot of research has been reported about applications of various advanced robust control techniques for chemical processes, but very few operations are presented for wastewater treatment system.

The paper is organized as follows. In the next section, the dynamical model of the activated sludge process is presented. In Section 3, it is shown how the uncertainties can be modeled and how this leads to the required $F_U(M,\Delta)$. In Section 4, the $H\infty$ analysis design by solving the pair of Riccati equations is introduced. In Section 5, the performance of both control algorithms are illustrated by extensive numerical simulations performed for a recycled aerobic wastewater treatment bioprocess. Finally, Section 6 concludes the paper.

PROCESS MODELLING

The wastewater treatment process with activated sludge is a biological process for biodegradable pollutants removal, see Figure 1. The basic set-up consists of an aeration tank where the specific micro-organism's population (the sludge) are produced aiming to remove the pollution from wastewater and a settler tank where the clear effluent can be separated from the sludge. Since the amount of microorganisms needs to be kept at a high level, sludge is recirculated as shown in Figure 1, while the rest is removed as waste sludge.

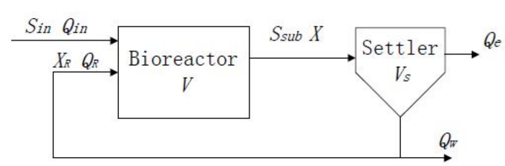

Figure 1 : Schematic diagram of an activated sludge process

Over the years there are various kinds of models developed and a lot of efforts have been spent to get the wastewater treatment processes by active sludge. The most widely used model is Activated Sludge Model No. 1 (ASM1). The ASM1 contains 13 state variables, such that various control strategies are unusable. The robust control strategy is developed based on a reduced ASM1 model in this work. The mass balance equations for the substrate, biomass and recycled biomass concentrations in state space form, are given by[12].

$$\dot{X}(t)=\left(\mu(S_{sub})-\frac{Q_{in}(t)+Q_R(t)}{V}\right)X(t)+\frac{Q_R(t)}{V}X_R(t) \qquad \textbf{(1)}$$

$$\dot{S}_{sub}(t) = -\frac{1}{Y}\mu(S_{sub})X(t) + \frac{Q_{in}(t)}{V}S_{in} - \frac{Q_{in}(t)+Q_R(t)}{V}S_{sub}(t) \quad (2)$$

$$\dot{X}(t) = \left(\frac{Q_{in}(t)+Q_R(t)}{V_S}\right)X(t) - \left(\frac{Q_W(t)+Q_R(t)}{V_S}\right)X_R(t) \quad (3)$$

Where: X, $X_R(t)$ and S_{sub} are the concentrations of the active microbial biomass, the biomass in the recycle stream and the substrate. $Q_{in}(t)$, $Q_R(t)$ and $Q_W(t)$ is influent flow rate, recycle flow rate (manipulated variable) and the waste flow rate. V is the bioreactor volume, V_S is the settler volume. $\mu(\cdot)$ is the specific growth rate, which is modeled by a Monod-type equation.

$$\mu(S(t)) = \frac{\mu_m(t)S(t)}{K_m(t)+S(t)} \quad (4)$$

Where: μ_m and K_m are the variables representing the specific maximum growth rate and the half-saturation constant.

$X_R(t)$ depends on the quality of settler type used and its characteristics. As the dynamics in settler are much faster than the dynamics in bioreactor, the dynamics in settler are first order. Moreover, assuming that the relationship of input to output solids concentration is unknown but bounded. The settler characteristics can be approximated[13] by.

$$X_R(t) = r(t)X(t) \quad (5)$$

where $r(t) > 1$ for all $r(t) \geq 0$. The controlled and manipulated variables are the biomass in the recycle stream and recycle flow rate. The relationship between the controlled and manipulated variable can be obtained by substituting equation (5) into equation (1),

$$\dot{X}_R(t) = (\mu(S_{sub}) - \frac{Q_{in}(t)}{V})X_R(t) + \frac{X_R(t)}{V}(r(t)-1)Q_R(t) \quad (6)$$

The sensor which measures the recycle biomass concentration is modeled by the first order transfer function,

$$X_m(s) = \frac{k_m}{T_m s+1}X_R(s) + d(s) \quad (7)$$

Where $d(s)$ is a random measurement noise. The recycle biomass concentration in can be inferred base on the CO_2 and O_2 concentration in the activated sludge process.

