Movement trajectory mathematical simulation of backward sliding shot put under mechanical model

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Abstract

Shot is one of the throwing items in Track and field, and this event has a more significant role on physical enhancement, development of trunk and upper and lower limbs strength. Expectations of coaches and athletes are throwing the shot farther under the premise of not detrimental to health. The throwing distance of shot is related to the releasing speed and releasing position. The expression of speed includes the velocity size and velocity direction, the expression of position includes the coordinates in the horizontal direction and the vertical direction. This paper uses the principle of sports biomechanics, analyzes the whole process of backward sliding shot put and the movement condition after releasing shot, researches the promotion program for shot releasing speed and performance through biomechanical parameters, and establishes the mathematical model to solve the optimal angle of shot throwing. It uses Mathematical software to conduct numerical modeling and simulation on the movement trajectory after releasing shot and to verify the reasonableness of the optimal angle, uses the studying results to validate the existing backward sliding shot put technique and puts forward rationalization proposals. © 2013 Trade Science Inc. - INDIA

Keywords
Sports biomechanics; Differential equations; Mathematical simulation; Shot.

Introduction

The kinematic parameters of backward sliding shot put include releasing condition parameter, reasonable movement trajectory of shot before releasing and sport characteristics of various physical links. The dynamics characteristics of movement include the force characteristics of the shot and the force characteristics of the human body. Use Mathematical software to run numerical modeling on the movement trajectory of shot after releasing, simulate its motion feature, and validate the theoretical rationality of the best throwing angle. For these described motion parameters many people have studied, and with the unremitting efforts of these researchers this game has improved and achieved good results.

In this paper, through biomechanics theory, it conducts detailed mechanical analysis for the movement process of backward sliding shot, studies the releasing speed and performance to promote the shot, establishes a mathematical optimization model of the best shot throwing angle, and provides reasonable suggestions
for the project’s training and theoretical basis for the coaches and athletes.

DYNAMICS ANALYSIS OF SHOT MOVEMENT

Motion analysis before releasing shot

The velocity vector sum that shot obtained during each movement stage determines the releasing speed of shot. Since the backward sliding shot put way leads to the velocity vector direction of shot at sliding stage and final force stage is different, according to the kinetic principle it is known when the speed is changed it is accompanied by a combined force of not zero acting. If the change rate of shot is in linear or coherence, the energy loss rate can be smaller and the transformed shot kinetic energy can increase. Therefore, we should pay particular attention to increase the body’s acting distance on shot and acting efficiency, minimize the initial position height of shot, maintain the projection of shot trajectory in the vertical plane of force form a straight line, which will help improve the velocity vector sum obtained at various shot stages. In the course of movement, if the body can keep the centroid continued and steady rise, then it can reduce the internal energy loss of human body and increase growth-style overlay of the velocity vector, as shown in Figure 2, the movement trajectory of the shot centroid and body centroid trajectories of Di Moer (grades 22.62m) and Kong Beiniu Si (grades 21.22m) during shot put process.

In Figure 1, (1) indicates the movement trajectory of the shot centroid held by Di Moer, (2) indicates the movement trajectory of the shot centroid held by Kong Beiniu Si, (3) represents indicates the movement trajectory of the body centroid of Di Moer, (4) represents indicates the movement trajectory of the body centroid of Kong Beiniu Si.

According to the comparison of (1) and (2) in Figure 2 it shows the shot centroid held by Di Moer has a bigger stationary degree and magnitude than the shot centroid held by Kong Beiniu Si. The same comparison result of (3) and (4) is that Di Moer is better than Kong Beiniu Si in both sides. According to the two persons’ results, Di Moer trajectories contribute to the promotion of its achievement.

The motion analysis of shot after releasing

After releasing the shot only suffers the force of gravity and air resistance. Since during exercise gravity shot receives is far greater than the air resistance, this paper only consider the gravity factor. Assuming that the height when releasing shot is \( h(0) \), during exercise the vertical height of the shot is \( h(t) \), the horizontal displacement when releasing shot is \( s(t_0) = 0 \), the horizontal displacement during exercise is \( s(t) \), the emission angle when releasing shot is \( \alpha \), releasing speed is \( v(0) \), shot’s moving trajectory is shown in Figure 2.