The influent wastewater flow rate has generally periodic characteristic. The reference concentration of the biomass in the recycle flow to be tracked is proportional to the influent flow rate:[14]

$$X_{ref}(t) = k_{ref}Q_{in}(t) \quad k_{ref} > 0 \quad (8)$$

and assuming that the reference signal is measurable and the output X_R can tracks asymptotically reference signal $X_{ref}(t)$. The main goal is to develop robust controller to keep tracking a desired reference signal with disturbances and measurement noise.

LFT PROCESS MODEL WITH PARAMETER UNCERTIANTY

In order to apply the robust control approach, the process dynamics (equation (6)) is simplified as equation (9) with uncertain parameters,

$$\dot{X}_R(t) = aX_R(t) + bQ_R(t) \quad \textbf{(9)}$$

Where

$$a = (\mu - \frac{Q_{in}}{V}),\ b = \frac{X_R}{V}(r-1)$$

All parameters included in a and b are unknown but bounded. Consequently, that is,

$$a = \bar{a}(1+\rho_a\delta_a),\ -1 \le \delta_a \le 1 \quad \textbf{(10)}$$

$$b = \bar{b}(1+\rho_b\delta_b),\ -1 \le \delta_b \le 1 \quad \textbf{(11)}$$

Where

$$a^{\min} = \frac{\mu_{\mathrm{m}}^{\min} S_{sub}^{\min}}{K_m^{\max} + S_{sub}^{\min}} - \frac{Q_{in}^{\max}}{V} \quad \textbf{(12)}$$

$$a^{\max} = \frac{\mu_{\mathrm{m}}^{\max} S_{sub}^{\max}}{K_m^{\min} + S_{sub}^{\max}} - \frac{Q_{in}^{\min}}{V} \quad \textbf{(13)}$$

$$b^{\min} = \frac{X_R^{\min}}{V}(r^{\min}-1) \quad \textbf{(14)}$$

$$b^{\max} = \frac{X_R^{\max}}{V}(r^{\max}-1) \quad \textbf{(15)}$$

$$\bar{a} = \frac{a^{\min} + a^{\max}}{2} \quad \textbf{(16)}$$

$$\bar{b} = \frac{b^{\min} + b^{\max}}{2} \quad \textbf{(17)}$$

$$\rho_a = \frac{a^{\max}}{\bar{a}} - 1 \quad \textbf{(18)}$$

$$\rho_b = \frac{b^{\max}}{\bar{b}} - 1 \quad \textbf{(19)}$$

the parameter $a = \bar{a}(1+\rho_a\delta_a)$ can be represented via the upper LFT in δ_a

$$a = F_U(M_a, \delta_a) \quad \textbf{(20)}$$

With

$$M_a = \begin{bmatrix} 0 & \bar{a} \\ \rho_a & \bar{a} \end{bmatrix}$$

and the parameter $b = \bar{b}(1+\rho_b\delta_b)$ can be represented via the upper LFT in δ_b,

$$b = F_U(M_b, \delta_b) \tag{21}$$

With

$$M_b = \begin{bmatrix} 0 & \bar{b} \\ \rho_b & \bar{b} \end{bmatrix}$$

All these LFTs block diagrams are depicted as shown in Figure 2.

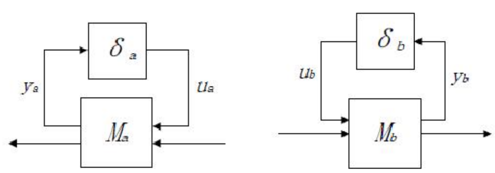

Figure 2 : Diagram representations of $F_U(M_a, \delta_a)$ **and** $F_U(M_b, \delta_b)$

To further represent the active sludge model with uncertainty parameters δ_a and δ_b, the inputs and outputs of δ_a and δ_b as y_a, y_b and u_a, u_b, respectively, are denoted as shown in Figure 3.

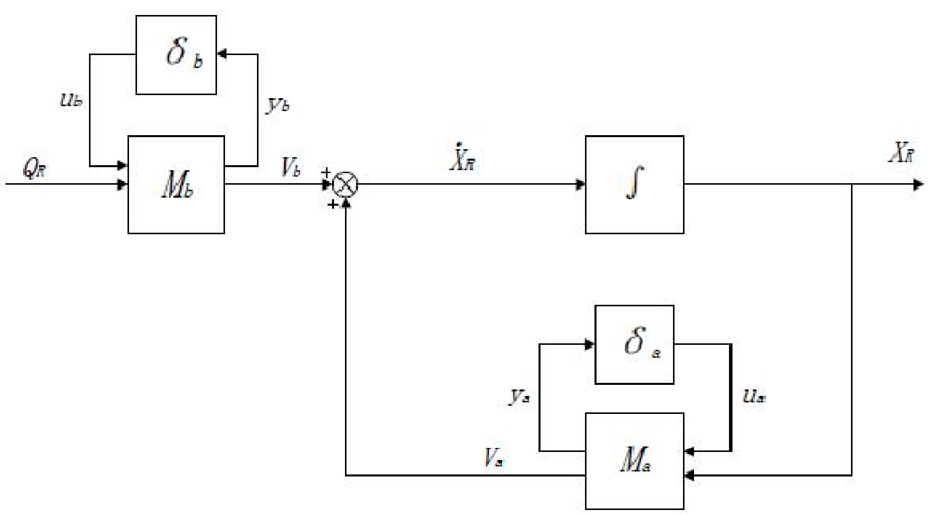

Figure 3 : Diagram of the wastewater treatment system with uncertain parameters

The equation relations between "inputs" and corresponding "outputs" with uncertainty parameters can be obtained with the Figure 3.

$$\begin{bmatrix} y_b \\ V_b \end{bmatrix} = \begin{bmatrix} 0 & \bar{b} \\ \rho_a & \bar{b} \end{bmatrix} \begin{bmatrix} u_b \\ Q_R \end{bmatrix} \quad \textbf{(22)}$$

$$\begin{bmatrix} y_a \\ V_a \end{bmatrix} = \begin{bmatrix} 0 & \bar{a} \\ \rho_a & \bar{a} \end{bmatrix} \begin{bmatrix} u_a \\ X_R \end{bmatrix} \quad \textbf{(23)}$$

As a result, the following equations are obtained

$$V_a = \rho_a u_a + \bar{a} X_R \quad \textbf{(24)}$$

$$V_b = \rho_b u_b + \bar{b} Q_{in} \quad \textbf{(25)}$$

$$y_a = \bar{a} X_R \quad \textbf{(26)}$$

$$y_b = \bar{b} Q_R \quad \textbf{(27)}$$

$$\dot{X}_R = V_b - V_a = \rho_b u_b + \bar{b} Q_{in} - \rho_a u_a - \bar{a} X \quad \textbf{(28)}$$

The state space representation of LFT in matrice form is given by eliminating the variables V_a and V_b, as description in equation (29) and (30).

$$\begin{bmatrix} \dot{X}_R \\ \hline y_a \\ y_b \\ \hline X_R \end{bmatrix} = \left[\begin{array}{c:cc:c} -\bar{a} & -\rho_a & \rho_b & \bar{b} \\ \hdashline \bar{a} & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{b} \\ \hdashline 1 & 0 & 0 & 0 \end{array}\right] \begin{bmatrix} X_R \\ \hline u_a \\ u_b \\ \hline Q_R \end{bmatrix} \quad \textbf{(29)}$$

$$\begin{bmatrix} u_a \\ u_b \end{bmatrix} = \begin{bmatrix} \delta_a & 0 \\ 0 & \sigma_b \end{bmatrix} \begin{bmatrix} y_a \\ y_b \end{bmatrix} \quad \textbf{(30)}$$

Let G_{wts} denote the input/output dynamics of wastewater treatment system, which contains the uncertain parameters in the matrix as shown in Figure 4. G_{wts} has three inputs (u_a; u_b; Q_R), three outputs (y_a; y_b; X_R)

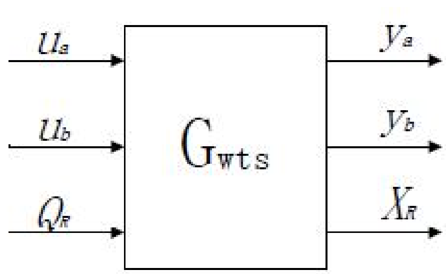

Figure 4 : Block diagram of the t system's input/output

The augmented matrice G_{wts} is

$$G_{wts} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right] \quad \textbf{(31)}$$