The transmitting speed is resolved in the horizontal direction and vertical direction, use to mean the sub-speed in the horizontal direction during the movement, use to mean the sub-speed in the vertical direction during the movement. Then we have formula (1):

\[
\begin{align*}
    v_x(t) & = v(0) \cos \alpha \\
    v_y(t) & = v(0) \sin \alpha - gt
\end{align*}
\]

According to the relationship between displacement and velocity, the differential expression of \( h(t) \) and is shown in the formula (2):

\[
\begin{align*}
    \frac{dh(t)}{dt} & = v_y(t) = v(0) \sin \alpha - gt \\
    \frac{ds(t)}{dt} & = v_x(t) = v(0) \cos \alpha
\end{align*}
\]

The coordinate expression of the shot’s moving trajectory is shown in formula (3):

\[
(s(t), h(t)) = \left( \int_0^t v_x(t) \, dt, \int_0^t v_y(t) \, dt \right)
\]
By equation (1), (2) and (3) we can solve the horizontal displacement (throwing distance degrees) as shown in formula (4) when: \( h(0) = 0 \)

\[
s = \sqrt{\frac{2h(0)v(0)\cos \alpha - v^2(0)\sin 2\alpha + \frac{v^2(0)\sin 2\alpha}{2g}}{g}}
\]

The formula (4) shows the related factors with shot throwing distance are the releasing angle \( \alpha \), releasing speed \( v(0) \) and releasing height \( h(0) \); when these factors change the throwing distance degree will also change.

**The optimal solution of throwing angle**

When the releasing position and the kinetic energy that shot has are the same, we believe the useful work that players do to the shot at earlier stage are the same. Assuming that in this condition how to select the best releasing angle, the formula (4) shows the function equation of throw distance and the releasing angle. We assume that \( s = f(\alpha) \) and solve \( f(\alpha)_{\text{max}} \). According to the best value principle conduct derivation on \( f(\alpha) \) and obtain the expression of \( \alpha \) when the derivative value is 0, the derivation detail is in formula (5):

\[
f'(\alpha) = \frac{1}{2} \frac{v^2(0)\sin(4\alpha) - 2gh(0)\sin(2\alpha)}{g\sqrt{\frac{8gh(0)\cos^2 \alpha + v^2(0)\sin^2(2\alpha)}{v^2(0) + \sin^2(2\alpha)}}}
\]

When \( f'(\alpha) = 0 \) we have formula (6):

\[
\tan \alpha = \frac{v(0)}{\sqrt{gh(0) + v^2(0)}}
\]

Optimum releasing angle is \( \alpha = \arctan \left( \frac{v(0)}{\sqrt{gh(0) + v^2(0)}} \right) \), because, \( \frac{v(0)}{\sqrt{gh(0) + v^2(0)}} < 1 \) so the releasing angle should be less than \( \frac{\pi}{4} \) (45 degrees); but with the increasing of the \( v(0) \) best releasing angle increases toward 45 degrees, but cannot go beyond that value. Meanwhile the optimal releasing angle is also related with the acceleration of gravity and releasing position.

When the gravitational acceleration decreases the releasing angle should increase a bit correspondingly, which could help improve performance, when the acceleration of gravity increases it decreases conversely; when the gravitational acceleration increases the releasing angle should decrease a bit correspondingly, which could help improve performance, when the acceleration of gravity decreases it increases conversely.

**PEAK VELOCITY DATA ANALYSIS OF VARIOUS LINKS AT THE FINAL STAGE**

Previous studies showed that about 80% of the effects on the shot releasing speed come from the final force, so research on the biomechanical characteristics of human various links at the final stage is necessary. In order to facilitate research, this article divided the final force stage of backward sliding shot put into the actions of three time periods, [T1, T2] represents a transition phase, [T2, T3] represents the right leg kicking phase, [T3, T4] represents the upper limb pushing the ball stage; four times respectively are the moments right foot just landed, the moment left foot just landed, the moment right foot just moved and the moment shot just released.

By studying the four athletes of men’s shot final and test, eight players of women’s shot final, three athletes of the decathlon third shot competition in 2006 National Track and Field Grand Prix in Zhaoqing and the former five athletes of men’s shot final in 2005 the fourth East Asian Games, the peak velocity change table of physical various links during optimal force process is shown in TABLE 1:

Table 1: Table showing peak velocity change of various links during optimal force process.

<table>
<thead>
<tr>
<th>Link</th>
<th>Peak Velocity Change</th>
<th>Optimal Releasing Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hip joint</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shoulder joint</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elbow joint</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wrist joint</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hand</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Seen from TABLE 1, in the final process of force, athletes’ each link speed accumulates and gets the maximum speed when passed to the releasing point, as shown in Figure 3:

Figure 3 shows that at the final stage of the backward sliding shot putting, the peak velocity of each body link from the trend point of view is bottom-up, and finally the speed of the releasing shot reaches the maximum.

For the transfer mode of peak speed in TABLE 1: hip joint -> shoulder joint -> elbow joint -> wrist joint -> hand -> shot. In the transfer process, we need to study the transfer effect of which link is the best. Sup-
pose the peak speed of the \( j \)-th links for the \( i \)-th athlete is, then can form a matrix of in the formula (7) below.

\[
\begin{pmatrix}
0.20 & 0.21 & 0.32 & 0.32 & 0.32 & 0.32 & 0.32 & 0.37 & 0.32 & 0.37 \\
0.26 & 0.26 & 0.26 & 0.26 & 0.26 & 0.26 & 0.26 & 0.32 & 0.32 & 0.32 \\
0.36 & 0.36 & 0.36 & 0.36 & 0.36 & 0.36 & 0.36 & 0.42 & 0.42 & 0.42 \\
0.28 & 0.28 & 0.28 & 0.28 & 0.28 & 0.28 & 0.28 & 0.35 & 0.35 & 0.35 \\
0.23 & 0.23 & 0.23 & 0.23 & 0.23 & 0.23 & 0.23 & 0.32 & 0.32 & 0.32 \\
0.31 & 0.31 & 0.31 & 0.31 & 0.31 & 0.31 & 0.31 & 0.39 & 0.39 & 0.39 \\
0.30 & 0.30 & 0.30 & 0.30 & 0.30 & 0.30 & 0.30 & 0.35 & 0.35 & 0.35 \\
0.32 & 0.32 & 0.32 & 0.32 & 0.32 & 0.32 & 0.32 & 0.39 & 0.39 & 0.39 \\
0.38 & 0.38 & 0.38 & 0.38 & 0.38 & 0.38 & 0.38 & 0.44 & 0.44 & 0.44 \\
0.28 & 0.28 & 0.28 & 0.28 & 0.28 & 0.28 & 0.28 & 0.35 & 0.35 & 0.35 \\
0.28 & 0.28 & 0.28 & 0.28 & 0.28 & 0.28 & 0.28 & 0.35 & 0.35 & 0.35 \\
0.37 & 0.37 & 0.37 & 0.37 & 0.37 & 0.37 & 0.37 & 0.42 & 0.42 & 0.42 \\
\end{pmatrix}
\]

Matrix (7) represents for the speed of 6 links for 11 athletes, the hip joint is the first link, the should joint is the second link, the elbow joint is the third link, the wrist joint is the fourth link, the hand is the five link, shot is the sixth link. Do on the matrix step by step, the first column is all reduced to 0 to obtain link speed and then incrementally increase coefficient matrix, denoted by.

\[
\begin{pmatrix}
-12.91 & 2.18 & 91.84 & 33.15 & 0.00 & 10.49 \\
-2.01 & 9.17 & 10.89 & 10.89 & 10.89 & 10.89 \\
-6.21 & 6.67 & 10.95 & 10.95 & 10.95 & 10.95 \\
-0.21 & 0.32 & 0.32 & 0.32 & 0.32 & 0.32 \\
-0.32 & 0.32 & 0.32 & 0.32 & 0.32 & 0.32 \\
-0.32 & 0.32 & 0.32 & 0.32 & 0.32 & 0.32 \\
\end{pmatrix}
\]

In matrix (8) represents the gradually increase coefficient matrix of peak velocity of all links, the matrix shows that... the forearm; during handheld shot process the swing of the forearm is small is not and is sensitive relative to...
the wrist speed transfer. Between the hip and the shoulder it can not only transfer speed, and shoulder has rotation relative to the hip; in the process of the whole body centroid rising, the hip joint has good promotion on the peak speed of shoulder joint. From the sixth column of the matrix, one can see the merits of athletic performance and the speed increase coefficient of the hand on the ball are correlated to a large extent, but do as. \( \alpha_i (i=0,1,2,\cdots; j=0,1,2,\cdots,9) \). Also in this section is referred to as releasing angles, this combination is also said as, for example \( A_{1j} = (11.00,1.651,0.722) \).