Where

$$A = -\bar{a} \,,\; B_1 = [-\rho_a \quad \rho_b] \,,\; B_2 = \bar{b} \,,$$

$$C_1 = \begin{bmatrix} \bar{a} \\ 0 \end{bmatrix}, C_2 = 1 \,, D_{11} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, D_{12} = \begin{bmatrix} 0 \\ \bar{b} \end{bmatrix}, D_{21} = \begin{bmatrix} 0 & 0 \end{bmatrix}, D_{22} = 0$$

The original system with the uncertain parameters can be presented by an upper LFT transformation

$$X_R = F_U(G_{wts}, \Delta) Q_R \quad \textbf{(32)}$$

where △= diag (δ_a;δ_b).), as shown in Figure 5.

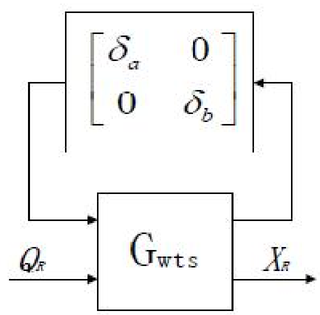

Figure 5 : LFT representation of the wastewater treatment system with uncertainties

H-INF ROBUST CONTROL DESIGN

H-inf robust control design can be described into a standard configuration as in Figure 6. Note that in Figure 6 w, z, y and u are all the exogenous inputs, the output signals, the vector of measurements and the manipulated signals. The control strategy is to minimize the output z by finding a stabilizing controller K. Thus, it is approximately equal to the H∞-norm of the transfer function matrix from w to z.

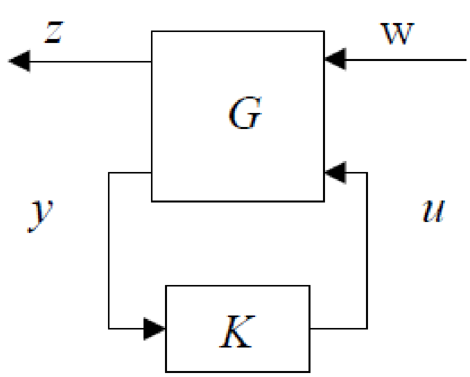

Figure 6 : The standard H-inf configuration

The state-space of generalized system G is described as

$$\dot{x}(t) = Ax(t) + B_1 w(t) + B_2 u(t) \quad \textbf{(33)}$$

$$z(t) = C_1 x(t) + D_{11} w(t) + D_{12} u(t) \quad \textbf{(34)}$$

$y(t) = C_2 x(t) + D_{12} w(t)$ (35) where $x(t) \in R^n$, $w(t) \in R^{m1}$, $u(t) \in R^{m2}$, $z(t) \in R^{p1}$ and $y(t) \in R^{p2}$ are the state vector, the exogenous input vector, the control input vector, the error (output) vector and the measurement vector, with $p_1 \geq m_2$ and $p_2 \leq m_1$. Then the $G(s)$ may be written as.

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right] \quad \textbf{(36)}$$

Supposing that the system $G(s)$ satisfy the following assumptions:

A1 (A, B_2) is stabilisable and (C_2, A) is detectable;

A2 D_{21} has full row rank and D_{12} has full column rank to ensure controller is proper;

A3 for $\forall$ ω, $\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix}$ has full column rank;

A4 for $\forall$ ω, $\begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix}$ has full row rank;

A5 $D_{22} = 0$

Define

$$R = D_{1\bullet}^T D_{1\bullet} - \begin{bmatrix} \gamma^2 I_{m1} & 0 \\ 0 & 0 \end{bmatrix}, \ D_{1\bullet} = \begin{bmatrix} D_{11} & D_{12} \end{bmatrix}$$

$$\tilde{R} = D_{\bullet 1} D_{\bullet 1}^T - \begin{bmatrix} \gamma^2 I_{m2} & 0 \\ 0 & 0 \end{bmatrix}, \ D_{\bullet 1} = \begin{bmatrix} D_{11} \\ D_{21} \end{bmatrix}$$

Assume that R and $\tilde{R}$ are nonsingular. We define two Hamiltonian matrices H and J as

$$H_\infty := \begin{bmatrix} A & 0 \\ -C_1^T C_1 & -A^T \end{bmatrix} - \begin{bmatrix} B_2 \\ -C_1^T D_{1\bullet} \end{bmatrix} R^{-1} \begin{bmatrix} D_{1\bullet}^T C_1 & B_2^T \end{bmatrix}$$

$$J_\infty := \begin{bmatrix} A^T & 0 \\ -B_1 B_1^T & -A \end{bmatrix} - \begin{bmatrix} C_2^T \\ -B_1 D_{\bullet 1}^T \end{bmatrix} \tilde{R}^{-1} \begin{bmatrix} D_{\bullet 1} B_1^T & C_2 \end{bmatrix}$$

Let

X:=Ric(H_∞)

Y:= Ric(J_∞)

Based on the solutions of the X and Y Riccati equations, the main result can be stated with the state feedback and observer gain matrices F and L.

$$F =: \begin{bmatrix} F_{1\infty} \\ F_{2\infty} \end{bmatrix} := -R^{-1}(D_{1\bullet}^{T} C_1 + \mathrm{B}^{T} X)$$

$$L =: \begin{bmatrix} L_{1\infty} & L_{2\infty} \end{bmatrix} := -(B_1 D_{\bullet 1}^{T} + Y C^{T}) \tilde{R}^{-1}$$

Partition D, $F_{1\infty}$, and $L_{1\infty}$ are as follows:

$$\left[\begin{array}{c|c} & F^T \\ \hline L^T & D \end{array}\right] = \left[\begin{array}{c|ccc} & F_{11\infty}^{*} & F_{12\infty}^{*} & F_{2\infty}^{*} \\ \hline L_{11\infty}^{*} & D_{1111} & D_{1112} & 0 \\ L_{12\infty}^{*} & D_{1121} & D_{1122} & I \\ L_{2\infty}^{*} & 0 & I & 0 \end{array}\right]$$

Theorem[15] Suppose G(s) satisfies the assumptions A1- A5.

(a) There exists an admissible controller K(s) such that $\|F_l(G,K)\|_\infty < \gamma$ if and only if

(Ⅰ) $\gamma > \max(\bar{\sigma}\begin{bmatrix} D_{1111} & D_{1112} \end{bmatrix}, \bar{\sigma}\begin{bmatrix} D_{1111}^{T} & D_{1121}^{T} \end{bmatrix})$

(Ⅱ) $H_\infty \in dom(Ric)$, with $X = Ric(H_\infty) \geq 0$;

(Ⅲ) $J_\infty \in dom(Ric)$, with $Y = Ric(J_\infty) \geq 0$;

(Ⅳ) $\rho(XY) < \gamma^2$

(b) Given that the conditions of part (a) are satisfied, then all rational internally stabilizing controllers *K(s)* satisfying $\|F_l(G,K)\|_\infty < \gamma$ are given by

$K(s) = F_l(M, Q)$ $\forall$ $Q(s) \in RH_\infty$, $\|Q(s)\|_\infty < \gamma$, where

$$M(s) = \left[\begin{array}{c|cc} \hat{A} & \hat{B}_1 & \hat{B}_2 \\ \hline \hat{C}_1 & \hat{D}_{11} & \hat{D}_{12} \\ \hat{C}_2 & \hat{D}_{21} & 0 \end{array}\right]$$

And

$$\hat{D}_{11} = -D_{1121} D_{1111}^{T} (\gamma^2 I - D_{1111} D_{1111}^{T})^{-1} D_{1112} - D_{1122}$$

$\hat{D}_{12} \in R^{m_2 \times m_2}$ and $\hat{D}_{21} \in R^{p_2 \times p_2}$ are any matrices satisfying

$$\hat{D}_{12} \hat{D}_{12}^{T} = I - D_{1121}(\gamma^2 I - D_{1111}^{T} D_{1111})^{-1} D_{1121}^{T}$$

$$\hat{D}_{21}^{T} \hat{D}_{21} = I - D_{1112}^{T}(\gamma^2 I - D_{1111} D_{1111}^{T})^{-1} D_{1112}$$