According to the initial value in TABLE 2, the simulated curves of nine combinations can be drawn as shown in Figure 4.

According to the initial value in TABLE 2, the simu-

### TABLE 2: Shot releasing speed

<table>
<thead>
<tr>
<th>Category</th>
<th>Shot releasing speed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>11.00</td>
</tr>
<tr>
<td>Releasing angle unit (rad)</td>
<td></td>
</tr>
<tr>
<td>1.651</td>
<td>0.722</td>
</tr>
<tr>
<td>1.700</td>
<td>0.720</td>
</tr>
<tr>
<td>1.752</td>
<td>0.719</td>
</tr>
<tr>
<td>1.799</td>
<td>0.718</td>
</tr>
<tr>
<td>1.853</td>
<td>0.717</td>
</tr>
<tr>
<td>1.900</td>
<td>0.717</td>
</tr>
</tbody>
</table>

not rule out some individual cases.

### THE SHOT MOVING TRAJECTORY SIMULATION AFTER RELEASING

As shown in TABLE 2, six athletes' height has been set, based on individual height factor and in the case of not changing releasing height carry out the record achievements with different releasing angles and different releasing speed.

The releasing angle in TABLE 2 is set in the form of coordinates, such as the 1st athlete’s releasing height is 1.651m, releasing speed is 11.00m/s, releasing angle is represented as, another example, the 2nd athlete’s releasing height is 1.700m, releasing speed is 11.50m/s, releasing angle is represented as, in turn expressed as \( \alpha_i (i=0,1,2,\cdots; j=0,1,2,\cdots,9) \). Also in this section is referred to as releasing angles, this combination is also said as, for example \( A_{1j} = (11.00,1.651,0.722) \).

According to the initial value in TABLE 2, the simulated curves of nine combinations can be drawn as shown in Figure 4.

According to the initial value in TABLE 2, the simu-

![Figure 4: The shot trajectory simulation of all combinations \( A_{ij} \) after releasing](image1)

![Figure 5: The shot trajectory simulation of all combinations \( A_{ij} \) after releasing](image2)

lated curves of nine combinations \( A_{21}, A_{22}, A_{23}, A_{24}, A_{25}, A_{26}, A_{27}, A_{28}, A_{29} \) can be drawn as shown in Figure 5.

Similarly, the shot trajectory simulation of all combinations \( A_{3j}, A_{4j}, A_{5j}, A_{6j} \) after releasing can be drawn, shown in Figure 6.

By the simulation images in Figure 4 ~ Figure 6, the releasing speed, releasing location and releasing angle have a decisive influence on the shot’s movement trajectory. The greater releasing speed and releasing height is, the better the score becomes; releasing angle controlled in interval \([41.5^\circ, 44^\circ]\) is the optimal shot angle; the greater releasing angle is, the releasing speed increases accordingly. Computer simulation well shows
CONCLUSIONS

The movement trajectory of the body centroid and the shot centroid should remain stable and large amplitude before releasing, helping to improve race performance; The shot releasing speed and the movement’s achievements are in positive linear correlation; The best releasing angle of shot is related to the gravity acceleration, releasing velocity and releasing position; the greater the releasing speed is the closer the releasing angle to 45 degrees; athletes constantly adjust the releasing angle according to their growing strength; when the releasing position changes, the optimum releasing angle will also change, when the releasing height increases the releasing angle should reduce accordingly, when the releasing height decreases vise versa; during the backward gliding shot put process, peak speed of various links of the human body increases in accordance with the order of bottom-up, in which the peak velocity of elbow joint on shoulder joint has the biggest role in promoting; In the complete backward gliding shot put technique, we should also pay attention to the technology of the left part of the body and the right part of the body, and coordination can promote achievements of athletes; to improve the smoothness of the movement trajectory of the body centroid, you need to open two feet wide, which helps to improve the stability angle of the body and makes the body increase the energy consumption; When the legs stand wide open, it increases the kicking efficiency of the right leg, also reduces the rotation inertia during the rotation period of the body, increases acceleration distance of the shot, and promotes the achievement; the computer simulation results well illustrate the relationship between releasing angle, releasing position, releasing speed and the movement trajectory.

REFERENCES