And

$$\hat{B}_2 = Z(B_2 + L_{12}) \hat{D}_{12}$$

$$\hat{C}_2 = -\hat{D}_{21}(C_2 + F_{12})$$

$$\hat{B}_1 = -Z L_2 + \hat{B}_2 \hat{D}_{12}^{-1} \hat{D}_{11}$$

$$= -Z L_2 + Z(B_2 + L_{12}) \hat{D}_{11}$$

$$\hat{C}_1 = F_2 + \hat{D}_{11} \hat{D}_{21}^{-1} \hat{C}_2$$

$$= F_2 - \hat{D}_{11}(C_2 + F_{12})$$

$$\hat{A} = A + BF + \hat{B}_1 \hat{D}_{21}^{-1} \hat{C}_2$$

$$= A + BF - \hat{B}_1 (C_2 + F_{12})$$

Where

$$Z = (I - \gamma^{-2} YX)^{-1}$$

SIMULATION AND DISCUSSION

The block diagram of the closed-loop feedback control system including the uncertainties in model and performance requirements is given in Figure 7. The uncertain parameter Δ satisfies the norm condition $\|\Delta\|_\infty < 1$. The parameter d is the various disturbances. The W_t and W_k are the weighting functions.

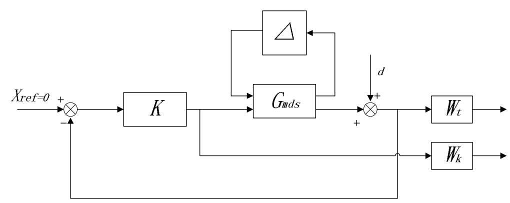

Figure 7 : Closed-loop system structure

The model constant and variable numerical data are given in TABLES I and II[16].

TABLE 1 : Constant numerical data

V	$T_m(h)$	k_m	$S_{in}(mgCOD/l)$	$k_{ref}(mgh/l^2)$
1.5×10^7	1/12	1	300	3.8×10^{-3}

TABLE 2 : Variable numerical data

Data	Nominal values	Min values	Max values
$\mu_m(h^{-1})$	0.2	0.1	0.3
$K_m(mg/l)$	90	60	120
Y	0.6	0.5	0.7
r	4	3	5
$F_{in}(1/h)$	1.9×10^6	$0.75\,F_{in}$	$1.25\,F_{in}$
$X_R(mg/l)$	7070	4000	10000
$S_{sub}(mgCOD/l)$	2	1	4

In choosing design weighting functions W_t and W_k, the system may obtain the acceptable performances and system stability. The tuning parameters of the weights W_t and W_k, are chosen as

$$W_t = \frac{3s+5}{10s+0.005} \quad W_k = 0.01$$

The reference signal is [0.8, 1, 0.8] and the transient responses to the reference input is shown in Figure 8. The disturbance signal is [1, 0, 1] and the transient responses to the disturbance input is shown in Figure 9. As shown in Figure 10, the sensitivity function lies below 1/Wt. The theoretical results verified that good performance is achieved robustly.

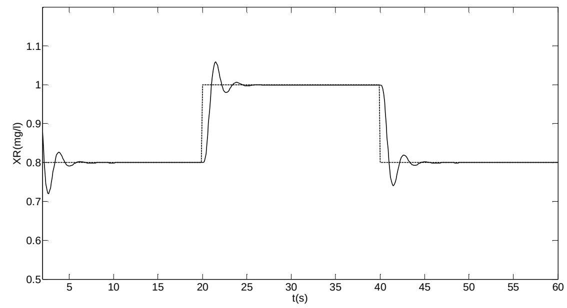

Figure 8 : Transient response to reference input

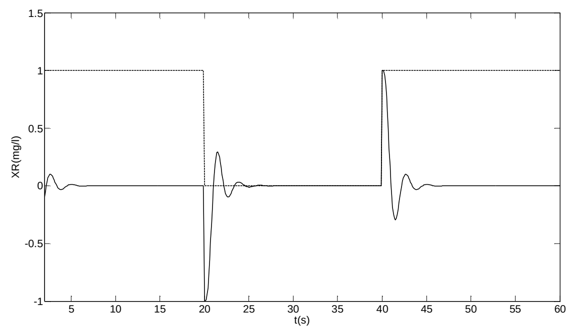

Figure 9 : Transient response to disturbance input

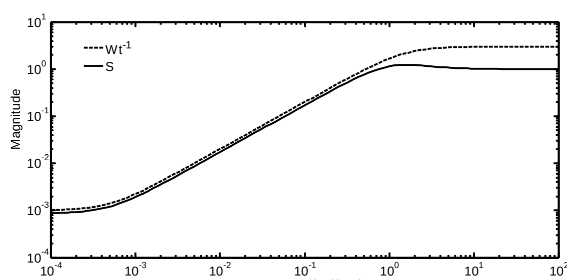

Figure 10 : Diagram of the sensitivity function

CONCLUSIONS

In this article implementation of robust control theory to activated sludge processes are presented. The robust control design is based on a simplified Activated Sludge Model No. 1 (ASM1) together with the description of the nonlinearities by bounded structured uncertainty. The state space model of active sludge processes is obtained in the framework of Linear Fractional Transformation (LFT). In the robust controller design, the weighting function W_t and W_k have been used to improve control performance and limit the controller gain. Simulation results have demonstrated that designed robust controller provides excellent robustness and performance with multiplesources of uncertainties.

ACKNOWLEDGEMENTS

This work is supported by The Natural Science Foundation of Guangdong Province (10151064101000075); the project is the Science &technology star of Zhujiang, Guangzhou city

(2011J2200084); the project on the Integration of Industry and Research of Huizhou city (2011C010002004).

REFERENCES

[1] R.E.Harabi, B.O.Bouamama, M.K.Ben Gayed, M.N.Abdelkrim; Robust fault diagnosis of chemical system by using uncertain bond graph model. Systems, Signals and Devices (SSD), 2011 8th International Multi-Conference on, 22-25, 1- 7 **(March 2011)**.

[2] F.Logist, B.Houska, M.Diehl, J.Van; Impe. Robust multi-objective optimal control of uncertain (bio) chemical processes. Chemical Engineering Science, **66**, 4670-4682, **(2011)**.

[3] Gustaf Olsson; "ICA and me – A subjective review," Water Research, **46(6)**, 1585–1624 **(15 April 2012)**.

[4] R.Vilanova, V.M.Alfaro, O.Arrieta, et al; Analysis of the claimed robustness for PI/PID robust tuning rules[C]//Control & Automation (MED), 2010 18th Mediterranean Conference on. IEEE, 658-662 **(2010)**.

[5] M.Ge, M.S.Chiu, Q.G.Wang; Robust PID controller design via LMI approach[J]. Journal of process control, **12(1)**, 3-13 **(2002)**.

[6] Y.Lucet, H.H.Bauschke, M.Trienis; The piecewise linear-quadratic model for computational convex analysis[J]. Computational Optimization and Applications, **43(1)**, 95-118 **(2009)**.

[7] E.Petre, D.Selisteanu, D.Sendrescu; Adaptive control strategies for a class of anaerobic depollution bioprocesses[C]//Automation, Quality and Testing, Robotics, 2008. AQTR 2008. IEEE International Conference on. IEEE, **2**, 159-164 **(2008)**.

[8] L.Mailleret, O.Bernard, J.P.Steyer; Nonlinear adaptive control for bioreactors with unknown kinetics[J]. Automatica, **40(8)**, 1379-1385 **(2004)**.

[9] K.Vasudeva, U.S.Bhalla; Adaptive stochastic-deterministic chemical kinetic simulations[J]. Bioinformatics, **20(1)**, 78-84 **(2004)**.

[10] M.Dalmau, I.Rodriguez-Roda, E.Ayesa, et al.; Development of a decision tree for the integrated operation of nutrient removal MBRs based on simulation studies and expert knowledge[J]. Chemical Engineering Journal, **(2012)**.

[11] D.Liu, D.Jiang, X.Chen, et al.; Research on fault identification for complex system based on generalized linear canonical correlation analysis[C]//Automation Science and Engineering (CASE), 2012 IEEE International Conference on. IEEE, 474-478 **(2012)**.

[12] P.G.Georgieva, S.Feyo De Azevedo; Robust control design of an activated sludge process, Int. J. Robust Nonlinear Control **9**, 949-967 **(1999)**.

[13] C.Schaper, D.Mellichamp, D.Seborg; "Robust control of a wastewater treatment system," Proc. 29th Conf. on Decision and Control, December, Honolulu, Hawaii, **(1990)**.

[14] P.Georgieva, A.Ilchmann; Adaptive λ-tracking control of activated sludge processes[J]. International Journal of Control, **74(12)**, 1247-1259 **(2001)**.

[15] K.Zhou, J.C.Doyle, K.Glover; Robust and Optimal Control. Prentice Hall, Upper Saddle River, NJ, **(1995)**.

[16] D.W.Sundstrom, H.E.Klei; Wastewater Treatment, Prentice-Hall, Englewood Cliffs, NJ, **(1979)**.